

3.1 Higher Order Derivatives. Velocity and Acceleration

A Higher Order Derivatives

Let consider the function $y = f(x)$.

The *first* derivative of f or “ f prime” is:

$$f'(x) = y' = \frac{dy}{dx}$$

The *second* derivative of f or “ f double prime” is:

$$f''(x) = (f'(x))' = y'' = (y')' = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d^2y}{dx^2}$$

The *third* derivative of f or “ f triple prime” is:

$$f'''(x) = (f''(x))' = y''' = (y'')' = \frac{d}{dx} \left(\frac{d^2y}{dx^2} \right) = \frac{d^3y}{dx^3}$$

The *fourth* derivative of f or “super 4” is:

$$f^{(4)}(x) = (f'''(x))' = y^{(4)} = (y''')' = \frac{d}{dx} \left(\frac{d^3y}{dx^3} \right) = \frac{d^4y}{dx^4}$$

The *n-th* derivative of f or “super n” is:

$$f^{(n)}(x) = (f^{(n-1)}(x))' = y^{(n)} = (y^{(n-1)})' = \frac{d}{dx} \left(\frac{d^{n-1}y}{dx^{n-1}} \right) = \frac{d^n y}{dx^n}$$

Notes.

1. f is *differentiable* at x if $f'(x)$ exists.
2. f is *double differentiable* at x if $f''(x)$ exists.
3. f is *n differentiable* at x if $f^{(n)}(x)$ exists.

Ex 1. Find f' , f'' , f''' , $f^{(4)}$, and $f^{(5)}$ for

$$f(x) = -x^4 + 3x^2 - x + 1.$$

Ex 2. Find f' , f'' , and f''' for $f(x) = \frac{x^3}{x^2 + 1}$.

Ex 3. Find $f^{(n)}$ for $f(x) = \frac{x}{x+1}$.

Ex 4. Show that $y = x^3 + 3x + 1$ satisfies $y''' + xy'' - 2y' = 0$.

B Velocity and Acceleration

Let $s(t)$ or $h(t)$ (height or altitude) be the *position function* of a particle.

$$\langle s \rangle_{IS} = \langle h \rangle_{IS} = m \text{ (meter)}$$

Displacement is the change in position over a time interval $[t_1, t_2]$:

$$\Delta s = s(t_2) - s(t_1) \quad \Delta h = h(t_2) - h(t_1)$$

Average Velocity is defined by:

$$AV = \frac{\Delta s}{\Delta t} = \frac{s(t_2) - s(t_1)}{t_2 - t_1} \text{ (m/s)}$$

Velocity function is the derivative of the position function:

$$v(t) = s'(t) = h'(t)$$

Speed function is the absolute value of velocity $|v(t)|$.

Acceleration function is the derivative of velocity:

$$a(t) = v'(t) = s''(t) \text{ (m/s}^2\text{)}$$

Jerk function is the derivative of acceleration:

$$j(t) = a'(t) = v''(t) = s'''(t) \text{ (m/s}^3\text{)}$$

Notes:

$s(t) = 0$ (particle is in the origin)

$h(t) = 0$ (particle is on the ground)

$s(t) > 0$ (particle is on the positive side of the origin)

$s(t) < 0$ (particle is on the negative side of the origin)

$h(t) > 0$ (particle is above the ground)

$h(t) < 0$ (particle is below the ground)

$v(t) = 0$ (particle is at rest, changes direction of movement)

$v(t) > 0$ (particle moves in the positive direction (right or up))

$v(t) < 0$ (particle moves in the negative direction (left or down))

$a(t) = 0$ (particle is not accelerated)

If $|s|$ is increasing then the particle is *moving away from the origin* (MAFO) and $s(t)v(t) > 0$.

If $|s|$ is decreasing then the particle is *moving towards the origin* (MTO) and $s(t)v(t) < 0$.

If $|v|$ is increasing then the particle is *speeding up* (SU) and $v(t)a(t) > 0$.

If $|v|$ is decreasing then the particle is *slowing down* (SD) and $v(t)a(t) < 0$.

Ex 4. Consider the following position function

$$s(t) = \frac{t^3 - 12t}{4}$$

a) Graph on the same grid $s(t)$, $v(t)$, $a(t)$.

b) Find the interval(s) when the particle is moving towards the origin.

c) Find the interval(s) when the particle is speeding up.

Homework:

a) $s(t) = (t^2 - 4)^2$ b) $s(t) = t^2(t^2 - 1)$

c) $s(t) = t^4 - 16$ d) $s(t) = t^3(t^2 - 1)$

e) $s(t) = t(t^4 - 1)$ f) $s(t) = t^4 - 2t^2$

Reading: Nelson Textbook, Pages 119-126

Homework: Nelson Textbook: Page 127 #2def, 5, 3be, 10, 11, 14, 18