1. Compute the following limit: $\lim_{x\to 1} \dfrac{x^2-1}{x^3-1}$

2. Compute the following limit: $\lim_{x\to -1} \dfrac{x^3+1}{x^2-1}$

3. Compute the following limit: $\lim_{x\to -2} \dfrac{x^4-16}{x^3+8}$

4. Compute the following limit: $\lim_{x\to 3} \dfrac{x^2-9}{x^4-81}$

5. Compute the following limit: $\lim_{x\to -3} \dfrac{x^4-81}{x^3+27}$

6. Compute the following limit: $\lim_{x\to -3} \dfrac{x^2-9}{x^3+27}$

7. Compute the following limit: $\lim_{x\to 1} \dfrac{x-1}{x^2-1}$

8. Compute the following limit: $\lim_{x\to 1} \dfrac{x^2-1}{x^4-1}$

9. Compute the following limit: $\lim_{x\to -3} \dfrac{x^4-81}{x^2-9}$

10. Compute the following limit: $\lim_{x\to 1} \dfrac{x^3-1}{x^4-1}$

Answers: 1. $\frac{2}{3}$ 2. $\frac{-3}{2}$ 3. $\frac{-8}{3}$ 4. $\frac{1}{18}$ 5. -4 6. $\frac{-2}{9}$ 7. $\frac{1}{2}$ 8. $\frac{1}{2}$ 9. 18 10. $\frac{3}{4}$

Solutions:

1. $\lim_{x\to 1} \dfrac{x^2-1}{x^3-1}$ ▶ Factorize:

$= \lim_{x\to 1} \dfrac{(x-1)(x+1)}{(x-1)(x^2+x+1)}$ ▶ Simplify the common factor:

$= \lim_{x\to 1} \dfrac{x+1}{x^2+x+1}$ ▶ Use substitution to compute the limit:

$= \dfrac{1+1}{(1)^2+(1)(1)+1}$ ▶ Simplify:

$= \dfrac{2}{3}$

2. $\lim_{x\to -1} \dfrac{x^3+1}{x^2-1}$ ▶ Factorize:

$= \lim_{x\to -1} \dfrac{(x+1)(x^2-x+1)}{(x+1)(x-1)}$ ▶ Simplify the common factor:

$= \lim_{x\to -1} \dfrac{x^2-x+1}{x-1}$ ▶ Use substitution to compute the limit:

$= \dfrac{(-1)^2+(-1)(-1)+1}{-1-1}$ ▶ Simplify:

$= \dfrac{-3}{2}$

3. $\lim_{x\to -2} \dfrac{x^4-16}{x^3+8}$ ▶ Factorize:

$= \lim_{x\to -2} \dfrac{(x+2)(x-2)(x^2+4)}{(x+2)(x^2-2x+4)}$ ▶ Simplify the common factor:

$= \lim_{x\to -2} \dfrac{(x-2)(x^2+4)}{x^2-2x+4}$ ▶ Use substitution to compute the limit:

$= \dfrac{(-2-2)((-2)^2+4)}{(-2)^2+(-2)(-2)+4}$ ▶ Simplify:

$= \dfrac{-8}{3}$

4. $\lim_{x\to 3} \dfrac{x^2-9}{x^4-81}$ ▶ Factorize:

$= \lim_{x\to 3} \dfrac{(x-3)(x+3)}{(x-3)(x+3)(x^2+9)}$ ▶ Simplify the common factor:

$= \lim_{x\to 3} \dfrac{x+3}{(x+3)(x^2+9)}$ ▶ Use substitution to compute the limit:

$= \dfrac{3+3}{(3+3)((3)^2+9)}$ ▶ Simplify:

$= \dfrac{1}{18}$

5. $\lim_{x\to -3} \dfrac{x^4-81}{x^3+27}$ ▶ Factorize:

$= \lim_{x\to -3} \dfrac{(x+3)(x-3)(x^2+9)}{(x+3)(x^2-3x+9)}$ ▶ Simplify the common factor:

$= \lim_{x\to -3} \dfrac{(x-3)(x^2+9)}{x^2-3x+9}$ ▶ Use substitution to compute the limit:

$= \dfrac{(-3-3)((-3)^2+9)}{(-3)^2+(-3)(-3)+9}$ ▶ Simplify:

$= -4$

6. $\lim_{x\to -3} \dfrac{x^2-9}{x^3+27}$ ▶ Factorize:

$= \lim_{x\to -3} \dfrac{(x+3)(x-3)}{(x+3)(x^2-3x+9)}$ ▶ Simplify the common factor:

$= \lim_{x\to -3} \dfrac{x-3}{x^2-3x+9}$ ▶ Use substitution to compute the limit:

$= \dfrac{-3-3}{(-3)^2+(-3)(-3)+9}$ ▶ Simplify:

$= \dfrac{-2}{9}$

7. $\lim_{x\to 1} \dfrac{x-1}{x^2-1}$ ▶ Factorize:

$= \lim_{x\to 1} \dfrac{x-1}{(x-1)(x+1)}$ ▶ Simplify the common factor:

$= \lim_{x\to 1} \dfrac{1}{x+1}$ ▶ Use substitution to compute the limit:

$= \dfrac{1}{1+1}$ ▶ Simplify:

$= \dfrac{1}{2}$

8. $\lim_{x\to 1} \dfrac{x^2-1}{x^4-1}$ ▶ Factorize:

$= \lim_{x\to 1} \dfrac{(x-1)(x+1)}{(x-1)(x+1)(x^2+1)}$ ▶ Simplify the common factor:

$= \lim_{x\to 1} \dfrac{x+1}{(x+1)(x^2+1)}$ ▶ Use substitution to compute the limit:

$= \dfrac{1+1}{(1+1)((1)^2+1)}$ ▶ Simplify:

$= \dfrac{1}{2}$

9. $\lim_{x\to -3} \dfrac{x^4-81}{x^2-9}$ ▶ Factorize:

$= \lim_{x\to -3} \dfrac{(x+3)(x-3)(x^2+9)}{(x+3)(x-3)}$ ▶ Simplify the common factor:

$= \lim_{x \to -3} \dfrac{(x-3)(x^2+9)}{x-3}$ ▶ Use substitution to compute the limit:

$= \dfrac{(-3-3)((-3)^2+9)}{-3-3}$ ▶ Simplify:

$= 18$

10. $\lim_{x \to 1} \dfrac{x^3-1}{x^4-1}$ ▶ Factorize:

$= \lim_{x \to 1} \dfrac{(x-1)(x^2+x+1)}{(x-1)(x+1)(x^2+1)}$ ▶ Simplify the common factor:

$= \lim_{x \to 1} \dfrac{x^2+x+1}{(x+1)(x^2+1)}$ ▶ Use substitution to compute the limit:

$= \dfrac{(1)^2+(1)(1)+1}{(1+1)((1)^2+1)}$ ▶ Simplify:

$= \dfrac{3}{4}$

