5.4 5.5 Derivative of Trigonometric Functions

A Review of Trigonometric Functions

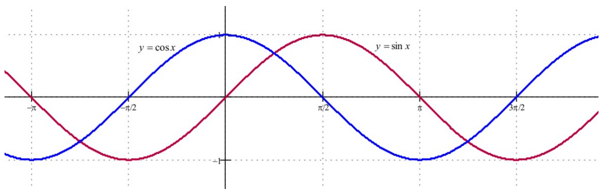
$\sin x : R \to [-1,1]$ $\quad$ $\sin(x+2\pi) = \sin x$

$\cos x : R \to [-1,1]$ $\quad$ $\cos(x+2\pi) = \cos x$

$\sin(x+\pi/2) = \cos x$ $\quad$ $\sin(x+\pi) = -\sin x$

$\sin(2x) = 2\sin x \cos x$ $\quad$ $\cos(2x) = \cos^2 x - \sin^2 x$

$\tan x = \dfrac{\sin x}{\cos x}$ $\quad$ $\tan x : R \setminus \left\{\dfrac{\pi}{2} + n\pi, n \in Z\right\} \to R$



Ex 1. Compute the following limits.

a) $\lim_{x\to\infty} \sin x$ (use the graph of $\sin x$)

b) $\lim_{x\to\pi/2} \tan x$

Ex 2. Use technology to evaluate each limit.

a) $\lim_{x\to 0} \dfrac{\sin x}{x}$ (*)

b) $\lim_{x\to 0} \dfrac{\cos x - 1}{x}$ (**)

B Derivative of $\sin x$

$$(\sin x)' = \cos x \qquad (1)$$

Proof:

$$(\sin x)' = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \sin x \lim_{h\to 0} \frac{\cos h - 1}{h} + \cos x \lim_{h\to 0} \frac{\sin h}{h}$$

Use now the limits (*) and (**):

$$(\sin x)' = \sin x(0) + \cos x(1) = \cos x$$

Ex 3. Differentiate.

a) $x^3 \sin x$

b) $\dfrac{e^x}{\sin x}$

c) $\ln \sin x$

C Derivative of $\sin f(x)$

Using (1) and chain rule:

$$[\sin f(x)]' = (\cos f(x)) f'(x) \qquad (2)$$

Ex 5. Differentiate.

a) $\sin x^2$

b) $\sin e^x$

c) $\sin \dfrac{1}{\ln x}$

D Derivative of $\cos x$

$$(\cos x)' = -\sin x \quad (3)$$

Proof:

$(\cos x)' = [\sin(x + \pi/2)]' = [\cos(x + \pi/2)](x + \pi/2)'$

$= -\sin x$

Ex 5. Differentiate.

a) $e^{-x}\cos x$

b) $\cos^4 x$

E Derivative of $\cos f(x)$

Using (3) and chain rule:

$$[\cos f(x)]' = -[\sin f(x)] f'(x) \quad (4)$$

Ex 6. Differentiate.

a) $\cos(\sin x)$

b) $\cos\frac{1}{x^2}$

c) $\cos(e^{x^2})$

F Derivative of $\tan x$

$$(\tan x)' = \frac{1}{\cos^2 x} = \sec^2 x$$

$$[\tan f(x)]' = \frac{f'(x)}{\cos^2 f(x)} = \sec^2 f(x) f'(x)$$

Proof:

$(\tan x)' = \left(\frac{\sin x}{\cos x}\right)' = \frac{(\sin x)'(\cos x) - (\sin x)(\cos x)'}{\cos^2 x}$

$= \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$= \frac{1}{\cos^2 x} = \sec^2 x$

Ex 7. Differentiate.

a) $x^3 \tan x$

b) $\cot x$

c) $\tan\sqrt{x^2+1}$

Ex 8. Find the equation of the tangent line to the graph of $y = f(x) = \sin x$ at $(\pi/4, 1/\sqrt{2})$.	Ex 9. Find the points of inflection for $f(x) = \cos^2 x$ over the interval $[0, \pi]$.
Ex 10. Find the global extrema for $f(x) = x + \cos x, \quad 0 \le x \le 2\pi$.	Ex 11. Consider the following position function: $s(t) = 5\sin\left(\frac{\pi t}{10}\right)$. Prove that: $a(t) + \frac{\pi^2}{100} s(t) = 0$.

Reading: Nelson Textbook, Pages 250-256
Homework: Nelson Textbook: Page 256 #1cfj, 2acd, 3b, 5ad, 6c, 11, 12, 14

Reading: Nelson Textbook, Pages 258-259
Homework: Nelson Textbook: Page 260 #1ade, 2b, 3abf, 4a, 6, 7, 8

