1. Compute the following limit: \( \lim_{x \to 1} \frac{x^2 - 1}{x^3 - 1} \)

2. Compute the following limit: \( \lim_{x \to 1} \frac{x^3 + 1}{x^2 - 1} \)

3. Compute the following limit: \( \lim_{x \to -2} \frac{x^4 - 16}{x^3 + 8} \)

4. Compute the following limit: \( \lim_{x \to 3} \frac{x^2 - 9}{x^4 - 81} \)

5. Compute the following limit: \( \lim_{x \to -3} \frac{x^4 - 81}{x^3 + 27} \)

6. Compute the following limit: \( \lim_{x \to -3} \frac{x^2 - 9}{x^3 + 27} \)

7. Compute the following limit: \( \lim_{x \to 1} \frac{x - 1}{x^2 - 1} \)

8. Compute the following limit: \( \lim_{x \to 1} \frac{x^2 - 1}{x^4 - 1} \)

9. Compute the following limit: \( \lim_{x \to -3} \frac{x^4 - 81}{x^2 - 9} \)

10. Compute the following limit: \( \lim_{x \to -3} \frac{x^3 - 1}{x^4 - 1} \)
Solutions:

1. \( \lim_{x \to 1} \frac{x^2 - 1}{x^3 - 1} \)  
   ▶ Factorize:
   \[
   = \lim_{x \to 1} \frac{(x - 1)(x + 1)}{(x - 1)(x^2 + x + 1)}
   \]  
   ▶ Simplify the common factor:
   \[
   = \lim_{x \to 1} \frac{x + 1}{x^2 + x + 1}
   \]  
   ▶ Use substitution to compute the limit:
   \[
   = \frac{2}{3}
   \]

2. \( \lim_{x \to -1} \frac{x^3 + 1}{x^2 - 1} \)  
   ▶ Factorize:
   \[
   = \lim_{x \to -1} \frac{(x + 1)(x^2 - x + 1)}{(x + 1)(x - 1)}
   \]  
   ▶ Simplify the common factor:
   \[
   = \lim_{x \to -1} \frac{x^2 - x + 1}{x - 1}
   \]  
   ▶ Use substitution to compute the limit:
   \[
   = \frac{(-1)^2 + (-1)(-1) + 1}{-1 - 1}
   \]  
   ▶ Simplify:
   \[
   = -\frac{3}{2}
   \]

3. \( \lim_{x \to -2} \frac{x^4 - 16}{x^3 + 8} \)  
   ▶ Factorize:
   \[
   = \lim_{x \to -2} \frac{(x + 2)(x - 2)(x^2 + 4)}{(x + 2)(x^2 - 2x + 4)}
   \]  
   ▶ Simplify the common factor:
   \[
   = \lim_{x \to -2} \frac{x^2 + 4}{x^2 - 2x + 4}
   \]  
   ▶ Use substitution to compute the limit:
   \[
   = \frac{(-2)^2 + (-2)^2 + 4}{(-2)^2 + (-2)(-2) + 4}
   \]  
   ▶ Simplify:
   \[
   = -\frac{8}{3}
   \]

4. \( \lim_{x \to -3} \frac{x^2 - 9}{x^4 - 81} \)  
   ▶ Factorize:
   \[
   = \lim_{x \to -3} \frac{(x - 3)(x + 3)}{(x - 3)(x + 3)(x^2 + 9)}
   \]  
   ▶ Simplify the common factor:
   \[
   = \lim_{x \to -3} \frac{x + 3}{(x + 3)(x^2 + 9)}
   \]  
   ▶ Use substitution to compute the limit:
   \[
   = \frac{3 + 3}{(3 + 3)((3)^2 + 9)}
   \]  
   ▶ Simplify:
   \[
   = \frac{1}{18}
   \]
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Limit (V)

5. \( \lim_{x \to -3} \frac{x^4 - 81}{x^3 + 27} \)
   - Factorize:
     \[
     = \lim_{x \to -3} \frac{(x + 3)(x - 3)(x^2 + 9)}{(x + 3)(x^2 - 3x + 9)}
     \]
   - Simplify the common factor:
     \[
     = \lim_{x \to -3} \frac{x - 3(x^2 + 9)}{x^2 - 3x + 9}
     \]
   - Use substitution to compute the limit:
     \[
     = \frac{(-3 - 3)((-3)^2 + 9)}{(-3)^2 + (-3)(-3) + 9}
     \]
   - Simplify:
     \[
     = -4
     \]

6. \( \lim_{x \to -3} \frac{x^2 - 9}{x^3 + 27} \)
   - Factorize:
     \[
     = \lim_{x \to -3} \frac{(x + 3)(x - 3)}{(x + 3)(x^2 - 3x + 9)}
     \]
   - Simplify the common factor:
     \[
     = \lim_{x \to -3} \frac{x - 3}{x^2 - 3x + 9}
     \]
   - Use substitution to compute the limit:
     \[
     = \frac{-3 - 3}{(-3)^2 + (-3)(-3) + 9}
     \]
   - Simplify:
     \[
     = -\frac{2}{9}
     \]

7. \( \lim_{x \to 1} \frac{x - 1}{x^2 - 1} \)
   - Factorize:
     \[
     = \lim_{x \to 1} \frac{x - 1}{(x - 1)(x + 1)}
     \]
   - Simplify the common factor:
     \[
     = \lim_{x \to 1} \frac{1}{x + 1}
     \]
   - Use substitution to compute the limit:
     \[
     = \frac{1}{1 + 1}
     \]
   - Simplify:
     \[
     = \frac{1}{2}
     \]

8. \( \lim_{x \to 1} \frac{x^2 - 1}{x^4 - 1} \)
   - Factorize:
     \[
     = \lim_{x \to 1} \frac{(x - 1)(x + 1)}{(x - 1)(x + 1)(x^2 + 1)}
     \]
   - Simplify the common factor:
     \[
     = \lim_{x \to 1} \frac{x + 1}{(x + 1)(x^2 + 1)}
     \]
   - Use substitution to compute the limit:
     \[
     = \frac{1 + 1}{(1 + 1)((1)^2 + 1)}
     \]
   - Simplify:
     \[
     = \frac{1}{2}
     \]

9. \( \lim_{x \to -3} \frac{x^4 - 81}{x^2 - 9} \)
   - Factorize:
     \[
     = \lim_{x \to -3} \frac{(x + 3)(x - 3)(x^2 + 9)}{(x + 3)(x - 3)}
     \]
   - Simplify the common factor:
\[
\lim_{x \to -3} \frac{(x - 3)(x^2 + 9)}{x - 3} \\
= \frac{(-3 - 3)((-3)^2 + 9)}{-3 - 3} \\
= 18
\]

10. \(\lim_{x \to 1} \frac{x^3 - 1}{x^4 - 1}\) ▶ Factorize:
\[
= \lim_{x \to 1} \frac{(x - 1)(x^2 + x + 1)}{(x - 1)(x + 1)(x^2 + 1)} \\
= \lim_{x \to 1} \frac{x^2 + x + 1}{(x + 1)(x^2 + 1)} \\
= \frac{(1)^2 + (1)(1) + 1}{(1 + 1)((1)^2 + 1)} \\
= \frac{3}{4}
\]