2.4 Quotient Rule

A Quotient Rule

If f and g are differentiable at x and $g(x) \neq 0$ then so is $\frac{f}{g}$ and:

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{[f(x)]^2}$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{[g(x)]^2}$$

$$\frac{d}{dx}\frac{u}{v} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Proof:

$$\left(\frac{f}{g}\right)' = (fg^{-1})' = f'g^{-1} + f(-1)g^{-2}g'$$

$$= \frac{f'}{g} - \frac{fg'}{g^2} = \frac{f'g - fg'}{g^2}$$

Ex 1. Differentiate. Simplify the answer.

$$f(x) = \frac{x^2 - 1}{x^3 + 1}$$

Ex 2. Given that $f(2) = 1$, $f'(2) = -1$, $g(2) = 2$, and $g'(2) = -2$ find $\left(\frac{g}{f+g}\right)'(2)$.

Ex 3. Let $f(x) = \frac{x^3}{(1+x)^2}$. Find the point(s) on the graph of $y = f(x)$ where the tangent line is horizontal.

Ex 4. Consider the position function $s(t)=\frac{\sqrt{t}}{t^2+1}, t\geq 0$.

Find the moment(s) of time when the particle is at rest.

Ex 5. Let $y=f(x)=\frac{\sqrt[3]{x^2}}{x^2+1}$.

a) Differentiate. Simplify the answer.

b) Find the points where the function is not differentiable.

c) Find the numbers x where the tangent line is horizontal.

d) Use technology to graph the function.

Reading: Nelson Textbook, Pages 94-95
Homework: Nelson Textbook: Page 95 #4f, 5b, 8, 9a, 12, 14, 15, 16