

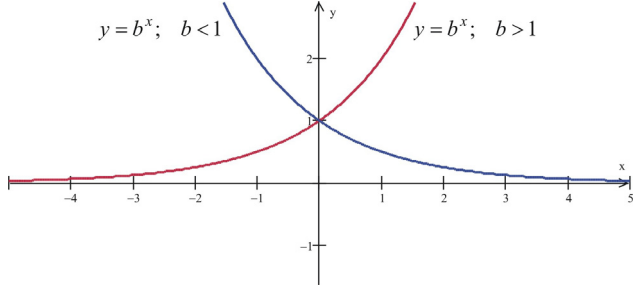
5.1 5.2 Derivative of Exponential Function

A Review of Exponential Functions

The exponential function is defined as:

$$y = f(x) = b^x; \quad b > 0, b \neq 1$$

The graph of the exponential function is represented below:



The x-axis ($y = 0$) is a horizontal asymptote.

Ex 1. Use the graph of the exponential function to evaluate each limit.

a) $\lim_{x \rightarrow \infty} 2^x$

b) $\lim_{x \rightarrow -\infty} 2^x$

c) $\lim_{x \rightarrow \infty} 0.2^x$

d) $\lim_{x \rightarrow -\infty} 0.2^x$

B Number e

The number e is defined by:

$$e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n \quad (1)$$

which can be written also as:

$$e = \lim_{u \rightarrow 0} (1 + u)^{\frac{1}{u}} \quad (2)$$

Ex 2. Estimate the number e using formula (1) and by taking $n = 100000$.

C Exponential Function

The exponential function e^x may be evaluate using the limit:

$$e^x = \lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n \quad (3)$$

Proof:

$$\begin{aligned} \lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n &= \lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^{\frac{n}{x} \cdot x} = \left(\lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^{\frac{n}{x}} \right)^x = \\ &= \left(\lim_{u \rightarrow 0} (1 + u)^{\frac{1}{u}} \right)^x = e^x \end{aligned}$$

Ex 3. Estimate \sqrt{e} using formula (2) and by taking $n = 100000$.

D Derivative of e^x

$$\begin{aligned} (e^x)' &= e^x \\ \frac{d}{dx} e^x &= e^x \quad (4) \end{aligned}$$

Proof:

$$\begin{aligned} \left(1 + \frac{x}{n}\right)^n \rightarrow e^x \Rightarrow n \left(1 + \frac{x}{n}\right)^{n-1} \left(\frac{1}{n}\right) \rightarrow (e^x)' \\ \left(\left(1 + \frac{x}{n}\right)^n\right)^{\frac{n-1}{n}} \rightarrow \left(1 + \frac{x}{n}\right)^{n-1} \rightarrow e^x \Rightarrow \therefore (e^x)' = e^x \end{aligned}$$

Ex 4. Differentiate and simplify.

a) $x^2 e^x$

b) $e^{x/2}$

c) e^{-x}

<p>E Derivative of $e^{f(x)}$ Using (4) and the chain rule:</p> $(e^{f(x)})' = e^{f(x)} f'(x)$ $\frac{d}{dx} e^{f(x)} = e^{f(x)} f'(x) \quad (5)$	<p>Ex 5. Differentiate.</p> <p>a) e^{-3x}</p> <p>b) e^{-1/x^2}</p> <p>c) $e^{\sqrt{x^2+1}}$</p>
<p>Ex 6. The hyperbolic functions are defined by: $\sinh x = \frac{e^x - e^{-x}}{2}$, $\cosh x = \frac{e^x + e^{-x}}{2}$, $\tanh x = \frac{\sinh x}{\cosh x}$. Prove that:</p> <p>a) $\cosh^2 x - \sinh^2 x = 1$</p> <p>b) $(\sinh x)' = \cosh x$</p>	<p>c) $(\cosh x)' = \sinh x$</p> <p>d) $(\tanh x)' = \frac{1}{\cosh^2 x}$</p>
<p>F Derivative of b^x, $b > 0, b \neq 1$</p> $(b^x)' = (\ln b)b^x$ $\frac{d}{dx} b^x = (\ln b)b^x \quad (6)$ <p>Proof: $(b^x)' = (e^{x \ln b})' = e^{x \ln b} (\ln b) = (\ln b)b^x$</p>	<p>Ex 7. Differentiate.</p> <p>a) 3^x</p> <p>b) $x^2 2^x$</p> <p>c) $(4^x + x^4)^3$</p>
<p>G Derivative of $b^{f(x)}$ Using (6) and the chain rule:</p> $(b^{f(x)})' = (\ln b)b^{f(x)} f'(x)$ $\frac{d}{dx} b^{f(x)} = (\ln b)b^{f(x)} f'(x) \quad (7)$	<p>Ex 8. Differentiate.</p> <p>a) 2^{-x^3}</p> <p>b) $10^{\sqrt{e^x - x^2}}$</p>

Ex 9. Find the equation of the tangent line to the graph of $y = f(x) = x(2^{-x})$ at $(0,0)$.

Ex 10. Find the local extrema for $y = f(x) = x^2 e^{-x^2}$.

Ex 11. Find the points of inflection for $y = f(x) = e^{-x^2}$.

Ex 12. Find the global extrema for $f(x) = x^3 10^{-x}$ over $[-1,2]$.

Reading: Nelson Textbook, Pages 227-232

Homework: Nelson Textbook: Page 232 #2ef, 3cdf, 4abc, 5a, 8, 10a, 13, 16, 17

Reading: Nelson Textbook, Pages 235-239

Homework: Nelson Textbook: Page 240 #1bd, 2abcd, 3, 4, 6, 8, 9