

1. Compute the following limit: $\lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x - 1}$
2. Compute the following limit: $\lim_{x \rightarrow \frac{25}{9}} \frac{\sqrt{x} - \frac{5}{3}}{x - \frac{25}{9}}$
3. Compute the following limit: $\lim_{x \rightarrow \frac{25}{4}} \frac{\sqrt{x} - \frac{5}{2}}{x - \frac{25}{4}}$
4. Compute the following limit: $\lim_{x \rightarrow \frac{1}{16}} \frac{\sqrt{x} - \frac{1}{4}}{x - \frac{1}{16}}$
5. Compute the following limit: $\lim_{x \rightarrow \frac{1}{9}} \frac{\sqrt{x} - \frac{1}{3}}{x - \frac{1}{9}}$
6. Compute the following limit: $\lim_{x \rightarrow \frac{4}{9}} \frac{\sqrt{x} - \frac{2}{3}}{x - \frac{4}{9}}$
7. Compute the following limit: $\lim_{x \rightarrow \frac{16}{9}} \frac{\sqrt{x} - \frac{4}{3}}{x - \frac{16}{9}}$
8. Compute the following limit: $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4}$
9. Compute the following limit: $\lim_{x \rightarrow \frac{9}{4}} \frac{\sqrt{x} - \frac{3}{2}}{x - \frac{9}{4}}$
10. Compute the following limit: $\lim_{x \rightarrow \frac{1}{4}} \frac{\sqrt{x} - \frac{1}{2}}{x - \frac{1}{4}}$

Answers: 1. $\frac{2}{1}$ 2. $\frac{10}{3}$ 3. $\frac{5}{1}$ 4. 2 5. $\frac{2}{3}$ 6. $\frac{4}{3}$ 7. $\frac{8}{3}$ 8. $\frac{4}{1}$ 9. $\frac{3}{1}$ 10. 1

Solutions:

$$\begin{aligned}
1. \quad & \lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x - 1} \quad \blacktriangleright \text{Multiply by the conjugate radical:} \\
& = \lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x - 1} \times \frac{\sqrt{x} + 1}{\sqrt{x} + 1} \quad \blacktriangleright \text{Use: } (a - b)(a + b) = a^2 - b^2 \quad \text{and} \quad (\sqrt{x})^2 = x \\
& = \lim_{x \rightarrow 1} \frac{x - 1}{(x - 1)(\sqrt{x} + 1)} \quad \blacktriangleright \text{Simplify the common factor:} \\
& = \lim_{x \rightarrow 1} \frac{1}{\sqrt{x} + 1} \quad \blacktriangleright \text{Use substitution to compute the limit:} \\
& = \frac{1}{\sqrt{1} + 1} \quad \blacktriangleright \text{Simplify:} \\
& = \frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
2. \quad & \lim_{x \rightarrow \frac{25}{9}} \frac{\sqrt{x} - \frac{5}{3}}{x - \frac{25}{9}} \quad \blacktriangleright \text{Multiply by the conjugate radical:} \\
& = \lim_{x \rightarrow \frac{25}{9}} \frac{\sqrt{x} - \frac{5}{3}}{x - \frac{25}{9}} \times \frac{\sqrt{x} + \frac{5}{3}}{\sqrt{x} + \frac{5}{3}} \quad \blacktriangleright \text{Use: } (a - b)(a + b) = a^2 - b^2 \quad \text{and} \quad (\sqrt{x})^2 = x \\
& = \lim_{x \rightarrow \frac{25}{9}} \frac{x - \frac{25}{9}}{(x - \frac{25}{9})(\sqrt{x} + \frac{5}{3})} \quad \blacktriangleright \text{Simplify the common factor:} \\
& = \lim_{x \rightarrow \frac{25}{9}} \frac{1}{\sqrt{x} + \frac{5}{3}} \quad \blacktriangleright \text{Use substitution to compute the limit:} \\
& = \frac{1}{\sqrt{\frac{25}{9}} + \frac{5}{3}} \quad \blacktriangleright \text{Simplify:} \\
& = \frac{3}{10}
\end{aligned}$$

$$\begin{aligned}
3. \quad & \lim_{x \rightarrow \frac{25}{4}} \frac{\sqrt{x} - \frac{5}{2}}{x - \frac{25}{4}} \quad \blacktriangleright \text{Multiply by the conjugate radical:} \\
& = \lim_{x \rightarrow \frac{25}{4}} \frac{\sqrt{x} - \frac{5}{2}}{x - \frac{25}{4}} \times \frac{\sqrt{x} + \frac{5}{2}}{\sqrt{x} + \frac{5}{2}} \quad \blacktriangleright \text{Use: } (a - b)(a + b) = a^2 - b^2 \quad \text{and} \quad (\sqrt{x})^2 = x \\
& = \lim_{x \rightarrow \frac{25}{4}} \frac{x - \frac{25}{4}}{(x - \frac{25}{4})(\sqrt{x} + \frac{5}{2})} \quad \blacktriangleright \text{Simplify the common factor:} \\
& = \lim_{x \rightarrow \frac{25}{4}} \frac{1}{\sqrt{x} + \frac{5}{2}} \quad \blacktriangleright \text{Use substitution to compute the limit:} \\
& = \frac{1}{\sqrt{\frac{25}{4}} + \frac{5}{2}} \quad \blacktriangleright \text{Simplify:} \\
& = \frac{1}{5}
\end{aligned}$$

$$4. \quad \lim_{x \rightarrow \frac{1}{16}} \frac{\sqrt{x} - \frac{1}{4}}{x - \frac{1}{16}} \quad \blacktriangleright \text{Multiply by the conjugate radical:}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow \frac{1}{16}} \frac{\sqrt{x} - \frac{1}{4}}{x - \frac{1}{16}} \times \frac{\sqrt{x} + \frac{1}{4}}{\sqrt{x} + \frac{1}{4}} && \blacktriangleright \text{Use: } (a - b)(a + b) = a^2 - b^2 && \text{and } (\sqrt{x})^2 = x \\
 &= \lim_{x \rightarrow \frac{1}{16}} \frac{x - \frac{1}{16}}{(x - \frac{1}{16})(\sqrt{x} + \frac{1}{4})} && \blacktriangleright \text{Simplify the common factor:} \\
 &= \lim_{x \rightarrow \frac{1}{16}} \frac{1}{\sqrt{x} + \frac{1}{4}} && \blacktriangleright \text{Use substitution to compute the limit:} \\
 &= \frac{1}{\sqrt{\frac{1}{16}} + \frac{1}{4}} && \blacktriangleright \text{Simplify:} \\
 &= 2
 \end{aligned}$$

$$\begin{aligned}
 5. \quad &\lim_{x \rightarrow \frac{1}{9}} \frac{\sqrt{x} - \frac{1}{3}}{x - \frac{1}{9}} && \blacktriangleright \text{Multiply by the conjugate radical:} \\
 &= \lim_{x \rightarrow \frac{1}{9}} \frac{\sqrt{x} - \frac{1}{3}}{x - \frac{1}{9}} \times \frac{\sqrt{x} + \frac{1}{3}}{\sqrt{x} + \frac{1}{3}} && \blacktriangleright \text{Use: } (a - b)(a + b) = a^2 - b^2 && \text{and } (\sqrt{x})^2 = x \\
 &= \lim_{x \rightarrow \frac{1}{9}} \frac{x - \frac{1}{9}}{(x - \frac{1}{9})(\sqrt{x} + \frac{1}{3})} && \blacktriangleright \text{Simplify the common factor:} \\
 &= \lim_{x \rightarrow \frac{1}{9}} \frac{1}{\sqrt{x} + \frac{1}{3}} && \blacktriangleright \text{Use substitution to compute the limit:} \\
 &= \frac{1}{\sqrt{\frac{1}{9}} + \frac{1}{3}} && \blacktriangleright \text{Simplify:} \\
 &= \frac{3}{2}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad &\lim_{x \rightarrow \frac{4}{9}} \frac{\sqrt{x} - \frac{2}{3}}{x - \frac{4}{9}} && \blacktriangleright \text{Multiply by the conjugate radical:} \\
 &= \lim_{x \rightarrow \frac{4}{9}} \frac{\sqrt{x} - \frac{2}{3}}{x - \frac{4}{9}} \times \frac{\sqrt{x} + \frac{2}{3}}{\sqrt{x} + \frac{2}{3}} && \blacktriangleright \text{Use: } (a - b)(a + b) = a^2 - b^2 && \text{and } (\sqrt{x})^2 = x \\
 &= \lim_{x \rightarrow \frac{4}{9}} \frac{x - \frac{4}{9}}{(x - \frac{4}{9})(\sqrt{x} + \frac{2}{3})} && \blacktriangleright \text{Simplify the common factor:} \\
 &= \lim_{x \rightarrow \frac{4}{9}} \frac{1}{\sqrt{x} + \frac{2}{3}} && \blacktriangleright \text{Use substitution to compute the limit:} \\
 &= \frac{1}{\sqrt{\frac{4}{9}} + \frac{2}{3}} && \blacktriangleright \text{Simplify:} \\
 &= \frac{3}{4}
 \end{aligned}$$

$$\begin{aligned}
 7. \quad &\lim_{x \rightarrow \frac{16}{9}} \frac{\sqrt{x} - \frac{4}{3}}{x - \frac{16}{9}} && \blacktriangleright \text{Multiply by the conjugate radical:} \\
 &= \lim_{x \rightarrow \frac{16}{9}} \frac{\sqrt{x} - \frac{4}{3}}{x - \frac{16}{9}} \times \frac{\sqrt{x} + \frac{4}{3}}{\sqrt{x} + \frac{4}{3}} && \blacktriangleright \text{Use: } (a - b)(a + b) = a^2 - b^2 && \text{and } (\sqrt{x})^2 = x
 \end{aligned}$$

$$= \lim_{x \rightarrow \frac{16}{9}} \frac{x - \frac{16}{9}}{(x - \frac{16}{9})(\sqrt{x} + \frac{4}{3})} \quad \blacktriangleright \text{Simplify the common factor:}$$

$$= \lim_{x \rightarrow \frac{16}{9}} \frac{1}{\sqrt{x} + \frac{4}{3}} \quad \blacktriangleright \text{Use substitution to compute the limit:}$$

$$= \frac{1}{\sqrt{\frac{16}{9} + \frac{4}{3}}} \quad \blacktriangleright \text{Simplify:}$$

$$= \frac{3}{8}$$

$$8. \lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4} \quad \blacktriangleright \text{Multiply by the conjugate radical:}$$

$$= \lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4} \times \frac{\sqrt{x} + 2}{\sqrt{x} + 2} \quad \blacktriangleright \text{Use: } (a - b)(a + b) = a^2 - b^2 \quad \text{and} \quad (\sqrt{x})^2 = x$$

$$= \lim_{x \rightarrow 4} \frac{x - 4}{(x - 4)(\sqrt{x} + 2)} \quad \blacktriangleright \text{Simplify the common factor:}$$

$$= \lim_{x \rightarrow 4} \frac{1}{\sqrt{x} + 2} \quad \blacktriangleright \text{Use substitution to compute the limit:}$$

$$= \frac{1}{\sqrt{4} + 2} \quad \blacktriangleright \text{Simplify:}$$

$$= \frac{1}{4}$$

$$9. \lim_{x \rightarrow \frac{9}{4}} \frac{\sqrt{x} - \frac{3}{2}}{x - \frac{9}{4}} \quad \blacktriangleright \text{Multiply by the conjugate radical:}$$

$$= \lim_{x \rightarrow \frac{9}{4}} \frac{\sqrt{x} - \frac{3}{2}}{x - \frac{9}{4}} \times \frac{\sqrt{x} + \frac{3}{2}}{\sqrt{x} + \frac{3}{2}} \quad \blacktriangleright \text{Use: } (a - b)(a + b) = a^2 - b^2 \quad \text{and} \quad (\sqrt{x})^2 = x$$

$$= \lim_{x \rightarrow \frac{9}{4}} \frac{x - \frac{9}{4}}{(x - \frac{9}{4})(\sqrt{x} + \frac{3}{2})} \quad \blacktriangleright \text{Simplify the common factor:}$$

$$= \lim_{x \rightarrow \frac{9}{4}} \frac{1}{\sqrt{x} + \frac{3}{2}} \quad \blacktriangleright \text{Use substitution to compute the limit:}$$

$$= \frac{1}{\sqrt{\frac{9}{4} + \frac{3}{2}}} \quad \blacktriangleright \text{Simplify:}$$

$$= \frac{1}{3}$$

$$10. \lim_{x \rightarrow \frac{1}{4}} \frac{\sqrt{x} - \frac{1}{2}}{x - \frac{1}{4}} \quad \blacktriangleright \text{Multiply by the conjugate radical:}$$

$$= \lim_{x \rightarrow \frac{1}{4}} \frac{\sqrt{x} - \frac{1}{2}}{x - \frac{1}{4}} \times \frac{\sqrt{x} + \frac{1}{2}}{\sqrt{x} + \frac{1}{2}} \quad \blacktriangleright \text{Use: } (a - b)(a + b) = a^2 - b^2 \quad \text{and} \quad (\sqrt{x})^2 = x$$

$$= \lim_{x \rightarrow \frac{1}{4}} \frac{x - \frac{1}{4}}{(x - \frac{1}{4})(\sqrt{x} + \frac{1}{2})} \quad \blacktriangleright \text{Simplify the common factor:}$$

$$= \lim_{x \rightarrow \frac{1}{4}} \frac{1}{\sqrt{x} + \frac{1}{2}} \quad \blacktriangleright \text{Use substitution to compute the limit:}$$

$$= \frac{1}{\sqrt{\frac{1}{4} + \frac{1}{2}}}$$

► Simplify:

$$= 1$$