3.3 3.4 Optimization

A Algorithm for Solving Optimization Problems

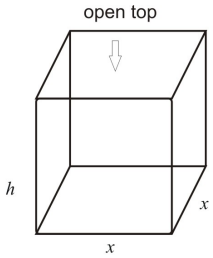
1. Read and understand the problem's text.
2. Draw a diagram (if necessary).
3. Assign variables to the quantities involved and state restrictions according to the situation.
4. Write relations between these variables.
5. Identify the variable that is minimized or maximized. This is the dependant variable.
6. Use the other relations (called constraints) to express the dependent variable (the one which is minimized or maximized) as a function of one single variable (the independent variable).
7. Find extrema (maximum or minimum) for the dependant variable (using global extrema algorithm, first derivative test or the second derivative test).
8. Check if extrema satisfy the conditions of the application.
9. Find the value of other variables at extrema (if necessary).
10. Write the conclusion statement.

B Optimization Problems Involving Numbers

Ex 1. Find two positive numbers with a product equal to 200 such that the sum of one number and twice the other number is as small as possible. What is the minimum value of the sum?

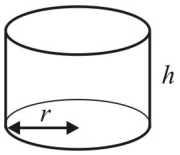
C Maximize the Volume given the Shape and Area

Ex 2. If $2700cm^2$ of material is available to make a box with *a square base and an open top*, find the dimensions (length, width, and height) of the box that give the largest volume of the box. What is the maximum volume of the box?



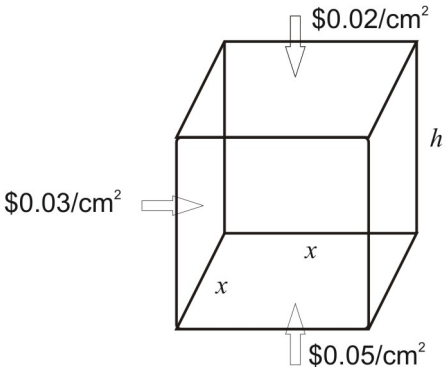
D Minimize the Area given the Shape and Volume

Ex 3. A cylindrical can is to be made to hold $1000cm^3$ (one litre) of oil. Find the dimensions (radius and height) of the can that will minimize the cost of the metal to make the can.



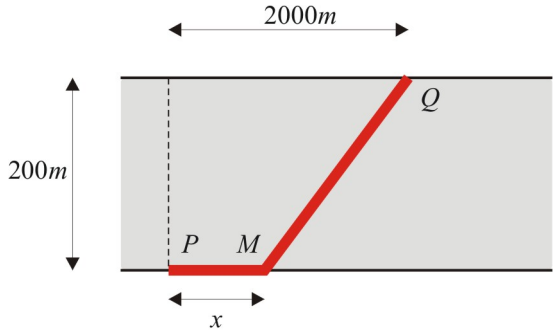
E Minimize the Cost given the Shape and Volume

Ex 4. A closed box with a square base is to contain $252cm^3$. The bottom costs $\$0.05/cm^2$, the top costs $\$0.02/cm^2$ and the sides costs $\$0.03/cm^2$. Find the dimensions (base side and height) that will minimize the cost.



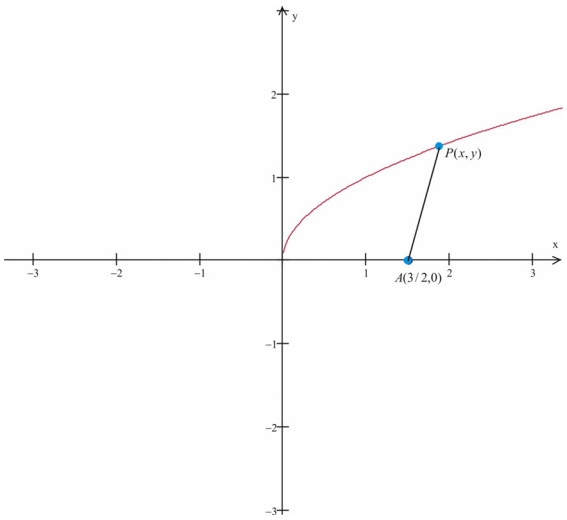
F Minimize the Cost for an Underground/Underwater Cable

Ex 5. A cable television company is laying cable in an area with underground utilities. Two subdivisions are located on opposite sides of a $200m$ wide river. The company has to connect points P and Q with cable, where P is on the South bank and Q is on the North bank $2000m$ East of P. It cost $\$8/m$ to lay cable underground and $\$10/m$ to lay cable underwater.
What is the least expensive way to lay the cable and what is the minimum cost?

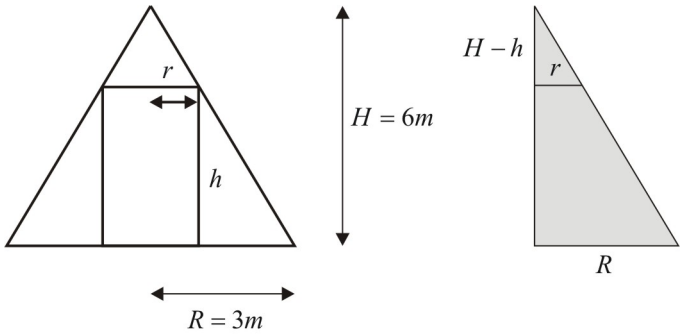


G Geometry

Ex 6. How close does the curve $y = \sqrt{x}$ come to the point $(3/2,0)$?



Ex 7. Find the dimensions of the largest right-cylinder that can be inscribed in a cone of radius $R = 3m$ and height $H = 6m$.



Reading: Nelson Textbook, Pages 141-144 (Optimization Problems)
Homework: Nelson Textbook: Page 145 #1, 4, 7, 10, 16, 20
Reading: Nelson Textbook, Pages 148-150 (Optimization Problems in Economics and Science)
Homework: Nelson Textbook: Page 151 #5, 10, 12, 17