### 7.5 Scalar and Vector Projections

#### A Scalar Projection

The **scalar projection** of the vector \( \vec{a} \) onto the vector \( \vec{b} \) is a scalar defined as:

\[
S_{\text{Proj}}(\vec{a} \text{ onto } \vec{b}) = ||\vec{a}|| \cos \theta \quad \text{where} \quad \theta = \angle(\vec{a}, \vec{b})
\]

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#### Ex 1. Given two vectors with the magnitudes \( ||\vec{a}|| = 10 \) and \( ||\vec{b}|| = 16 \) respectively, and the angle between them equal to \( \theta = 120^\circ \), find the scalar projection

a) of the vector \( \vec{a} \) onto the vector \( \vec{b} \)

b) of the vector \( \vec{b} \) onto the vector \( \vec{a} \)

#### B Special Cases

Consider two vectors \( \vec{a} \) and \( \vec{b} \).

- a) If \( \vec{a} \parallel \vec{b} \) (\( \cos \theta = 1 \)), then \( S_{\text{Proj}}(\vec{a} \text{ onto } \vec{b}) = ||\vec{a}|| \)
- b) If \( \vec{a} \perp \vec{b} \) (\( \cos \theta = -1 \)), then \( S_{\text{Proj}}(\vec{a} \text{ onto } \vec{b}) = -||\vec{a}|| \)
- c) If \( \vec{a} \perp \vec{b} \) then \( S_{\text{Proj}}(\vec{a} \text{ onto } \vec{b}) = 0 \)

#### Ex 2. Find the scalar projection of the vector \( \vec{a} \) onto:

- a) itself
- b) the opposite vector \( -\vec{a} \)

#### C Dot Product and Scalar Projection

Recall that the **dot product** of the vectors \( \vec{a} \) and \( \vec{b} \) is defined as:

\[
\vec{a} \cdot \vec{b} = ||\vec{a}|| ||\vec{b}|| \cos \theta
\]

So, the scalar projection of the vector \( \vec{a} \) onto the vector \( \vec{b} \) can be written as:

\[
S_{\text{Proj}}(\vec{a} \text{ onto } \vec{b}) = ||\vec{a}|| \cos \theta = \frac{\vec{a} \cdot \vec{b}}{||\vec{b}||}
\]

Note:

For a Cartesian (Rectangular) coordinate system, the *scalar components* \( a_x, a_y, \) and \( a_z \) of a vector \( \vec{a} = (a_x, a_y, a_z) \) are the *scalar projections* of the vector \( \vec{a} \) onto the unit vectors \( i, j, \) and \( k \).

**Proof:**

\[
S_{\text{Proj}}(\vec{a} \text{ onto } i) = \frac{\vec{a} \cdot i}{||i||} = \frac{(a_x, a_y, a_z) \cdot (1,0,0)}{1} = a_x
\]

#### Ex 3. Given the vector \( \vec{a} = (2, -3, 4) \), find the scalar projection:

- a) of \( \vec{a} \) onto the unit vector \( i \)
- b) of \( \vec{a} \) onto the vector \( i - j \)
- c) of \( \vec{a} \) onto the vector \( \vec{b} = -i + 2j + k \)
- d) of the unit vector \( i \) onto the vector \( \vec{a} \)

#### D Vector Projection

The **vector projection** of the vector \( \vec{a} \) onto the vector \( \vec{b} \) is a vector defined as:

\[
V_{\text{Proj}}(\vec{a} \text{ onto } \vec{b}) = ||\vec{a}|| \cos \theta \frac{\vec{b}}{||\vec{b}||}
\]

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#### E Dot Product and Vector Projection

The vector projection of the vector \( \vec{a} \) onto the vector \( \vec{b} \) can be written using the dot product as:

\[
V_{\text{Proj}}(\vec{a} \text{ onto } \vec{b}) = \frac{(\vec{a} \cdot \vec{b})\vec{b}}{||\vec{b}||^2}
\]

Note:

For a Cartesian (Rectangular) coordinate system, the *vector components* \( a_x, a_y, a_z \) of a vector \( \vec{a} = (a_x, a_y, a_z) \) are the *vector projections* of the vector \( \vec{a} \) onto the unit vectors \( i, j, \) and \( k \).
Ex 4. Given two vectors $\vec{a} = (0,1,-2)$ and $\vec{b} = (-1,0,3)$, find:

a) the vector projection of the vector $\vec{a}$ onto the vector $\vec{b}$

b) the vector projection of the vector $\vec{b}$ onto the vector $\vec{a}$

c) the vector projection of the vector $\vec{a}$ onto the unit vector $\vec{k}$

d) the vector projection of the vector $\vec{i}$ onto the vector $\vec{a}$

Ex 5. Find an expression using the dot product of the vector components of the vector $\vec{a}$ along to the vector $\vec{b}$ and along to a direction perpendicular to the direction of the vector $\vec{b}$ but in the same plan containing the vectors $\vec{a}$ and $\vec{b}$.

Reading: Nelson Textbook, Pages 390-398
Homework: Nelson Textbook: Page 399 # 6, 11, 13, 14