

5A Derivative of Logarithmic Function

A Review of Logarithmic Function

$$y = b^x \Leftrightarrow x = \log_b y$$

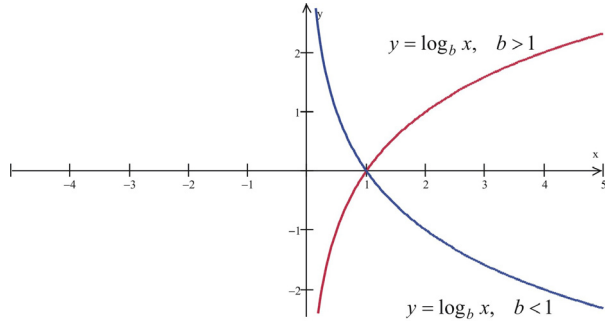
$$y = f(x) = \log_b x, \quad b > 0, b \neq 1, x > 0$$

$$\log_b(xy) = \log_b x + \log_b y \qquad \log_b x^n = n \log_b x$$

$$\log_b \frac{x}{y} = \log_b x - \log_b y \qquad \log_b x = \frac{\log_a x}{\log_a b}$$

$$\ln x = \log_e x \qquad \log_b 1 = 0$$

$$\log x = \log_{10} x \qquad \log_b b = 1$$



Ex 1. Use the graph of the logarithmic function to evaluate each limit.

a) $\lim_{x \rightarrow 0^+} \ln x$

b) $\lim_{x \rightarrow 0^+} \log_{0.5} x$

c) $\lim_{x \rightarrow \infty} \log x$

d) $\lim_{x \rightarrow \infty} \log_{0.1} x$

B Derivative of $\ln x$

$$(\ln x)' = \frac{1}{x} \qquad (1)$$

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

Proof:

$$y = \ln x \Rightarrow x = e^y \Rightarrow (x)' = (e^y)' \Rightarrow$$

$$1 = e^y y' \Rightarrow y' = \frac{1}{e^y} \Rightarrow y' = \frac{1}{x} \Rightarrow \therefore (\ln x)' = \frac{1}{x}$$

Ex 2. Differentiate and simplify.

a) $x^2 \ln x$

b) $\frac{\ln x}{x}$

c) $e^x \ln x$

C Derivative of $\ln f(x)$

Using (1) and the chain rule:

$$[\ln f(x)]' = \frac{f'(x)}{f(x)} \qquad (2)$$

$$\frac{d}{dx} \ln f(x) = \frac{f'(x)}{f(x)}$$

Ex 3. Differentiate and simplify.

a) $\ln(x^3 + x^2)$

b) $\ln \frac{x-1}{x+1}$

c) $\ln(e^x + e^{-x})$

<p>D Derivative of $\log_b x$</p> $(\log_b x)' = \frac{1}{(\ln b)x} \quad (3)$ $\frac{d}{dx} \log_b x = \frac{1}{(\ln b)x}$ <p>Proof:</p> $(\log_b x)' = \left(\frac{\ln x}{\ln b} \right)' = \frac{1}{\ln b} (\ln x)' = \frac{1}{(\ln b)x}$	<p>Ex 4. Differentiate.</p> <p>a) $\log x$</p> <p>b) $x^2 \log_3 x$</p> <p>c) $\frac{\log x}{10^x}$</p>
<p>E Derivative of $\log_b f(x)$</p> <p>Using (3) and the chain rule:</p> $[\log_b f(x)]' = \frac{f'(x)}{(\ln b)f(x)} \quad (4)$ $\frac{d}{dx} \log_b f(x) = \frac{f'(x)}{(\ln b)f(x)}$	<p>Ex 5. Differentiate.</p> <p>a) $\log(x^2 + 1)$</p> <p>b) $\log_2(x^2 2^x)$</p> <p>c) $\log \ln x$</p>
<p>Ex 6. Find the equation of the tangent line to the curve $y = f(x) = e^{-x} \ln x$ at the point $P(1,0)$.</p>	<p>Ex 7. Find local extrema points for $f(x) = \frac{\ln x}{x}$.</p>

Ex 8. Find the inflection points for $f(x) = x^2 \ln x$.

Ex 9. Find the global extrema for $f(x) = \frac{\log x}{x}$ over $[1,10]$.

Ex 10. Differentiate.

a) $y = x^x$

b) $y = \ln |x|$

Reading: Nelson Textbook, Pages 571-574

Homework: Nelson Textbook: Page 575 #3ef, 4ace, 5ab, 6abc, 9a, 10, 11

Reading: Nelson Textbook, Pages 576-577

Homework: Nelson Textbook: Page 578 #1ad, 2cf, 3a, 4bd, 5, 8, 10