

3.2 Maximum and Minimum on an Interval. Extreme Values

A Global Maximum

A function f has a *global (absolute) maximum* at $x = c$ if $f(x) \leq f(c)$ for all $x \in D_f$.

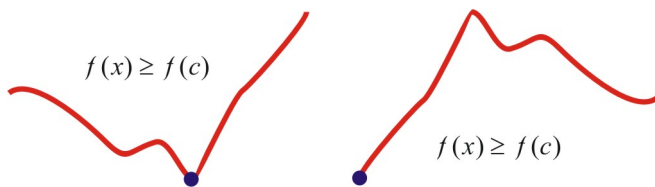
$f(c)$ is called the *global (absolute) maximum value*.
 $(c, f(c))$ is called the *global (absolute) maximum point*.



B Global Minimum

A function f has a *global (absolute) minimum* at $x = c$ if $f(x) \geq f(c)$ for all $x \in D_f$.

$f(c)$ is called the *global (absolute) minimum value*.
 $(c, f(c))$ is called the *global (absolute) minimum point*.



C Extremum and Extrema

An *extremum* is either a minimum or a maximum (value, point, local or global).

Extrema is the plural of extremum.

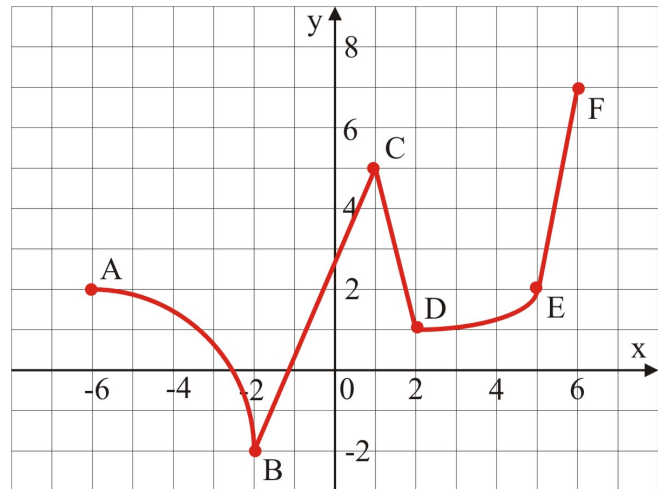
D Global (Absolute) Extrema Algorithm

To find the global (absolute) extrema for a *continuous* function f over a closed interval $[a, b]$:

- 1) identify the *critical* numbers over (a, b)
- 2) find the *values* of the function $f(c)$ at each critical number c in (a, b)
- 3) find the *values* $f(a)$ and $f(b)$
- 4) from the values obtained at part 2) and 3):
 - the *largest* represents the *global (absolute) maximum* value
 - the *least* represents the *global (absolute) minimum* value

Note. c is a critical number if either $f'(c) = 0$ or $f'(c)$ DNE

Ex 1. Find extrema for the function represented in the figure below by its graph.



Ex 2. Find extrema for $f(x) = 3x^4 - 4x^3$ over $[-1, 2]$.

Ex 3. Find extrema for $f(x) = \frac{x^2 - 1}{x^2 + 1}$ over $[-1, 3]$.

Ex 4. Find extrema for $f(x) = (x + 1)^2 \sqrt[3]{x - 1}$ over $[0, 1/2]$.

Reading: Nelson Textbook, Pages 130-134

Homework: Nelson Textbook: Page 136 #3ace, 4abe, 7ab, 10, 11