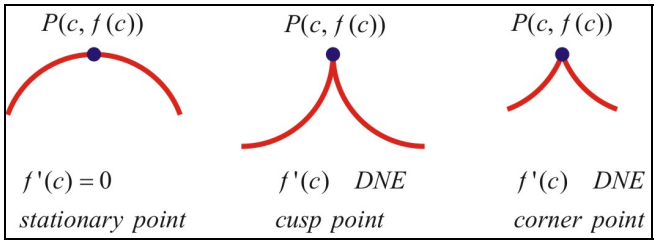
4.2 Critical Points. Local Maxima and Minima

A Local Maximum

A function f has a *local (relative) maximum* at $x = c$ if $f(x) \le f(c)$ when x is sufficiently close to c (from both sides).

$f(c)$ is called the local (relative) maximum *value*.

$(c, f(c))$ is called the local (relative) maximum *point*.

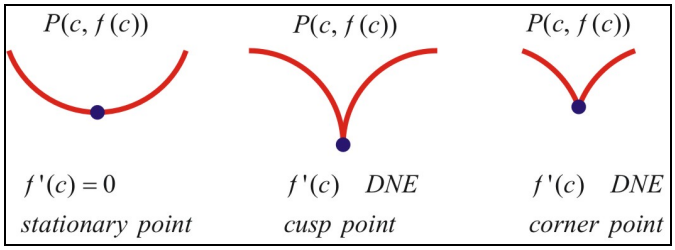


B Local Minimum

A function f has a *local (relative) minimum* at $x = c$ if $f(x) \ge f(c)$ when x is sufficiently close to c (from both sides).

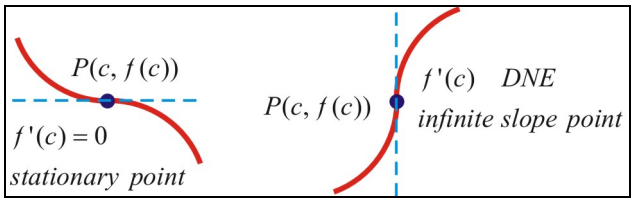
$f(c)$ is called the local (relative) minimum *value*.

$(c, f(c))$ is called the local (relative) minimum *point*.

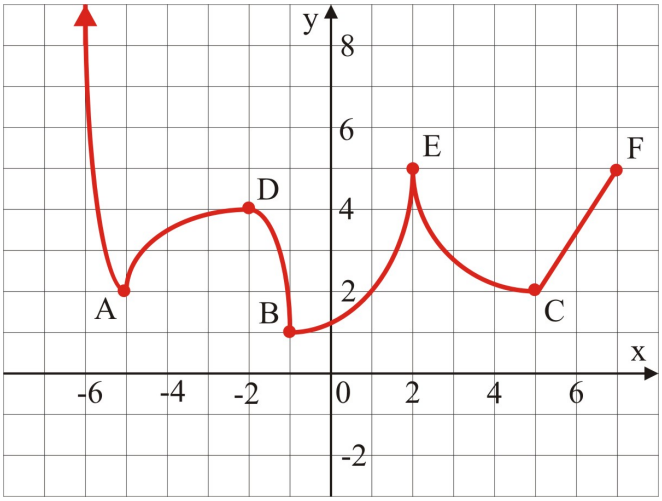


C Note

The following points are neither local minimum or maximum points.



Ex 1. A function is defined by the following graph. Find the local minimum or maximum points.

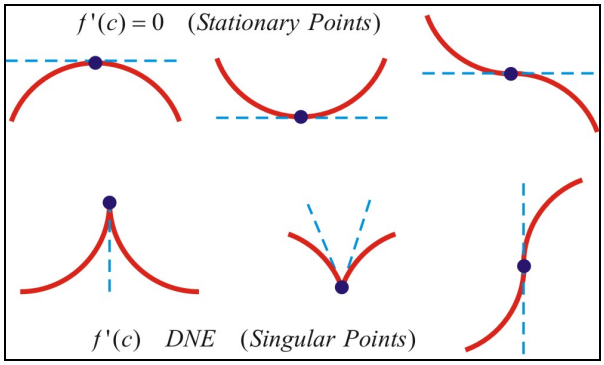


D Critical Numbers and Critical Points

The number $c \in D_f$ is a *critical number* if either

$$f'(c) = 0 \text{ or } f'(c) \text{ DNE}$$

The point $P(c, f(c))$ is called *critical point*.



Ex 2. Find the critical points for:

a) $f(x) = 2x^3 + 3x^2$

b) $f(x) = \sqrt[3]{x^2 - 1}$

E Fermat's Theorem

If f has a local extremum (minimum or maximum) at $x=c$ then c is a critical number for f. So, at a local extremum:

$$f'(c)=0 \text{ or } f'(c)\ DNE$$

Note.

If

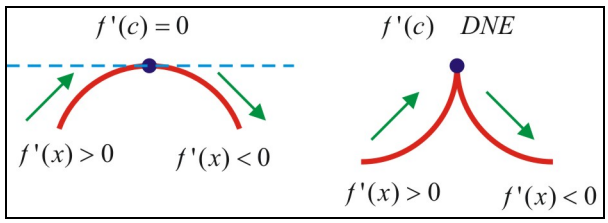
$$f'(c)=0 \text{ or } f'(c)\ DNE$$

then the function f may have or not a local extremum (minimum or maximum) at $x=c$.

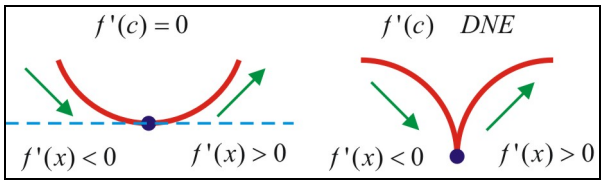
F First Derivative Test

Let c be a critical number of a *continuous* function f.

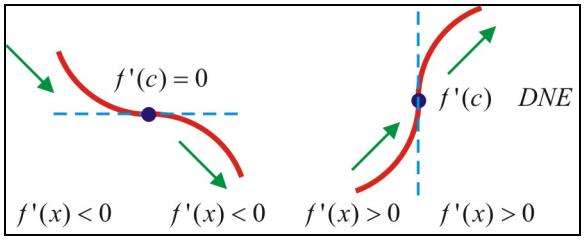
If $f'(x)$ changes sign from positive to negative at c then f has a *local maximum* at c.



If $f'(x)$ changes sign from negative to positive at c then f has a *local minimum* at c.



If $f'(x)$ does not change sign at c then f has no local extremum (minimum or maximum) at c.



Ex 3. Find the local extrema points for the functions in the previous example.

a) $f(x)=2x^3+3x^2$

b) $f(x)=\sqrt[3]{x^2-1}$

Ex 4. Find a function of the form $f(x)=ax^4+bx^2+cx+d$ with a local maximum at $(0,-6)$ and a local minimum at $(1,-8)$.

Reading: Nelson Textbook, Pages 172-178
Homework: Nelson Textbook: Page 178 #3ab, 5ad, 7adf, 10, 13, 15

