7.4 Dot Product of Algebraic Vectors

A Dot Product for Standard Unit Vectors
The *dot product* of the *standard unit vectors* is given by:

$\vec{i}\cdot\vec{i}=1 \qquad \vec{j}\cdot\vec{j}=1 \qquad \vec{k}\cdot\vec{k}=1$

$\vec{i}\cdot\vec{j}=0 \qquad \vec{j}\cdot\vec{k}=0 \qquad \vec{k}\cdot\vec{i}=0$

Proof:

$\vec{i}\cdot\vec{i}=\|\vec{i}\|\|\vec{i}\|\cos 0^\circ=(1)(1)(1)=1$

$\vec{i}\cdot\vec{j}=\|\vec{i}\|\|\vec{j}\|\cos 90^\circ=(1)(1)(0)=0$

B Dot Product for two Algebraic Vectors
The *dot product* of two *algebraic* vectors
$\vec{a}=(a_x,a_y,a_z)=a_x\vec{i}+a_y\vec{j}+a_z\vec{k}$ and
$\vec{b}=(b_x,b_y,b_z)=b_x\vec{i}+b_y\vec{j}+b_z\vec{k}$ is given by:

$$\vec{a}\cdot\vec{b}=a_xb_x+a_yb_y+a_zb_z$$

Proof:

$\vec{a}\cdot\vec{b}=(a_x\vec{i}+a_y\vec{j}+a_z\vec{k})\cdot(b_x\vec{i}+b_y\vec{j}+b_z\vec{k})$

$=(a_xb_x)(\vec{i}\cdot\vec{i})+(a_xb_y(\vec{i}\cdot\vec{j})+(a_xb_z)(\vec{i}\cdot\vec{k})+$

$+(a_yb_x)(\vec{j}\cdot\vec{i})+(a_yb_y(\vec{j}\cdot\vec{j})+(a_yb_z)(\vec{j}\cdot\vec{k})+$

$+(a_zb_x)(\vec{k}\cdot\vec{i})+(a_zb_y(\vec{k}\cdot\vec{j})+(a_zb_z)(\vec{k}\cdot\vec{k})$

$=a_xb_x+a_yb_y+a_zb_z$

Ex 1. For each case, find the dot product of the vectors $\vec{a}$ and $\vec{b}$.

a) $\vec{a}=(1,-2,0)$, $\vec{b}=(0,-1,2)$

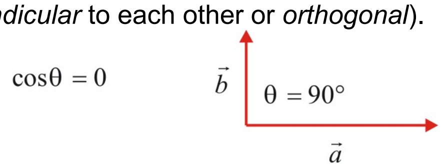
b) $\vec{a}=-\vec{i}+2\vec{j}$, $\vec{b}=\vec{i}-2\vec{j}-\vec{k}$

c) $\vec{a}=(-1,1,-1)$, $\vec{b}=-\vec{i}+2\vec{j}-2\vec{k}$

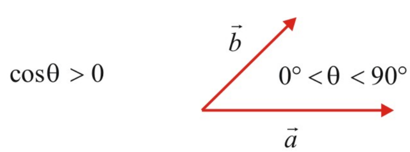
C Angle between two Vectors
The angle $\theta=\angle(\vec{a},\vec{b})$ between two vectors $\vec{a}$ and $\vec{b}$ (when positioned tail to tail) is given by:

$$\cos\theta=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}=\frac{a_xb_x+a_yb_y+a_zb_z}{\sqrt{a_x^2+a_y^2+a_z^2}\sqrt{b_x^2+b_y^2+b_z^2}}$$

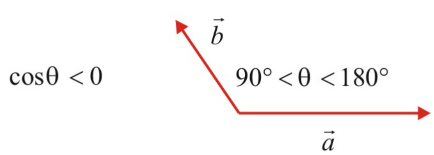
Notes:
1. If $\cos\theta=1$ then $\vec{a}\uparrow\uparrow\vec{b}$ (vectors are *parallel* and have *same direction*).
2. If $\cos\theta=-1$ then $\vec{a}\uparrow\downarrow\vec{b}$ (vectors are *parallel* but have *opposite direction*).
3. If $\cos\theta=0$ then $\vec{a}\perp\vec{b}$ (vectors are *perpendicular* to each other or *orthogonal*).



4. If $\cos\theta>0$ then $0^\circ<\theta<90^\circ$ (θ is an *acute* angle).



5. If $\cos\theta<0$ then $90^\circ<\theta<180^\circ$ (θ is an *obtuse* angle).



Ex 2. For each case, find the angle between the vectors $\vec{a}$ and $\vec{b}$.

a) $\vec{a}=(1,-2,-1)$, $\vec{b}=(0,-1,2)$

b) $\vec{a}=-\vec{i}-2\vec{k}$, $\vec{b}=-2\vec{j}+\vec{k}$

Ex 3. Find a non zero vector perpendicular to each of the vectors $\vec{a}=(1,5,-1)$ and $\vec{b}=(-3,1,2)$.

Ex 4. A triangle is defined by three points $A(0,1,2)$, $B(1,0,2)$, and $C(-1,2,0)$. Find the angles $\angle A$ of this triangle.

Ex 5. Find the angles between the vector $\vec{a} = -2\vec{i} + \vec{j} + 3\vec{k}$ and the coordinate axes.

Ex 6. For what values of *k* are the vectors $\vec{a} = (k,-2,3)$ and $\vec{b} = (2,2k-6,6)$

a) perpendicular (orthogonal)?

b) parallel (collinear)?

c) in opposite direction?

Reading: Nelson Textbook, Pages 379-385
Homework: Nelson Textbook: Page #2c, 6d, 7a, 10, 13, 14, 18, 19

