### A Symmetric Equation
The parametric equations of a line in $\mathbb{R}^2$:

\[
\begin{align*}
&x = x_0 + tu_x, \\
y = y_0 + tu_y,
\end{align*}
\]

may be written as:

\[
\frac{x - x_0}{u_x} = \frac{y - y_0}{u_y} = t, \quad t \in \mathbb{R}
\]

The symmetric equation of the line is (if exists):

\[
\frac{x - x_0}{u_x} = \frac{y - y_0}{u_y}
\]

Note. The symmetric equation does exist if $u_x \neq 0$ and $u_y \neq 0$.

#### Ex 1. Convert the vector or parametric equations to the symmetric equation (if exists).

a) \[
\begin{align*}
x &= -3 + 2t \\
y &= -5t
\end{align*}
\]

b) $\vec{r} = (0, 1) + t(1, -2)$, $t \in \mathbb{R}$

c) $\vec{r} = (-2, 3) + t(2, 0)$, $t \in \mathbb{R}$

#### Ex 2. Convert the symmetric equation of a line $L$:

\[
\frac{x + 2}{-1} = \frac{y - 3}{2}
\]
to the vector equation.

### B Normal Equation
Let consider a line $L$ that passes through the specific point $P_0(x_0, y_0)$ and has the direction vector $\vec{u} = (u_x, u_y)$.

The vectors $\vec{n} = (-u_y, u_x) = (A, B)$ or $\vec{n} = (u_y, -u_x) = (A, B)$ are perpendicular to the vector $\vec{u}$ and so they are perpendicular to the line $L$.

These are called normal vectors to the line $L$.

Let $P(x, y)$ be a generic point on the line $L$. So:

\[
\frac{\overrightarrow{P_0P} \parallel \vec{u}}{\overrightarrow{P_0P} \perp \vec{n}} \Rightarrow \overrightarrow{P_0P} \cdot \vec{n} = 0
\]

The normal equation of a line is given by:

\[
(Ax + By + C = 0)
\]

where $\vec{n} = (A, B)$ is a normal vector and the constant $C$ depends on a specific point of the line.

### C Cartesian Equation
The normal equation can be written as:

\[
\begin{align*}
&\vec{r} \cdot \vec{n} - \vec{n_0} \cdot \vec{n} = 0 \\
&(x, y) \cdot (A, B) - (x_0, y_0) \cdot (A, B) = 0 \\
&Ax + By - Ax_0 - By_0 = 0 \\
&Ax + By + C = 0 \text{ where } C = -Ax_0 - By_0
\end{align*}
\]

The Cartesian equation of a line is given by:

\[
Ax + By + C = 0
\]
### Ex 3. Convert the vector equation of the line $L : \vec{r} = (1,2) + t(3,-2), \quad t \in \mathbb{R}$ to the Cartesian equation.

### Ex 4. Convert the Cartesian equation to the parametric equations and then to the vector equation. $L : -x + 2y + 3 = 0$

### D Slope y-intercept Equation

Let solve the symmetric equation of a line:

$$\frac{x-x_0}{u_x} = \frac{y-y_0}{u_y} = t, \quad t \in \mathbb{R}$$

for $y$:

$$y - y_0 = u_y \frac{x - x_0}{u_x}$$

$$y = u_y \frac{x + y_0}{u_x} - u_y x_0$$

The **slope y-intercept equation** of a line in $\mathbb{R}^2$ is given by:

$$y = mx + n$$

$$m = \frac{u_y}{u_x}$$

where $m$ is the slope and $n$ is the y-intercept which depends on a specific point of the line.

### Ex 5. Convert the vector equation to the slope y-intercept equation: $L : \vec{r} = (-2,3) + t(1,-2)$

### Ex 6. Convert the slope y-intercept equation to the vector equation.

$L : y = -\frac{2}{3}x + 4$.

### E Angle between two Lines

The angle between two lines is determined by the angle between the direction vectors:

$$\cos \theta = \frac{\vec{u}_1 \cdot \vec{u}_2}{\| \vec{u}_1 \|\| \vec{u}_2 \|}$$

Note. There are two pairs of equal angles between two lines (see the figure below).

**Note:** $\theta_1 + \theta_2 = 180^\circ$

### Ex 7. Find the acute angle between each pair of lines.

- **a)** $L_1 : \vec{r} = (0,1) + t(1,-3), \quad t \in \mathbb{R}; \quad L_2 : \frac{x-2}{3} = \frac{y+1}{-2}$

- **b)** $L_1 : 2x - 3y + 6 = 0; \quad L_2 : y = -\frac{1}{3}x + 2$

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**Reading:** Nelson Textbook, Pages 435-442  
**Homework:** Nelson Textbook: Page 443 #1, 3, 4, 5, 6, 7, 8, 10ab, 11, 14