

1. Compute the following limit: $\lim_{x \rightarrow 4} \frac{\sqrt{x-1} - \sqrt{3}}{x-4}$

2. Compute the following limit: $\lim_{x \rightarrow -2} \frac{\sqrt{x+9} - \sqrt{7}}{x+2}$

3. Compute the following limit: $\lim_{x \rightarrow -2} \frac{\sqrt{x+7} - \sqrt{5}}{x+2}$

4. Compute the following limit: $\lim_{x \rightarrow 4} \frac{\sqrt{x-3} - \sqrt{1}}{x-4}$

5. Compute the following limit: $\lim_{x \rightarrow -5} \frac{\sqrt{x+7} - \sqrt{2}}{x+5}$

6. Compute the following limit: $\lim_{x \rightarrow 4} \frac{\sqrt{x+7} - \sqrt{11}}{x-4}$

7. Compute the following limit: $\lim_{x \rightarrow -3} \frac{\sqrt{x+13} - \sqrt{10}}{x+3}$

8. Compute the following limit: $\lim_{x \rightarrow -3} \frac{\sqrt{x+8} - \sqrt{5}}{x+3}$

9. Compute the following limit: $\lim_{x \rightarrow 0} \frac{\sqrt{x+15} - \sqrt{15}}{x-0}$

10. Compute the following limit: $\lim_{x \rightarrow -1} \frac{\sqrt{x+9} - \sqrt{8}}{x+1}$

Answers:
 1. $\frac{2\sqrt{3}}{1}$
 2. $\frac{2\sqrt{7}}{1}$
 3. $\frac{2\sqrt{5}}{1}$
 4. $\frac{2\sqrt{1}}{1}$
 5. $\frac{2\sqrt{2}}{1}$
 6. $\frac{2\sqrt{11}}{1}$
 7. $\frac{2\sqrt{10}}{1}$
 8. $\frac{2\sqrt{5}}{1}$
 9. $\frac{2\sqrt{15}}{1}$
 10. $\frac{2\sqrt{8}}{1}$

Solutions:

$$\begin{aligned}
 1. \quad & \lim_{x \rightarrow 4} \frac{\sqrt{x-1} - \sqrt{3}}{x-4} \quad \blacktriangleright \text{Multiply by the conjugate radical:} \\
 &= \lim_{x \rightarrow 4} \frac{\sqrt{x-1} - \sqrt{3}}{x-4} \times \frac{\sqrt{x-1} + \sqrt{3}}{\sqrt{x-1} + \sqrt{3}} \quad \blacktriangleright \text{Use: } (a-b)(a+b) = a^2 - b^2 \quad \text{and} \quad (\sqrt{a})^2 = a \\
 &= \lim_{x \rightarrow 4} \frac{(x-1) - 3}{(x-4)(\sqrt{x-1} + \sqrt{3})} \quad \blacktriangleright \text{Remove brackets and simplify:} \\
 &= \lim_{x \rightarrow 4} \frac{x-4}{(x-4)(\sqrt{x-1} + \sqrt{3})} \quad \blacktriangleright \text{Simplify the common factor:} \\
 &= \lim_{x \rightarrow 4} \frac{1}{\sqrt{x-1} + \sqrt{3}} \quad \blacktriangleright \text{Use substitution to compute the limit:} \\
 &= \frac{1}{\sqrt{4-1} + \sqrt{3}} \quad \blacktriangleright \text{Simplify:} \\
 &= \frac{1}{2\sqrt{3}}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \lim_{x \rightarrow -2} \frac{\sqrt{x+9} - \sqrt{7}}{x+2} \quad \blacktriangleright \text{Multiply by the conjugate radical:} \\
 &= \lim_{x \rightarrow -2} \frac{\sqrt{x+9} - \sqrt{7}}{x+2} \times \frac{\sqrt{x+9} + \sqrt{7}}{\sqrt{x+9} + \sqrt{7}} \quad \blacktriangleright \text{Use: } (a-b)(a+b) = a^2 - b^2 \quad \text{and} \quad (\sqrt{a})^2 = a \\
 &= \lim_{x \rightarrow -2} \frac{(x+9) - 7}{(x+2)(\sqrt{x+9} + \sqrt{7})} \quad \blacktriangleright \text{Remove brackets and simplify:} \\
 &= \lim_{x \rightarrow -2} \frac{x+2}{(x+2)(\sqrt{x+9} + \sqrt{7})} \quad \blacktriangleright \text{Simplify the common factor:} \\
 &= \lim_{x \rightarrow -2} \frac{1}{\sqrt{x+9} + \sqrt{7}} \quad \blacktriangleright \text{Use substitution to compute the limit:} \\
 &= \frac{1}{\sqrt{-2+9} + \sqrt{7}} \quad \blacktriangleright \text{Simplify:} \\
 &= \frac{1}{2\sqrt{7}}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & \lim_{x \rightarrow -2} \frac{\sqrt{x+7} - \sqrt{5}}{x+2} \quad \blacktriangleright \text{Multiply by the conjugate radical:} \\
 &= \lim_{x \rightarrow -2} \frac{\sqrt{x+7} - \sqrt{5}}{x+2} \times \frac{\sqrt{x+7} + \sqrt{5}}{\sqrt{x+7} + \sqrt{5}} \quad \blacktriangleright \text{Use: } (a-b)(a+b) = a^2 - b^2 \quad \text{and} \quad (\sqrt{a})^2 = a \\
 &= \lim_{x \rightarrow -2} \frac{(x+7) - 5}{(x+2)(\sqrt{x+7} + \sqrt{5})} \quad \blacktriangleright \text{Remove brackets and simplify:} \\
 &= \lim_{x \rightarrow -2} \frac{x+2}{(x+2)(\sqrt{x+7} + \sqrt{5})} \quad \blacktriangleright \text{Simplify the common factor:} \\
 &= \lim_{x \rightarrow -2} \frac{1}{\sqrt{x+7} + \sqrt{5}} \quad \blacktriangleright \text{Use substitution to compute the limit:} \\
 &= \frac{1}{\sqrt{-2+7} + \sqrt{5}} \quad \blacktriangleright \text{Simplify:}
 \end{aligned}$$

$$= \frac{1}{2\sqrt{5}}$$

$$\begin{aligned}
 4. \quad & \lim_{x \rightarrow 4} \frac{\sqrt{x-3} - \sqrt{1}}{x-4} \quad \blacktriangleright \text{Multiply by the conjugate radical:} \\
 &= \lim_{x \rightarrow 4} \frac{\sqrt{x-3} - \sqrt{1}}{x-4} \times \frac{\sqrt{x-3} + \sqrt{1}}{\sqrt{x-3} + \sqrt{1}} \quad \blacktriangleright \text{Use: } (a-b)(a+b) = a^2 - b^2 \quad \text{and} \quad (\sqrt{a})^2 = a \\
 &= \lim_{x \rightarrow 4} \frac{(x-3) - 1}{(x-4)(\sqrt{x-3} + \sqrt{1})} \quad \blacktriangleright \text{Remove brackets and simplify:} \\
 &= \lim_{x \rightarrow 4} \frac{x-4}{(x-4)(\sqrt{x-3} + \sqrt{1})} \quad \blacktriangleright \text{Simplify the common factor:} \\
 &= \lim_{x \rightarrow 4} \frac{1}{\sqrt{x-3} + \sqrt{1}} \quad \blacktriangleright \text{Use substitution to compute the limit:} \\
 &= \frac{1}{\sqrt{4-3} + \sqrt{1}} \quad \blacktriangleright \text{Simplify:} \\
 &= \frac{1}{2\sqrt{1}}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad & \lim_{x \rightarrow -5} \frac{\sqrt{x+7} - \sqrt{2}}{x+5} \quad \blacktriangleright \text{Multiply by the conjugate radical:} \\
 &= \lim_{x \rightarrow -5} \frac{\sqrt{x+7} - \sqrt{2}}{x+5} \times \frac{\sqrt{x+7} + \sqrt{2}}{\sqrt{x+7} + \sqrt{2}} \quad \blacktriangleright \text{Use: } (a-b)(a+b) = a^2 - b^2 \quad \text{and} \quad (\sqrt{a})^2 = a \\
 &= \lim_{x \rightarrow -5} \frac{(x+7) - 2}{(x+5)(\sqrt{x+7} + \sqrt{2})} \quad \blacktriangleright \text{Remove brackets and simplify:} \\
 &= \lim_{x \rightarrow -5} \frac{x+5}{(x+5)(\sqrt{x+7} + \sqrt{2})} \quad \blacktriangleright \text{Simplify the common factor:} \\
 &= \lim_{x \rightarrow -5} \frac{1}{\sqrt{x+7} + \sqrt{2}} \quad \blacktriangleright \text{Use substitution to compute the limit:} \\
 &= \frac{1}{\sqrt{-5+7} + \sqrt{2}} \quad \blacktriangleright \text{Simplify:} \\
 &= \frac{1}{2\sqrt{2}}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & \lim_{x \rightarrow 4} \frac{\sqrt{x+7} - \sqrt{11}}{x-4} \quad \blacktriangleright \text{Multiply by the conjugate radical:} \\
 &= \lim_{x \rightarrow 4} \frac{\sqrt{x+7} - \sqrt{11}}{x-4} \times \frac{\sqrt{x+7} + \sqrt{11}}{\sqrt{x+7} + \sqrt{11}} \quad \blacktriangleright \text{Use: } (a-b)(a+b) = a^2 - b^2 \quad \text{and} \quad (\sqrt{a})^2 = a \\
 &= \lim_{x \rightarrow 4} \frac{(x+7) - 11}{(x-4)(\sqrt{x+7} + \sqrt{11})} \quad \blacktriangleright \text{Remove brackets and simplify:} \\
 &= \lim_{x \rightarrow 4} \frac{x-4}{(x-4)(\sqrt{x+7} + \sqrt{11})} \quad \blacktriangleright \text{Simplify the common factor:} \\
 &= \lim_{x \rightarrow 4} \frac{1}{\sqrt{x+7} + \sqrt{11}} \quad \blacktriangleright \text{Use substitution to compute the limit:} \\
 &= \frac{1}{\sqrt{4+7} + \sqrt{11}} \quad \blacktriangleright \text{Simplify:}
 \end{aligned}$$

$$= \frac{1}{2\sqrt{11}}$$

7. $\lim_{x \rightarrow -3} \frac{\sqrt{x+13} - \sqrt{10}}{x+3}$ ▶ Multiply by the conjugate radical:

$$= \lim_{x \rightarrow -3} \frac{\sqrt{x+13} - \sqrt{10}}{x+3} \times \frac{\sqrt{x+13} + \sqrt{10}}{\sqrt{x+13} + \sqrt{10}} \quad \text{▶ Use: } (a-b)(a+b) = a^2 - b^2 \quad \text{and} \quad (\sqrt{a})^2 = a$$

$$= \lim_{x \rightarrow -3} \frac{(x+13) - 10}{(x+3)(\sqrt{x+13} + \sqrt{10})} \quad \text{▶ Remove brackets and simplify:}$$

$$= \lim_{x \rightarrow -3} \frac{x+3}{(x+3)(\sqrt{x+13} + \sqrt{10})} \quad \text{▶ Simplify the common factor:}$$

$$= \lim_{x \rightarrow -3} \frac{1}{\sqrt{x+13} + \sqrt{10}} \quad \text{▶ Use substitution to compute the limit:}$$

$$= \frac{1}{\sqrt{-3+13} + \sqrt{10}} \quad \text{▶ Simplify:}$$

$$= \frac{1}{2\sqrt{10}}$$

8. $\lim_{x \rightarrow -3} \frac{\sqrt{x+8} - \sqrt{5}}{x+3}$ ▶ Multiply by the conjugate radical:

$$= \lim_{x \rightarrow -3} \frac{\sqrt{x+8} - \sqrt{5}}{x+3} \times \frac{\sqrt{x+8} + \sqrt{5}}{\sqrt{x+8} + \sqrt{5}} \quad \text{▶ Use: } (a-b)(a+b) = a^2 - b^2 \quad \text{and} \quad (\sqrt{a})^2 = a$$

$$= \lim_{x \rightarrow -3} \frac{(x+8) - 5}{(x+3)(\sqrt{x+8} + \sqrt{5})} \quad \text{▶ Remove brackets and simplify:}$$

$$= \lim_{x \rightarrow -3} \frac{x+3}{(x+3)(\sqrt{x+8} + \sqrt{5})} \quad \text{▶ Simplify the common factor:}$$

$$= \lim_{x \rightarrow -3} \frac{1}{\sqrt{x+8} + \sqrt{5}} \quad \text{▶ Use substitution to compute the limit:}$$

$$= \frac{1}{\sqrt{-3+8} + \sqrt{5}} \quad \text{▶ Simplify:}$$

$$= \frac{1}{2\sqrt{5}}$$

9. $\lim_{x \rightarrow 0} \frac{\sqrt{x+15} - \sqrt{15}}{x-0}$ ▶ Multiply by the conjugate radical:

$$= \lim_{x \rightarrow 0} \frac{\sqrt{x+15} - \sqrt{15}}{x-0} \times \frac{\sqrt{x+15} + \sqrt{15}}{\sqrt{x+15} + \sqrt{15}} \quad \text{▶ Use: } (a-b)(a+b) = a^2 - b^2 \quad \text{and} \quad (\sqrt{a})^2 = a$$

$$= \lim_{x \rightarrow 0} \frac{(x+15) - 15}{(x-0)(\sqrt{x+15} + \sqrt{15})} \quad \text{▶ Remove brackets and simplify:}$$

$$= \lim_{x \rightarrow 0} \frac{x-0}{(x-0)(\sqrt{x+15} + \sqrt{15})} \quad \text{▶ Simplify the common factor:}$$

$$= \lim_{x \rightarrow 0} \frac{1}{\sqrt{x+15} + \sqrt{15}} \quad \text{▶ Use substitution to compute the limit:}$$

$$= \frac{1}{\sqrt{0+15} + \sqrt{15}} \quad \text{▶ Simplify:}$$

$$= \frac{1}{2\sqrt{15}}$$

$$10. \lim_{x \rightarrow -1} \frac{\sqrt{x+9} - \sqrt{8}}{x+1} \quad \blacktriangleright \text{Multiply by the conjugate radical:}$$

$$= \lim_{x \rightarrow -1} \frac{\sqrt{x+9} - \sqrt{8}}{x+1} \times \frac{\sqrt{x+9} + \sqrt{8}}{\sqrt{x+9} + \sqrt{8}} \quad \blacktriangleright \text{Use: } (a-b)(a+b) = a^2 - b^2 \quad \text{and} \quad (\sqrt{a})^2 = a$$

$$= \lim_{x \rightarrow -1} \frac{(x+9) - 8}{(x+1)(\sqrt{x+9} + \sqrt{8})} \quad \blacktriangleright \text{Remove brackets and simplify:}$$

$$= \lim_{x \rightarrow -1} \frac{x+1}{(x+1)(\sqrt{x+9} + \sqrt{8})} \quad \blacktriangleright \text{Simplify the common factor:}$$

$$= \lim_{x \rightarrow -1} \frac{1}{\sqrt{x+9} + \sqrt{8}} \quad \blacktriangleright \text{Use substitution to compute the limit:}$$

$$= \frac{1}{\sqrt{-1+9} + \sqrt{8}} \quad \blacktriangleright \text{Simplify:}$$

$$= \frac{1}{2\sqrt{8}}$$