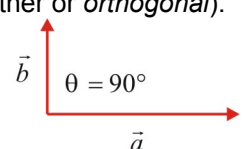
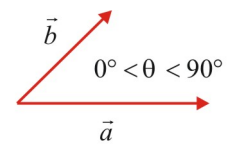
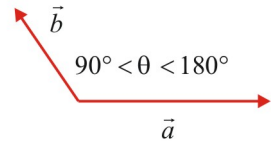


7.4 Dot Product of Algebraic Vectors

<p>A Dot Product for Standard Unit Vectors The <i>dot product</i> of the <i>standard unit vectors</i> is given by:</p> $\vec{i} \cdot \vec{i} = 1 \quad \vec{j} \cdot \vec{j} = 1 \quad \vec{k} \cdot \vec{k} = 1$ $\vec{i} \cdot \vec{j} = 0 \quad \vec{j} \cdot \vec{k} = 0 \quad \vec{k} \cdot \vec{i} = 0$	<p>Proof: $\vec{i} \cdot \vec{i} = \ \vec{i}\ \ \vec{i}\ \cos 0^\circ = (1)(1)(1) = 1$ $\vec{i} \cdot \vec{j} = \ \vec{i}\ \ \vec{j}\ \cos 90^\circ = (1)(1)(0) = 0$</p>
<p>B Dot Product for two Algebraic Vectors The <i>dot product</i> of two <i>algebraic vectors</i> $\vec{a} = (a_x, a_y, a_z) = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$ and $\vec{b} = (b_x, b_y, b_z) = b_x\vec{i} + b_y\vec{j} + b_z\vec{k}$ is given by:</p> $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$ <p>Proof: $\vec{a} \cdot \vec{b} = (a_x\vec{i} + a_y\vec{j} + a_z\vec{k}) \cdot (b_x\vec{i} + b_y\vec{j} + b_z\vec{k})$ $= (a_x b_x)(\vec{i} \cdot \vec{i}) + (a_x b_y)(\vec{i} \cdot \vec{j}) + (a_x b_z)(\vec{i} \cdot \vec{k}) +$ $+ (a_y b_x)(\vec{j} \cdot \vec{i}) + (a_y b_y)(\vec{j} \cdot \vec{j}) + (a_y b_z)(\vec{j} \cdot \vec{k}) +$ $+ (a_z b_x)(\vec{k} \cdot \vec{i}) + (a_z b_y)(\vec{k} \cdot \vec{j}) + (a_z b_z)(\vec{k} \cdot \vec{k})$ $= a_x b_x + a_y b_y + a_z b_z$</p>	<p>Ex 1. For each case, find the dot product of the vectors \vec{a} and \vec{b}.</p> <p>a) $\vec{a} = (1, -2, 0)$, $\vec{b} = (0, -1, 2)$</p> <p>b) $\vec{a} = -\vec{i} + 2\vec{j}$, $\vec{b} = \vec{i} - 2\vec{j} - \vec{k}$</p> <p>c) $\vec{a} = (-1, 1, -1)$, $\vec{b} = -\vec{i} + 2\vec{j} - 2\vec{k}$</p>
<p>C Angle between two Vectors The angle $\theta = \angle(\vec{a}, \vec{b})$ between two vectors \vec{a} and \vec{b} (when positioned tail to tail) is given by:</p> $\cos \theta = \frac{\vec{a} \cdot \vec{b}}{ \vec{a} \vec{b} } = \frac{a_x b_x + a_y b_y + a_z b_z}{\sqrt{a_x^2 + a_y^2 + a_z^2} \sqrt{b_x^2 + b_y^2 + b_z^2}}$ <p>Notes:</p> <ol style="list-style-type: none"> If $\cos \theta = 1$ then $\vec{a} \uparrow \uparrow \vec{b}$ (vectors are <i>parallel</i> and have <i>same direction</i>). If $\cos \theta = -1$ then $\vec{a} \uparrow \downarrow \vec{b}$ (vectors are <i>parallel</i> but have <i>opposite direction</i>). If $\cos \theta = 0$ then $\vec{a} \perp \vec{b}$ (vectors are <i>perpendicular</i> to each other or <i>orthogonal</i>). <div style="display: flex; align-items: center;"> <div style="margin-right: 20px;">$\cos \theta = 0$</div>  </div> <ol style="list-style-type: none"> If $\cos \theta > 0$ then $0^\circ < \theta < 90^\circ$ (θ is an <i>acute angle</i>). <div style="display: flex; align-items: center;"> <div style="margin-right: 20px;">$\cos \theta > 0$</div>  </div> <ol style="list-style-type: none"> If $\cos \theta < 0$ then $90^\circ < \theta < 180^\circ$ (θ is an <i>obtuse angle</i>). <div style="display: flex; align-items: center;"> <div style="margin-right: 20px;">$\cos \theta < 0$</div>  </div>	<p>Ex 2. For each case, find the angle between the vectors \vec{a} and \vec{b}.</p> <p>a) $\vec{a} = (1, -2, -1)$, $\vec{b} = (0, -1, 2)$</p> <p>b) $\vec{a} = -\vec{i} - 2\vec{k}$, $\vec{b} = -2\vec{j} + \vec{k}$</p> <p>Ex 3. Find a non zero vector perpendicular to each of the vectors $\vec{a} = (1, 5, -1)$ and $\vec{b} = (-3, 1, 2)$.</p>

Ex 4. A triangle is defined by three points $A(0,1,2)$, $B(1,0,2)$, and $C(-1,2,0)$. Find the angles $\angle A$ of this triangle.

Ex 5. Find the angles between the vector $\vec{a} = -2\vec{i} + \vec{j} + 3\vec{k}$ and the coordinate axes.

Ex 6. For what values of k are the vectors $\vec{a} = (k, -2, 3)$ and $\vec{b} = (2, 2k - 6, 6)$

a) perpendicular (orthogonal)?

b) parallel (collinear)?

c) in opposite direction?

Reading: Nelson Textbook, Pages 379-385

Homework: Nelson Textbook: Page #2c, 6d, 7a, 10, 13, 14, 18, 19