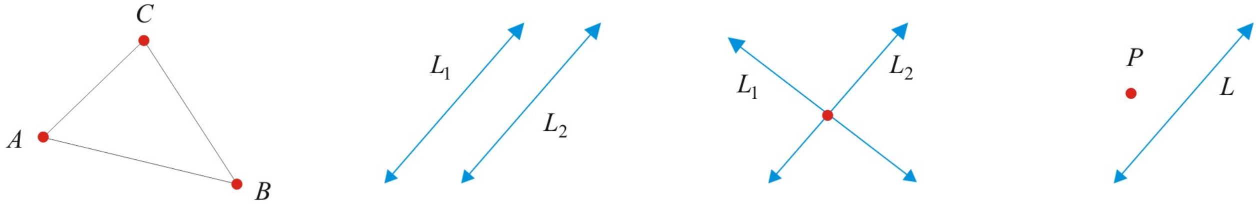
8.4 Vector and Parametric Equations of a Plane

A Planes

A plane may be determined by points and lines, There are four main possibilities as represented in the following figure:



a) plane determined by three points

b) plane determined by two parallel lines

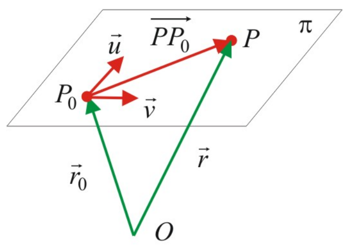
c) plane determined by two intersecting lines

d) plane determined by a line and a point

B Vector Equation of a Plane

Let consider a plane π .

Two vectors $\vec{u}$ and $\vec{v}$, parallel to the plane π but not parallel between them, are called *direction vectors* of the plane π .



The vector $\overrightarrow{P_0P}$ from a specific point $P_0(x_0, y_0, z_0)$ to a generic point $P(x, y, z)$ of the plane is a *linear combination* of direction vectors $\vec{u}$ and $\vec{v}$:

$$\overrightarrow{P_0P} = s\vec{u} + t\vec{v}; \quad s,t \in R$$

The *vector equation* of the plane is:

$$\pi : \vec{r} = \vec{r}_0 + s\vec{u} + t\vec{v}; \quad s,t \in R$$

Ex 1. A plane π is given by the following vector equation:

$\pi : \vec{r} = (-1,0,2) + s(0,0,1) + t(1,0,-1); \quad s,t \in R$

a) Find two points on this plane.

b) Find one line on this plane.

c) Find the vector equation of a line $L_\perp$ that passes through the origin and is perpendicular to this plane.

C Parametric Equations of a Plane

Let write vector equation of the plane as:

$(x, y, z) = (x_0, y_0, z_0) + s(u_x, u_y, u_z) + t(v_x, v_y, v_z)$

or:

$$\begin{cases} x = x_0 + su_x + tv_x \\ y = y_0 + su_y + tv_y \\ z = z_0 + su_z + tv_z \end{cases} ; \quad s,t \in R$$

These are the *parametric equations* of a line.

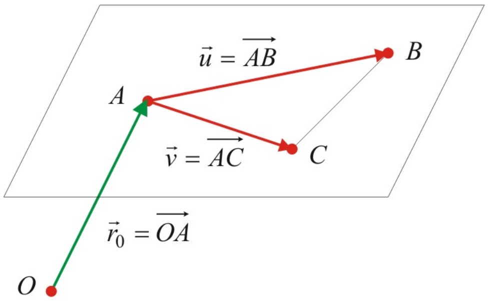
Ex 2. Convert the vector equation to the parametric equations.

$\vec{r} = (-1,0,2) + s(0,1,-1) + t(1,-2,0); \quad s,t \in R$

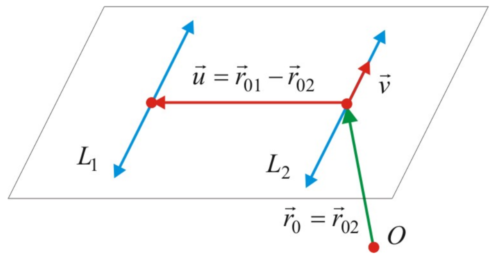
Ex 3. Convert the parametric equations to the vector equation.

$$\begin{cases} x = 1 + s - 2t \\ y = 3t \\ z = 4 - s \end{cases} ; \quad s,t \in R$$

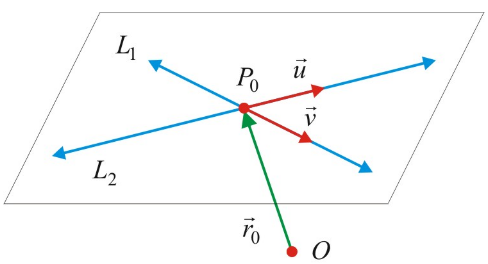
Ex 4. (Plane determined by three points)
Find the vector equation of the plane π that passes through the points $A(0,1,-1)$, $B(2,-1,0)$, and $C(0,0,1)$.



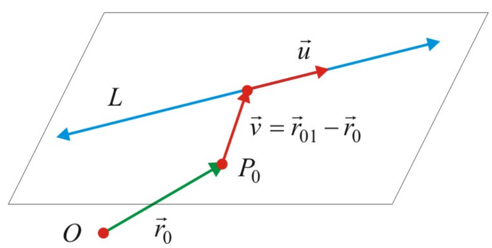
Ex 5. (Plane determined by two parallel and distinct lines)
Find the vector and parametric equations of the plane π that contains the following parallel and distinct lines:
$L_1 : \vec{r} = (1,2,1) + s(0,-1,-2); \quad s \in R$
$L_2 : \vec{r} = (3,4,0) + t(0,1,2); \quad t \in R$



Ex 6. (Plane determined by two intersecting lines)
Find the vector equation of the plane π determined by the following intersecting lines.
$L_1 : \vec{r} = (0,0,1) + s(-1,0,0); \quad s \in R$
$L_2 : \vec{r} = (-3,0,1) + t(0,0,2); \quad t \in R$



Ex 7. (Plane determined by a line and an external point)
Find the vector equation of the plane π that passes through the origin and contains the line
$L : \vec{r} = (0,1,2) + t(-1,0,3); \quad t \in R$.



Reading: Nelson Textbook, Pages 453-458
Homework: Nelson Textbook: Page 459 #1, 2, 4, 6b, 7, 9, 10, 15