

### 6.6 Operations with Algebraic Vectors in $\mathbb{R}^2$

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| <p><b>A 2D Algebraic Vectors</b><br/>                     A 2D Algebraic Vector may be written in <i>components form</i> as:</p> $\vec{v} = (v_x, v_y)$ <p>or in <i>terms of unit vectors</i> as:</p> $\vec{v} = v_x \vec{i} + v_y \vec{j}$ <p>and has a <i>magnitude</i> given by:</p> $\ \vec{v}\  = \sqrt{v_x^2 + v_y^2}$  | <p>Ex 1. Consider the vector <math>\vec{a} = -3\vec{i} + 4\vec{j}</math>.</p> <p>a) Write the vector in components form.</p> <p>b) Find the magnitude of the vector <math>\vec{a}</math>.</p>   |
| <p><b>B Addition of 2D Algebraic Vectors</b><br/>                     The sum of two 2D algebraic vectors <math>\vec{a} = (a_x, a_y) = a_x \vec{i} + a_y \vec{j}</math> and <math>\vec{b} = (b_x, b_y) = b_x \vec{i} + b_y \vec{j}</math> is a 2D algebraic vector given by:</p> $\vec{a} + \vec{b} = a_x \vec{i} + a_y \vec{j} + b_x \vec{i} + b_y \vec{j}$ $= (a_x + b_x) \vec{i} + (a_y + b_y) \vec{j} = (a_x + b_x, a_y + b_y)$             | <p>Ex 2. Find the sum of the vector <math>\vec{a} = 3\vec{i} - 5\vec{j}</math> and <math>\vec{b} = (-2, 7)</math>.</p>  |
| <p><b>C Subtraction of 2D Algebraic Vectors</b><br/>                     The difference of two 2D algebraic vectors <math>\vec{a} = (a_x, a_y) = a_x \vec{i} + a_y \vec{j}</math> and <math>\vec{b} = (b_x, b_y) = b_x \vec{i} + b_y \vec{j}</math> is a 2D algebraic vector given by:</p> $\vec{a} - \vec{b} = a_x \vec{i} + a_y \vec{j} - (b_x \vec{i} + b_y \vec{j})$ $= (a_x - b_x) \vec{i} + (a_y - b_y) \vec{j} = (a_x - b_x, a_y - b_y)$ | <p>Ex 3. Find the difference <math>\vec{d} = \vec{a} - \vec{b}</math> between the vector <math>\vec{a} = 3\vec{i} - 5\vec{j}</math> and <math>\vec{b} = (-2, 7)</math>.</p>   |
| <p><b>D Multiplication of 2D Algebraic Vector by a Scalar</b><br/>                     The multiplication of a 2D algebraic vector <math>\vec{a} = (a_x, a_y) = a_x \vec{i} + a_y \vec{j}</math> by a scalar <math>\lambda</math> is a 2D algebraic vector given by:</p> $\lambda \vec{a} = \lambda(a_x \vec{i} + a_y \vec{j}) = \lambda a_x \vec{i} + \lambda a_y \vec{j} = (\lambda a_x, \lambda a_y)$  | <p>Ex 4. For each case, multiply the given vector by the given scalar.</p> <p>a) <math>\vec{u} = (-1, 3)</math>, <math>\lambda = -2</math></p> <p>b) <math>\vec{v} = 4\vec{i} - 2\vec{j}</math>, <math>\mu = \frac{1}{2}</math></p> <p>Ex 5. Consider <math>\vec{a} = (3, -5)</math>, <math>\vec{b} = (0, -2)</math>, and <math>\vec{c} = (-1, 0)</math>. Find the vector <math>\vec{v} = -2\vec{a} + 3\vec{b} - 4\vec{c}</math>.</p> |
| <p><b>E Vector Equations</b><br/>                     Use backward operations to solve equations involving vectors:</p> $\vec{x} + \vec{a} = \vec{b} \Rightarrow \vec{x} = \vec{b} - \vec{a}$ $\vec{a} - \vec{x} = \vec{b} \Rightarrow \vec{x} = \vec{a} - \vec{b}$ $\lambda \vec{x} = \vec{a} \Rightarrow \vec{x} = \frac{1}{\lambda} \vec{a}$   | <p>Ex 6. Given <math>\vec{a} = (-2, 1)</math>, <math>\vec{b} = (1, -3)</math> solve for <math>\vec{x}</math> the following vector equation <math>2\vec{a} - 3\vec{x} = \vec{b}</math>.</p>  |

Ex 7. Find the perimeter of the triangle  $\triangle ABC$  where  $A(0,1)$ ,  $B(2,3)$ , and  $C(-1,-2)$ .

Ex 8. Find an unit vector parallel to the vector  $\vec{a} = (3,-4)$ .

Ex 9. Given  $A(-2,3)$ ,  $B(-1,-2)$ , and  $D(4,2)$ , find the point  $C$  such that the polygon  $ABCD$  is a parallelogram.

Ex 10. Find the angle between  $\vec{PQ}$  and  $\vec{PR}$  where  $P(2,0)$ ,  $Q(0,3)$ , and  $R(-3,-4)$ .

**Reading:** Nelson Textbook, Pages 319-324

**Homework:** Nelson Textbook: Page 324 #1, 3, 5, 7c, 11, 13b, 17