6.3 Multiplication of a Vector by a Scalar

A Multiplication of a Vector by a Scalar

By multiplying a vector \( \vec{v} \) by a scalar \( k \) we obtain a new vector noted \( k\vec{v} \) with the following properties:

a) \( k\vec{v} \) has the same direction as \( \vec{v} \) if \( k > 0 \) and the opposite direction if \( k < 0 \)

b) \( ||k\vec{v}|| = |k| \times ||\vec{v}|| \)

B Properties

The following properties apply for multiplication of a vector by a scalar:

\[ k(\vec{a} + \vec{b}) = k\vec{a} + k\vec{b} \]
\[ k(m\vec{a}) = (km)\vec{a} = km\vec{a} \]
\[ (k + m)\vec{a} = k\vec{a} + m\vec{a} \]

Ex 1. Given the vector \( \vec{v} \), draw the following vectors:

a) \( 2\vec{v} \)  

b) \( -3\vec{v} \)  

d) \( \frac{1}{2}\vec{v} \)  

e) \( -\frac{1}{4}\vec{v} \)

Ex 2. Given \( \vec{a} = 2\vec{i} - 3\vec{j} + \vec{k} \), \( \vec{b} = -\vec{i} + \vec{j} + 2\vec{k} \), write the following expressions in terms of the vectors \( \vec{i} \), \( \vec{j} \), and \( \vec{k} \).

a) \( \vec{a} + \vec{b} \)

b) \( 2\vec{a} - 3\vec{b} \)

C Vector Unit

An unit vector is a vector having a magnitude of 1. For any vector \( \vec{v} \), a unit vector parallel to \( \vec{v} \) is given by:

\[ \vec{u} = \frac{\vec{v}}{||\vec{v}||} \]

Ex 3. If \( \vec{x} \) and \( \vec{y} \) are two unit vectors with an angle of 30° between them, find the magnitude and direction of the vector \( 3\vec{x} - 5\vec{y} \).
Ex 4. Given $\| \vec{u} \| = 8m$ and $\| \vec{v} \| = 12m$, $\| \vec{u} + \vec{v} \| = 16$, determine the magnitude and the direction of the vector $2\vec{u} - 3\vec{v}$.

Reading: Nelson Textbook, Pages 293-298
Homework: Nelson Textbook: Page 298 #4, 9, 13, 15, 17, 18, 21, 22