6.2 Addition and Subtraction of Geometric Vectors

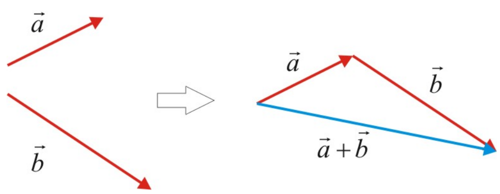
A Addition of two Vectors

The *vector addition* $\vec{s}$ of two vectors $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a}+\vec{b}$ and is called the *sum* or *resultant* of the two vectors. So:

$$\vec{s}=\vec{a}+\vec{b}$$

B Triangle Rule (Tail to Tip Rule)

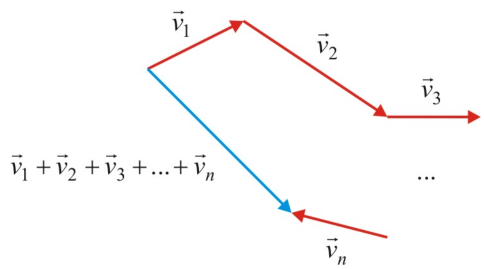
In order to find the sum (resultant) of two geometric vectors:
a) Place the second vector with its tail on the tip (head) of the first vector.
b) The sum (resultant) is a vector with the tail at the tail of the first vector and the head at the head of the second vector.



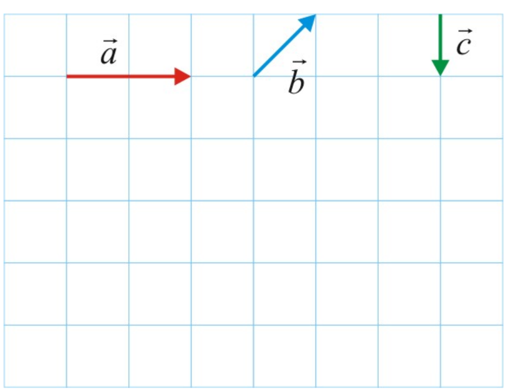
C Polygon Rule

In order to find the sum (resultant) of n geometric vectors:
a) Place the next vector with its tail on the tip (head) of the precedent vector.
b) The sum (resultant) is a vector with the tail at the tail of the first vector and the head at the head of the last vector.

$$\vec{s}=\overrightarrow{v_1}+\overrightarrow{v_2}+\ldots+\overrightarrow{v_n}$$

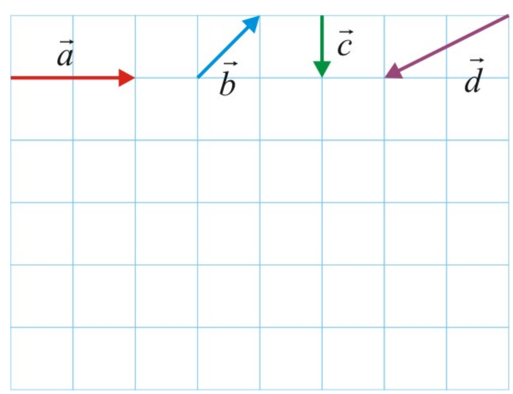


Ex 1. Use the following diagram and the triangle rule compute the required operations.



a) $\vec{a}+\vec{b}$ b) $\vec{b}+\vec{c}$ c) $\vec{a}+\vec{c}$

Ex 2. Use the following diagram and the triangle rule compute the required operations.



a) $\vec{a}+\vec{b}+\vec{c}$ b) $\vec{b}+\vec{c}+\vec{d}$ c) $\vec{a}+\vec{b}+\vec{c}+\vec{d}$

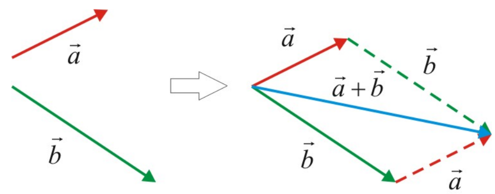
D Parallelogram Rule (Tail to Tail Rule)

To add two geometric vectors, the following rule can also be used:

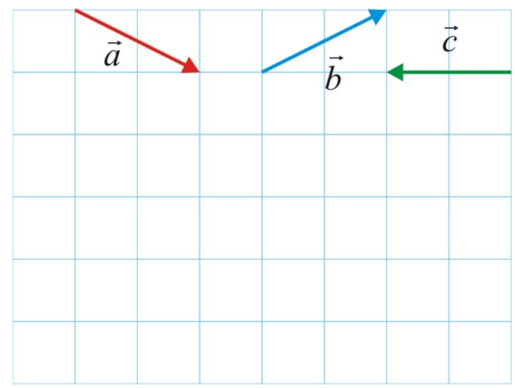
a) Position both vectors with their *tails* at the same point.

b) Build a *parallelogram* using the vectors as two sides.

c) The sum (resultant) is given by the *diagonal* of the parallelogram starting from the common tail point.



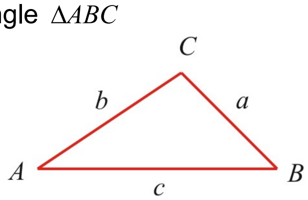
Ex 3. Use the parallelogram rule to compute the required operations:



a) $\vec{a}+\vec{b}$ b) $\vec{b}+\vec{c}$ c) $\vec{a}+\vec{c}$

E Sine Law

For any triangle ΔABC



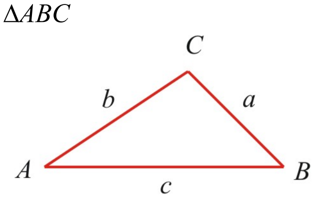
The following relation is true:

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$$

This is called the *sine law*.

F Cosine Law

For any triangle ΔABC



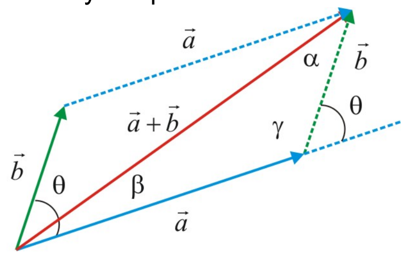
The following relations are true:

$$a^2=b^2+c^2-2bc\cos A$$
$$b^2=a^2+c^2-2ac\cos B$$
$$c^2=a^2+b^2-2ab\cos C$$

Any of these is called the *cosine law*.

G Magnitude and Direction for Vector Sum

Let $\theta=\angle(\vec{a},\vec{b})$ be the angle between the vectors $\vec{a}$ and $\vec{b}$ when they are placed tail to tail.



The magnitude of the vector sum is given by:

$$\|\vec{a}+\vec{b}\|^2=\|\vec{a}\|^2+\|\vec{b}\|^2-2\|\vec{a}\|\|\vec{b}\|\cos\gamma \qquad (1)$$
$$\gamma=180^\circ-\theta$$

The direction of the vector sum $\vec{s}=\vec{a}+\vec{b}$ is defined by the angles α and β formed by the vector sum and the vectors $\vec{b}$ and $\vec{a}$ respectively:

$$\frac{\|\vec{a}\|}{\sin\alpha}=\frac{\|\vec{b}\|}{\sin\beta}=\frac{\|\vec{a}+\vec{b}\|}{\sin\gamma} \qquad (2)$$

Ex 4. Analyze the following specific cases of addition of two vectors. Explain and make a diagram.

a) $\theta = 0$

b) $\theta = \pi / 2$

c) $\theta = \pi$

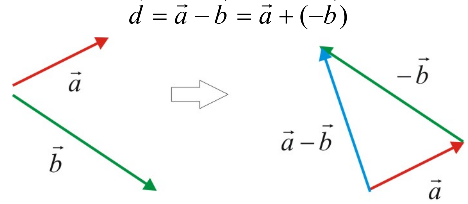
Ex 5. Given the magnitude of two vectors $\| \vec{a} \| = 4$ and $\| \vec{b} \| = 7$, and the angle between them when placed tail to tail as being $\theta = 60°$, find the magnitude of the vector sum $\vec{s} = \vec{a} + \vec{b}$ and the direction (the angles between the vector sum and each vector). Draw a diagram.

Ex 6. Given $\| \vec{a} \| = 4$, $\| \vec{b} \| = 6$, and $\| \vec{a} + \vec{b} \| = 8$, find the angle between the vectors $\vec{a}$ and $\vec{b}$ (when placed tail to tail) $\theta = \angle(\vec{a}, \vec{b})$.

H Vector Subtraction

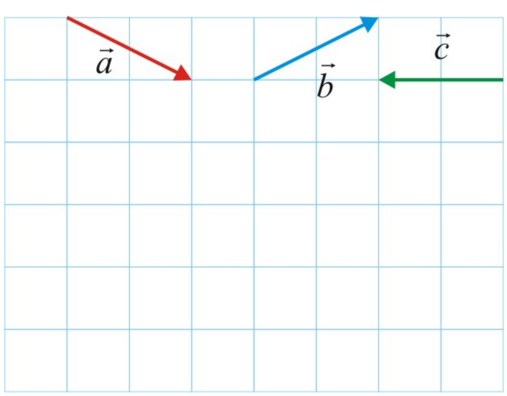
The subtraction operation between two vectors $\vec{a} - \vec{b}$ can be understood as a vector addition between the first vector and the opposite of the second vector:

$$\vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b})$$



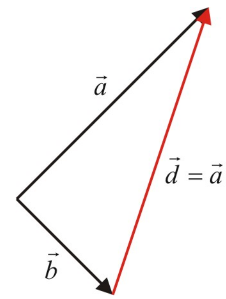
Ex 7. Compute the required operations.

a) $\vec{a} - \vec{b}$ b) $\vec{b} - \vec{c}$ c) $\vec{a} - \vec{c}$

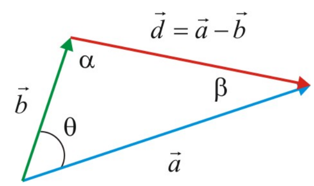


I Inverse Operation
The vector subtraction operation is the inverse o operation of the vector addition:

$\vec{d} = \vec{a} - \vec{b} \quad \Leftrightarrow \vec{a} = \vec{b} + \vec{d}$



J Magnitude and Direction for Vector Difference
Let $\vec{a}$ and $\vec{b}$ be two vectors and $\vec{d} = \vec{a} - \vec{b}$ be the vector difference. Let θ be the angle between the vectors $\vec{a}$ and $\vec{b}$ when they are placed tail to tail:



The magnitude of the *vector difference* is given by:

$$\| \vec{a} - \vec{b} \|^2 = \| \vec{a} \|^2 + \| \vec{b} \|^2 - 2 \| \vec{a} \| \| \vec{b} \| \cos\theta \quad (3)$$

The direction of the vector difference $\vec{d} = \vec{a} - \vec{b}$ is defined by the angles α and β formed by the vector sum and the vectors $\vec{b}$ and $\vec{a}$ respectively:

$$\frac{\| \vec{a} \|}{\sin\alpha} = \frac{\| \vec{b} \|}{\sin\beta} = \frac{\| \vec{a} - \vec{b} \|}{\sin\theta} \quad (4)$$

Ex 8. Given the magnitude of two vectors $\| \vec{a} \| = 10$ and $\| \vec{b} \| = 14$, and the angle between them when placed tail to tail as being $\theta = 120°$, find the magnitude of the vector difference $\vec{d} = \vec{a} - \vec{b}$ and the direction (the angles between the vector sum and each vector). Draw a diagram.

Ex 9. Given $\| \vec{a} \| = 10$, $\| \vec{b} \| = 15$, and $\| \vec{a} + \vec{b} \| = 20$, find $\| \vec{a} - \vec{b} \|$.

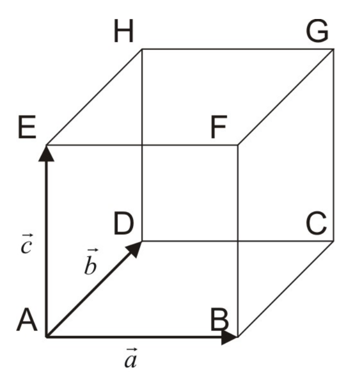
Ex 10. A cube is constructed from three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$, as shown in the left figure. Express in terms of $\vec{a}$, $\vec{b}$, and $\vec{c}$:

a) $\overrightarrow{AF}$

b) $\overrightarrow{CF}$

c) $\overrightarrow{AG}$

d) $\overrightarrow{BH}$



Reading: Nelson Textbook, Pages 282-289
Homework: Nelson Textbook: Page 290 #2, 5, 7, 12, 13, 14, 16

