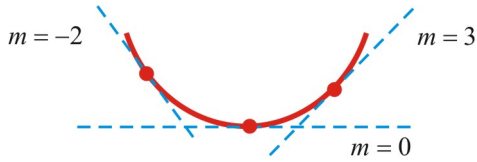


4.4 Concavity and Points of Inflection

A Concavity

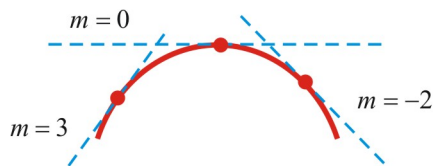
The graph of a function has a *concavity upward* if:

- Graph lies above all its tangents
- Tangents rotate counter-clockwise
- Slope of tangent lines increases
- $f'(x)$ *increases* or $f''(x) > 0$



The graph of a function has a *concavity downward* if:

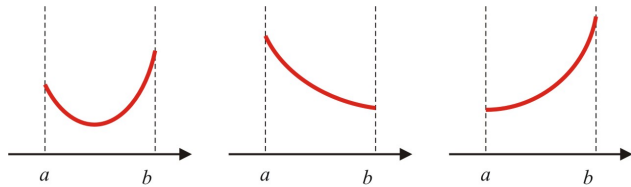
- Graph lies below all its tangents
- Tangents rotate clockwise
- Slope of tangent lines decreases
- $f'(x)$ *decreases* or $f''(x) < 0$



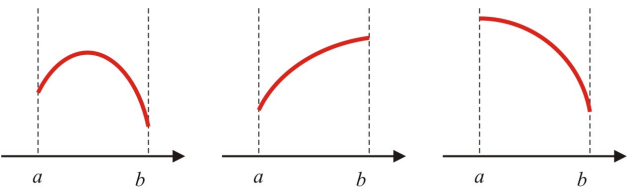
B Test for Concavity

Let f be a function *twice differentiable* ($f''(x)$ exists) over (a, b) .

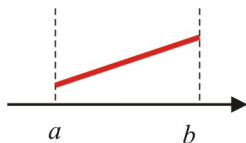
1. If $f''(x) > 0$ for all $x \in (a, b)$, then the graph of f is *concave upward* (has a *concavity upward*) over (a, b) .



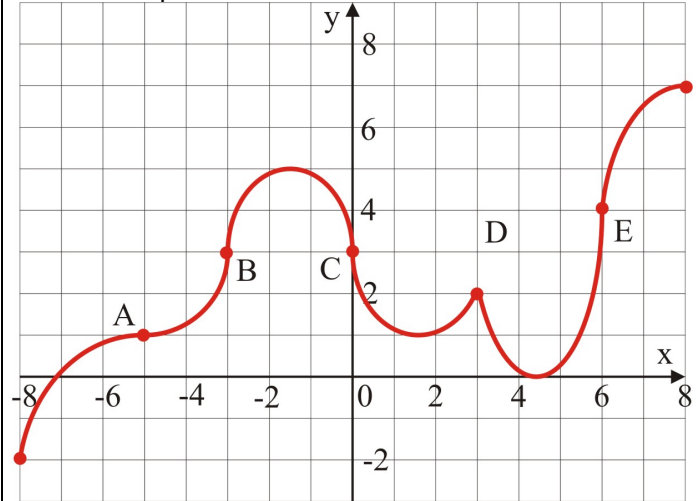
2. If $f''(x) < 0$ for all $x \in (a, b)$, then the graph of f is *concave downward* (has a *concavity downward*) over (a, b) .



3. 1. If $f''(x) = 0$ for all $x \in (a, b)$, then the graph of f has *no concavity* over (a, b) ($f'(x) = \text{const}$; the graph is a straight line).



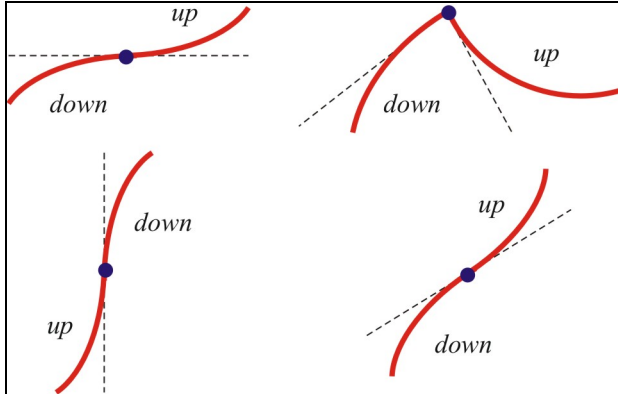
Ex 1. Find the intervals on which the graph, given below, is concave upward or downward.



Ex 2. Find the intervals of concavity for $f(x) = x^4 - 2x^3$.

C Point of Inflection

A point $P(i, f(i))$ on the graph of $y = f(x)$ is called *point of inflection* if the concavity of the graph changes at P (from concave upward to concave downward or from concave downward to concave upward).



Ex 2. Use the function given at Ex. 1 to identify the points of inflection.

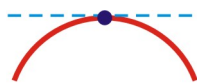
Ex 3. Use the sign chart obtained at Ex 2. to identify the points of inflection for $f(x) = x^4 - 2x^3$.

Ex 4. Find the intervals of concavity and the inflection points for $f(x) = \frac{x^2}{x^2 + 1}$.

D Second Derivative Test

Let f be a *twice differentiable* function over an open interval containing the *critical number* c and $f'(c) = 0$ ($(c, f(c))$ is a *stationary point*).

- If $f''(c) > 0$ then f has a *local minimum* at $x = c$.
- If $f''(c) < 0$ then f has a *local maximum* at $x = c$.

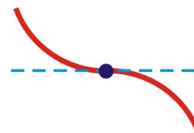


$f'(c) = 0$
 $f''(c) < 0$
 (maximum point)



$f'(c) = 0$
 $f''(c) > 0$
 (minimum point)

3. If $f''(c) = 0$ then the function may have a local minimum, maximum, or neither (inconclusive case). Use the first derivative test to conclude.



$f'(c) = 0$
 $f''(c) = 0$
 (no local extremum)



$f'(c) = 0$
 $f''(c) = 0$
 (minimum point)

Ex 6. Use the second derivative test to find the local extrema. If the second derivative test is not conclusive (fails), then use the first derivative test to conclude.

a) $f(x) = (x-1)^3$

b) $f(x) = (3-2x)^4$

Ex 5. Use the second derivative test to find the local extrema for $f(x) = \frac{x}{x^2+1}$.

Reading: Nelson Textbook, Pages 198-204

Homework: Nelson Textbook: Page 205 #2, 3, 5, 8iab, 9, 10, 11, 12