# 6.2 Addition and Subtraction of Geometric Vectors

## A Addition of two Vectors
The **vector addition** \( \vec{s} \) of two vectors \( \vec{a} \) and \( \vec{b} \) is denoted by \( \vec{a} + \vec{b} \) and is called the **sum or resultant** of the two vectors. So:

\[
\vec{s} = \vec{a} + \vec{b}
\]

## B Triangle Rule (Tail to Tip Rule)
In order to find the sum (resultant) of two geometric vectors:

a) Place the second vector with its tail on the tip (head) of the first vector.

b) The sum (resultant) is a vector with the tail at the tail of the first vector and the head at the head of the second vector.

## C Polygon Rule
In order to find the sum (resultant) of \( n \) geometric vectors:

a) Place the next vector with its tail on the tip (head) of the precedent vector.

b) The sum (resultant) is a vector with the tail at the tail of the first vector and the head at the head of the last vector.

\[
\vec{s} = \vec{v}_1 + \vec{v}_2 + \ldots + \vec{v}_n
\]

Ex 1. Use the following diagram and the triangle rule compute the required operations.

- a) \( \vec{a} + \vec{b} \)
- b) \( \vec{b} + \vec{c} \)
- c) \( \vec{a} + \vec{c} \)

Ex 2. Use the following diagram and the triangle rule compute the required operations.

- a) \( \vec{a} + \vec{b} + \vec{c} \)
- b) \( \vec{b} + \vec{c} + \vec{d} \)
- c) \( \vec{a} + \vec{b} + \vec{c} + \vec{d} \)
### D Parallelogram Rule (Tail to Tail Rule)
To add two geometric vectors, the following rule can also be used:
- a) Position both vectors with their tails at the same point.
- b) Build a parallelogram using the vectors as two sides.
- c) The sum (resultant) is given by the diagonal of the parallelogram starting from the common tail point.

![Parallelogram Diagram]

Ex 3. Use the parallelogram rule to compute the required operations:

a) \( \vec{a} + \vec{b} \)

b) \( \vec{b} + \vec{c} \)

c) \( \vec{a} + \vec{c} \)

### E Sine Law
For any triangle \( \triangle ABC \)

The following relation is true:

\[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
\]

This is called the sine law.

### F Cosine Law
For any triangle \( \triangle ABC \)

The following relations are true:

\[
a^2 = b^2 + c^2 - 2bc \cos A \\
b^2 = a^2 + c^2 - 2ac \cos B \\
c^2 = a^2 + b^2 - 2ab \cos C
\]

Any of these is called the cosine law.

### G Magnitude and Direction for Vector Sum
Let \( \theta = \angle(\vec{a}, \vec{b}) \) be the angle between the vectors \( \vec{a} \) and \( \vec{b} \) when they are placed tail to tail.

The magnitude of the vector sum is given by:

\[
\| \vec{a} + \vec{b} \|^2 = \| \vec{a} \|^2 + \| \vec{b} \|^2 - 2\| \vec{a} \| \| \vec{b} \| \cos \theta
\]

\[
\gamma = 180^\circ - \theta
\]

The direction of the vector sum \( \vec{s} = \vec{a} + \vec{b} \) is defined by the angles \( \alpha \) and \( \beta \) formed by the vector sum and the vectors \( \vec{b} \) and \( \vec{a} \) respectively:

\[
\frac{\| \vec{a} \|}{\sin \alpha} = \frac{\| \vec{b} \|}{\sin \beta} = \frac{\| \vec{a} + \vec{b} \|}{\sin \gamma}
\]
### Ex 4. Analyze the following specific cases of addition of two vectors. Explain and make a diagram.

a) $\theta = 0$

b) $\theta = \pi / 2$

c) $\theta = \pi$

### Ex 5. Given the magnitude of two vectors $\| \vec{a} \| = 4$ and $\| \vec{b} \| = 7$, and the angle between them when placed tail to tail as being $\theta = 60^\circ$, find the magnitude of the vector sum $\vec{s} = \vec{a} + \vec{b}$ and the direction (the angles between the vector sum and each vector). Draw a diagram.

### Ex 6. Given $\| \vec{a} \| = 4$, $\| \vec{b} \| = 6$, and $\| \vec{a} + \vec{b} \| = 8$, find the angle between the vectors $\vec{a}$ and $\vec{b}$ (when placed tail to tail) $\theta = \angle(\vec{a}, \vec{b})$.

### Ex 7. Compute the required operations.

a) $\vec{a} - \vec{b}$

b) $\vec{b} - \vec{c}$

c) $\vec{a} - \vec{c}$

### H Vector Subtraction

The subtraction operation between two vectors $\vec{a} - \vec{b}$ can be understood as a vector addition between the first vector and the opposite of the second vector: $\vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b})$.
I Inverse Operation
The vector subtraction operation is the inverse operation of the vector addition:
\[ \vec{d} = \vec{a} - \vec{b} \iff \vec{a} = \vec{b} + \vec{d} \]

J Magnitude and Direction for Vector Difference
Let \( \vec{a} \) and \( \vec{b} \) be two vectors and \( \vec{d} = \vec{a} - \vec{b} \) be the vector difference. Let \( \theta \) be the angle between the vectors \( \vec{a} \) and \( \vec{b} \) when they are placed tail to tail:

The magnitude of the vector difference is given by:
\[ ||\vec{a} - \vec{b}||^2 = ||\vec{a}||^2 + ||\vec{b}||^2 - 2 ||\vec{a}|| ||\vec{b}|| \cos \theta \] (3)

The direction of the vector difference \( \vec{d} = \vec{a} - \vec{b} \) is defined by the angles \( \alpha \) and \( \beta \) formed by the vector sum and the vectors \( \vec{b} \) and \( \vec{a} \) respectively:
\[ \frac{||\vec{a}||}{\sin \alpha} = \frac{||\vec{b}||}{\sin \beta} = \frac{||\vec{a} - \vec{b}||}{\sin \theta} \] (4)

Ex 8. Given the magnitude of two vectors \( ||\vec{a}|| = 10 \) and \( ||\vec{b}|| = 14 \), and the angle between them when placed tail to tail as being \( \theta = 120^\circ \), find the magnitude of the vector difference \( \vec{d} = \vec{a} - \vec{b} \) and the direction (the angles between the vector sum and each vector). Draw a diagram.

Ex 9. Given \( ||\vec{a}|| = 10 \), \( ||\vec{b}|| = 15 \), and \( ||\vec{a} + \vec{b}|| = 20 \), find \( ||\vec{a} - \vec{b}|| \).

Ex 10. A cube is constructed from three vectors \( \vec{a} \), \( \vec{b} \), and \( \vec{c} \), as shown in the left figure. Express in terms of \( \vec{a} \), \( \vec{b} \), and \( \vec{c} \):
   a) \( \vec{AF} \)
   b) \( \vec{CF} \)
   c) \( \vec{AG} \)
   d) \( \vec{BH} \)

Reading: Nelson Textbook, Pages 282-289
Homework: Nelson Textbook: Page 290 #2, 5, 7, 12, 13, 14, 16

6.2 Addition and Subtraction of Geometric Vectors ©2010 Iulia & Teodoru Gugoiu - Page 4 of 4