### 8.3 Vector, Parametric, and Symmetric Equations of a Line in $\mathbb{R}^3$

#### A Vector Equation

The vector equation of the line is:

$$\mathbf{r} = \mathbf{r_0} + t\mathbf{u}, \quad t \in \mathbb{R}$$

where:
- $\mathbf{r}$ is the position vector of a generic point $P$ on the line,
- $\mathbf{r}_0$ is the position vector of a specific point $P_0$ on the line,
- $\mathbf{u}$ is a vector parallel to the line called the **direction vector** of the line, and
- $t$ is a **real number** corresponding to the generic point $P$.

**Ex 1.** Find two vector equations of the line $L$ that passes through the points $A(1,2,3)$ and $B(2,-1,0)$.

#### B Specific Lines

A line is **parallel to the x-axis** if $\mathbf{u} = (u_x,0,0), u_x \neq 0$. In this case, the line is also **perpendicular to the yz-plane**.

A line with $\mathbf{u} = (0,u_y,u_z), u_y \neq 0, u_z \neq 0$ is **parallel to the yz-plane**.

**Ex 2.** Find the vector equation of a line $L_2$ that passes through the origin and is parallel to the line $L_1 : \mathbf{r} = (-2,0,3) + t(-1,0,2), \quad t \in \mathbb{R}$.

**Ex 3.** Find the vector equation of a line that:

a) passes through $A(3,-2,0)$ and is parallel to the y-axis

b) passes through $M(-1,0,4)$ and is perpendicular to the yz-plane

c) passes through $P(3,0,0)$ and is perpendicular to the x-axis
d) passes through the origin and is parallel to the xz-plane

#### C Parametric Equations

Let rewrite the vector equation of a line:

$$\mathbf{r} = \mathbf{r_0} + t\mathbf{u}, \quad t \in \mathbb{R}$$

as:

$$(x,y,z) = (x_0,y_0,z_0) + t(u_x,u_y,u_z), \quad t \in \mathbb{R}$$

The **parametric equations** of a line in $\mathbb{R}^3$ are:

$$\begin{align*}
x &= x_0 + tu_x \\
y &= y_0 + tu_y \\
z &= z_0 + tu_z
\end{align*}$$

**Ex 4.** Find the parametric equations of the line $L$ that passes through the points $A(0,-1,2)$ and $B(1,-1,3)$. Describe the line.
### D Symmetric Equations

The parametric equations of a line may be written as:

\[
\begin{align*}
  x - x_0 &= tu_x \\
  y - y_0 &= tu_y \\
  z - z_0 &= tu_z
\end{align*}
\]

where \( t \in \mathbb{R} \).

From here, the **symmetric equations** of the line are:

\[
\frac{x - x_0}{u_x} = \frac{y - y_0}{u_y} = \frac{z - z_0}{u_z}
\]

where \( u_x \neq 0, u_y \neq 0, u_z \neq 0 \).

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<table>
<thead>
<tr>
<th>Ex 5. Convert the vector equation of the line ( L: \vec{r} = (0,1,-3) + t(-1,2,0) ), ( t \in \mathbb{R} ) to the parametric and symmetric equations.</th>
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### Ex 6. Convert the symmetric equations for a line: \[
\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z}{4}
\] to the parametric and vector equations.

### Ex 7. For each case, find if the given point lies on the given line.

- **a)** \( L: \vec{r} = (1,2,-3) + t(0,1,-2) \); \( P(1,4,-7) \)

\[
\begin{align*}
  x &= -2 + 3t \\
  y &= t \\
  z &= 5
\end{align*}
\]

- **b)** \( L: \vec{r} = (0,1,5) \); \( P(0,1,5) \)

- **c)** \( L: \vec{r} = \frac{x+1}{-2} = \frac{y-2}{1} = \frac{z}{-3} \); \( P(-3,3,-3) \)

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### E Intersections

A line **intersects the x-axis** when \( y = z = 0 \).

A line **intersects the xy-plane** when \( z = 0 \).

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**Reading:** Nelson Textbook, Pages 445-448

**Homework:** Nelson Textbook: Page 449 #1abc, 5acf, 6, 8, 9, 12, 13, 14