A Plane
A plane may be determined by points and lines. There are four main possibilities as represented in the following figure:

- a) plane determined by three points
- b) plane determined by two parallel lines
- c) plane determined by two intersecting lines
- d) plane determined by a line and a point

B Vector Equation of a Plane
Let consider a plane \( \pi \).

Two vectors \( \mathbf{u} \) and \( \mathbf{v} \), parallel to the plane \( \pi \) but not parallel between them, are called direction vectors of the plane \( \pi \).

The vector \( \overrightarrow{P_0P} \) from a specific point \( P_0(x_0, y_0, z_0) \) to a generic point \( P(x, y, z) \) of the plane is a linear combination of direction vectors \( \mathbf{u} \) and \( \mathbf{v} \):

\[
\overrightarrow{P_0P} = s\mathbf{u} + t\mathbf{v} \quad s, t \in \mathbb{R}
\]

The vector equation of the plane is:

\[
\pi : \mathbf{r} = \mathbf{r}_0 + s\mathbf{u} + t\mathbf{v} \quad s, t \in \mathbb{R}
\]

C Parametric Equations of a Plane
Let write vector equation of the plane as:

\[
(x, y, z) = (x_0, y_0, z_0) + s(u_x, u_y, u_z) + t(v_x, v_y, v_z)
\]
or:

\[
\begin{align*}
x &= x_0 + su_x + tv_x \\
y &= y_0 + su_y + tv_y \\
z &= z_0 + su_z + tv_z
\end{align*}
\]

These are the parametric equations of a line.

Ex 1. A plane \( \pi \) is given by the following vector equation:

\[
\pi : \mathbf{r} = (-1,0,2) + s(0,0,1) + t(1,0,-1) \quad s, t \in \mathbb{R}
\]

a) Find two points on this plane.

b) Find one line on this plane.

c) Find the vector equation of a line \( L \) that passes through the origin and is perpendicular to this plane.

Ex 2. Convert the vector equation to the parametric equations.

\[
\mathbf{r} = (-1,0,2) + s(0,1,-1) + t(1,0,0) \quad s, t \in \mathbb{R}
\]

Ex 3. Convert the parametric equations to the vector equation.

\[
\begin{align*}
x &= 1 + s - 2t \\
y &= 3t \\
z &= 4 - s
\end{align*}
\]
Ex 4. (Plane determined by three points) Find the vector equation of the plane \( \pi \) that passes through the points \( A(0,1,-1) \), \( B(2,-1,0) \), and \( C(0,0,1) \).

Ex 5. (Plane determined by two parallel and distinct lines) Find the vector and parametric equations of the plane \( \pi \) that contains the following parallel and distinct lines:

\[ L_1 : \vec{r} = (1,2,1) + s(0,-1,-2) ; \quad s \in \mathbb{R} \]
\[ L_2 : \vec{r} = (3,4,0) + t(0,1,2) ; \quad t \in \mathbb{R} \]

Ex 6. (Plane determined by two intersecting lines) Find the vector equation of the plane \( \pi \) determined by the following intersecting lines.

\[ L_1 : \vec{r} = (0,0,1) + s(-1,0,0) ; \quad s \in \mathbb{R} \]
\[ L_2 : \vec{r} = (-3,0,1) + t(0,0,2) ; \quad t \in \mathbb{R} \]

Ex 7. (Plane determined by a line and an external point) Find the vector equation of the plane \( \pi \) that passes through the origin and contains the line

\[ L : \vec{r} = (0,1,2) + t(-1,0,3) ; \quad t \in \mathbb{R} \]

Reading: Nelson Textbook, Pages 453-458
Homework: Nelson Textbook: Page 459 #1, 2, 4, 6b, 7, 9, 10, 15