4.5 An Algorithm for Curve Sketching

A Algorithm for Curve Sketching **1. Domain** ⇨ $denominator \neq 0$ (rational functions) ⇨ $radicand \geq 0$ (even roots) ⇨ $logarithmic\ argument > 0$ (logarithmic functions) **2. Intercepts** ⇨ $f(x) = 0$ (x-intercepts or zeros) ⇨ $numerator = 0$ (for rational functions) ⇨ $y - \text{int} = f(0)$ (if exists) **3. Symmetry** ⇨ $f(-x) = f(x)$ (even functions are symmetric about the y-axis) ⇨ $f(-x) = -f(x)$ (odd functions are symmetric about the origin) ⇨ $f(x+T) = f(x)$ (periodic functions have cycles) **4. Asymptotes** ⇨ compute $\lim_{x \to \pm\infty} f(x)$ (horizontal asymptote) ⇨ compute $\lim_{x \to a} f(x)$ (vertical asymptote where a is a zero of the denominator but not of the numerator) ⇨ compute long division (to find the oblique asymptotes for rational functions) **5. First Derivative** ⇨ compute $f'(x)$ ⇨ find critical points ($f'(x) = 0$ or $f'(x)$ DNE) ⇨ create the sign chart for $f'(x)$ ⇨ find intervals of increase/decrease ⇨ find the local extrema (using first derivative test) and global extrema (if function is defined on a closed interval) **6. Second Derivative** ⇨ compute $f''(x)$ ⇨ find points where $f''(x) = 0$ or $f''(x)$ DNE ⇨ create the sign chart for $f''(x)$ ⇨ find points of inflection ⇨ find intervals of concavity upward/downward ⇨ check the local extrema using the second derivative test (if necessary) **7. Curve Sketching** ⇨ use broken lines to draw the asymptotes ⇨ plot x- and y- intercepts, extrema, and inflection points ⇨ draw the curve near the asymptotes ⇨ sketch the curve	Ex 1. Sketch the graph for $y = f(x) = 3x^5 - 5x^3$.

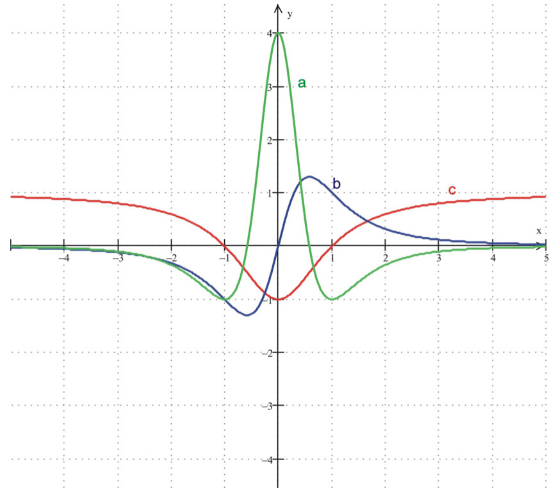
Ex 2. Sketch the graph for $y = f(x) = x^3 - 6x^2 + 9x + 1$.	Ex 3. Sketch the graph for $y = f(x) = \frac{4x}{x^2 + 1}$.

Ex 4. Sketch the graph for $y = f(x) = \frac{x^2}{x-1}$.	Ex 5. Sketch the graph for $y = f(x) = x(5-x)^{2/3}$.

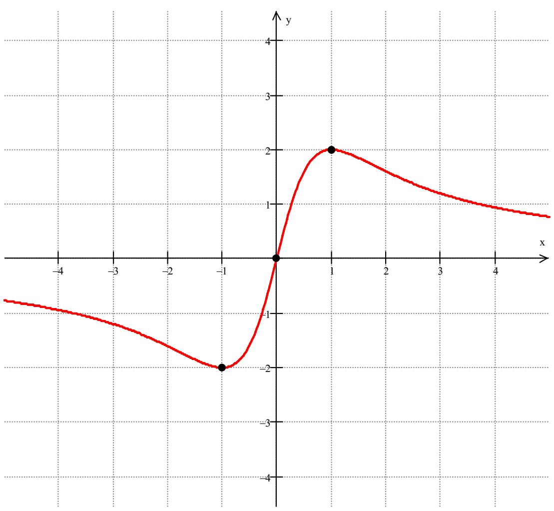
B Link between a function and its derivative
Consider a double differentiable function $y = f(x)$ ($f'(x)$ and $f''(x)$ exist). Then:

1. $f'(x)$ is the slope of the tangent at $P(x, f(x))$.
2. If $f'(x) = 0$, then $P(x, f(x))$ is a local extrema and tangent is horizontal.
3. If $f'(x) > 0$, then the function $y = f(x)$ is increasing.
4. If $f'(x) < 0$, then the function $y = f(x)$ is decreasing.
5. If $f''(x) = 0$, then $f'(x)$ has a local extrema and $y = f(x)$ has an inflection point.
6. If $f''(x) > 0$, then $f'(x)$ is increasing and $y = f(x)$ is concave upward.
7. If $f''(x) < 0$, then $f'(x)$ is decreasing and $y = f(x)$ is concave downward.

Ex 6. The graphs of a function and its first and second derivatives are represented on the same grid. Identify each of them.



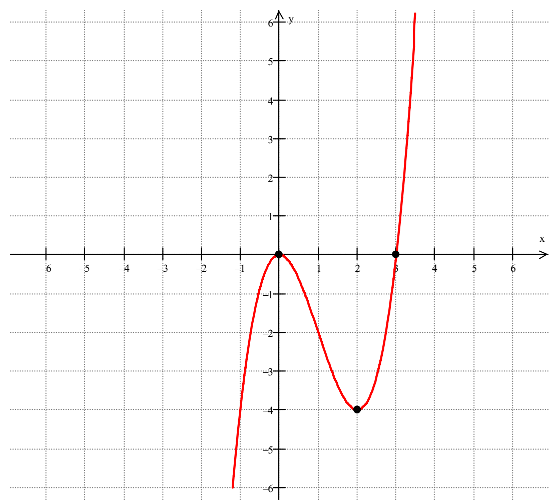
Ex 7. In the next figure is given the graph of a function $y = f(x)$.



a) Find the intervals where $f'(x)$ is positive and negative.

b) *Estimate* intervals where $f''(x)$ is positive and negative.

Ex 8. In the next figure is given the graph of the derivative $f'(x)$ of a function $f(x)$.



a) Find intervals where the function $f(x)$ is increasing or decreasing.

b) Find intervals where the graph of $f(x)$ is concave upward or downward.

Reading: Nelson Textbook, Pages 207-212
Homework: Nelson Textbook: Page 213 #4begij, 6, 7b, 9