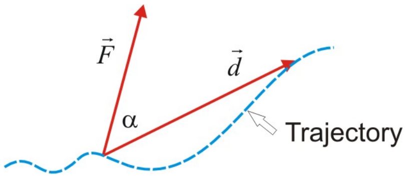
7.7 Applications of the Dot and Cross Product

A Work

The work W done by a *constant force* $\vec{F}$ acting on an object during a *displacement* $\vec{d}$ is given by:

$$W = \vec{F} \cdot \vec{d} = F\ d\ \cos\alpha$$



where

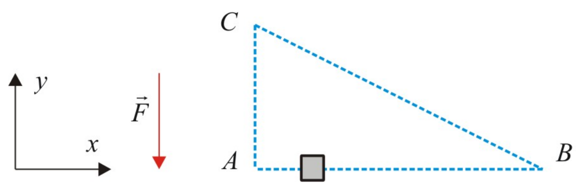
$$\alpha = \angle(\vec{F}, \vec{d})$$

$$[\vec{F}]_{SI} = N\ (Newton) \quad [\vec{d}]_{SI} = m\ (meter)$$

$$[W]_{SI} = J\ (Joule)$$

Ex 1. A box is pulled a horizontal distance of $100m$ by a force of $500N$ applied at an angle of $30°$ to the horizontal line. Calculate the amount of work done.

Ex 2. An object with a weight of $50N$ is moved, in vertical plane, along to the path $ABCA$ as presented in the next figure where $AB = 6m$ and $AC = 3m$.



Find the work done by the force of gravity:

a) from A to B

b) from B to C

c) from C to A

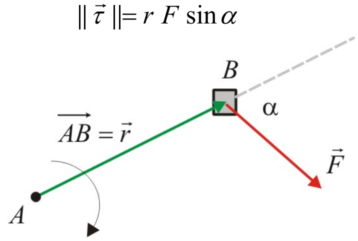
d) along to the path $ABCA$

B Torque

The torque (rotational or turning effect) about the point A, created by a force $\vec{F}$ acting on an object located at the point B is given by:

$$\vec{\tau} = \overrightarrow{AB} \times \vec{F} = \vec{r} \times \vec{F}$$

$$\|\vec{\tau}\| = r\ F\ \sin\alpha$$



where:

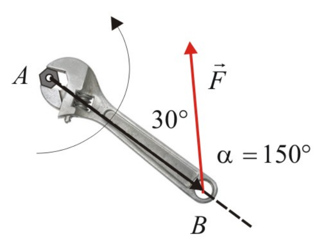
$$\alpha = \angle(\vec{F}, \vec{r})$$

$$\vec{r} = \overrightarrow{AB}\ (meter)$$

$$[\vec{F}]_{SI} = N\ (Newtom)$$

$$[\vec{\tau}]_{SI} = Nm$$

Ex 3. A wrench $30cm$ long is used to loose a bolt by applying a force of $20N$ (see the figure below). Find the magnitude of the torque.

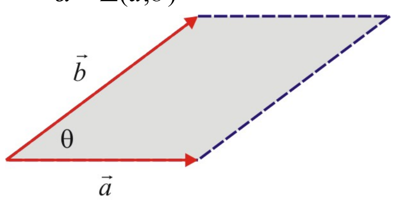


C Parallelogram Area

The *area* of a *parallelogram* defined by the vectors $\vec{a}$ and $\vec{b}$ is determined by the formula:

$$A = \|\vec{a}\times\vec{b}\| = \|\vec{a}\|\|\vec{b}\|\sin\alpha$$

$$\alpha = \angle(\vec{a},\vec{b})$$



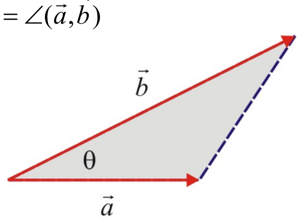
Ex 4. Find the area of the parallelogram defined by the vectors $\vec{a} = (1,-1,0)$ and $\vec{b} = (0,1,2)$.

D Triangle Area

The *area* of a *triangle* defined by the vectors $\vec{a}$ and $\vec{b}$ is given by:

$$A = \frac{1}{2}\|\vec{a}\times\vec{b}\| = \frac{1}{2}\|\vec{a}\|\|\vec{b}\|\sin\alpha$$

$$\alpha = \angle(\vec{a},\vec{b})$$

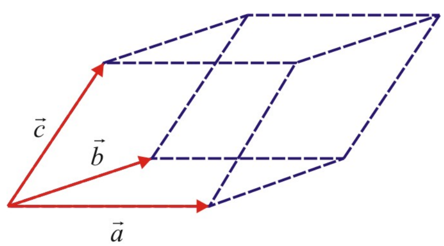


Ex 5. Find the area of the triangle ΔABC where $A(0,1,2)$, $B(-1,0,2)$, and $C(1,-2,0)$.

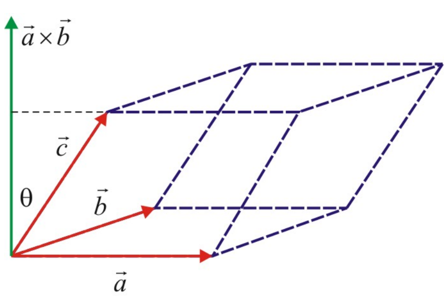
E Parallelepiped Volume

The *volume* of a *parallelepiped* defined by the vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ is given by:

$$V = |\vec{c}\cdot(\vec{a}\times\vec{b})| = |\vec{a}\cdot(\vec{b}\times\vec{c})| = |\vec{b}\cdot(\vec{c}\times\vec{a})|$$



Proof:



$$V = A_{base}\times h = \|\vec{a}\times\vec{b}\|\,|SProj(\vec{c}\ onto\ \vec{a}\times\vec{b})|$$

$$= \|\vec{a}\times\vec{b}\|\left|\frac{\vec{c}\cdot(\vec{a}\times\vec{b})}{\|\vec{a}\times\vec{b}\|}\right| = |\vec{c}\cdot(\vec{a}\times\vec{b})|$$

Ex 6. Find the volume of the parallelepiped defined by the vectors $\vec{a} = (0,1,-3)$, $\vec{b} = (1,2,3)$ and $\vec{c} = (-1,0,1)$.

Ex 7. Find an unit vector perpendicular to both $\vec{a} = (0,1,1)$ and $\vec{b} = (1,1,0)$.

Reading: Nelson Textbook, Pages 409-414

Homework: Nelson Textbook: Page 414 #3, 5a, 8, 10

