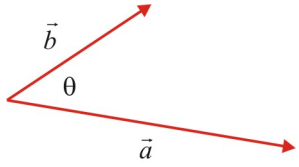
7.3 Dot Product of two Geometric Vectors

A Definition

The *Dot Product* of two *geometric* vectors $\vec{a}$ and $\vec{b}$ with an angle $\theta = \angle(\vec{a},\vec{b})$ between them (when positioned tail to tail) is a *scalar* defined by:

$$\vec{a}\cdot\vec{b} = \| \vec{a} \| \| \vec{b} \| \cos\theta$$

Note. By convention $0° \le \theta \le 180°$.

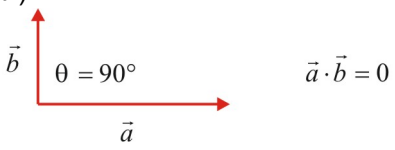


Ex 1. If $\| \vec{u} \| = 4$, $\| \vec{v} \| = 6$, and $\theta = \angle(\vec{u},\vec{v}) = 120°$, find $\vec{u}\cdot\vec{v}$.

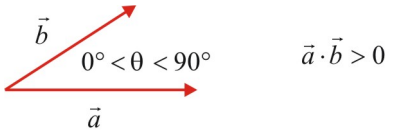
Ex 2. Find the angle between two unit vectors with a dot product equal to $1/\sqrt{2}$.

B Properties of Dot Product

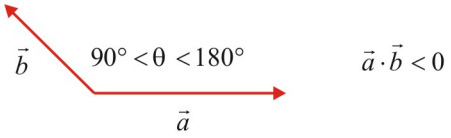
1. $\vec{a}\cdot\vec{b}$ is a scalar (a real number).
2. If $\vec{a} \perp \vec{b}$ then $\vec{a}\cdot\vec{b} = 0$ (because $\theta = 90°$ and $\cos 90° = 0$).



3. If $\vec{a}\cdot\vec{b} = 0$ then $\| \vec{a} \| = 0$ or $\| \vec{b} \| = 0$ or $\vec{a} \perp \vec{b}$.
4. If $0° < \theta < 90°$ then $\cos\theta > 0$ and $\vec{a}\cdot\vec{b} > 0$.



5. If $90° < \theta < 180°$ then $\cos\theta < 0$ and $\vec{a}\cdot\vec{b} < 0$.



6. If $\vec{a} \uparrow\uparrow \vec{b}$ then $\theta = 0°$, $\cos\theta = 1$, and $\vec{a}\cdot\vec{b} = \| \vec{a} \| \| \vec{b} \|$
7. If $\vec{a} \uparrow\downarrow \vec{b}$ then $\theta = 180°$, $\cos\theta = -1$, and $\vec{a}\cdot\vec{b} = -\| \vec{a} \| \| \vec{b} \|$
8. $\vec{a}\cdot\vec{a} = \| \vec{a} \|^2$
9. $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$ (commutative property)
10. $\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}$ (distributative property)
11. $(k\vec{a})\cdot\vec{b} = k(\vec{a}\cdot\vec{b}) = \vec{a}\cdot(k\vec{b})$
12. $\vec{a}\cdot\vec{0} = 0$

Note. $\vec{0}$ is the zero vector and 0 is the number zero.

Ex 3. Use the dot product to prove each relation:

a) $\| \vec{a}+\vec{b} \|^2 = \| \vec{a} \|^2 + \| \vec{b} \|^2 + 2\vec{a}\cdot\vec{b}$

b) $\| \vec{a}-\vec{b} \|^2 = \| \vec{a} \|^2 + \| \vec{b} \|^2 - 2\vec{a}\cdot\vec{b}$

c) $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = \| \vec{a} \|^2 - \| \vec{b} \|^2$

Ex 4. If the vectors $2\vec{a}+\vec{b}$ and $\frac{\vec{a}}{2}-\vec{b}$ are perpendicular to each other and $2\| \vec{b} \| = 3\| \vec{a} \|$ find the angle $\theta = \angle(\vec{a},\vec{b})$.

Reading: Nelson Textbook, Pages 371-376
Homework: Nelson Textbook: Page 377 #1, 2, 5, 6b, 7a, 9a, 11, 15, 17