2.4 Quotient Rule

**A Quotient Rule**

If \( f \) and \( g \) are differentiable at \( x \) and \( g(x) \neq 0 \) then so is \( \frac{f}{g} \) and:

\[
\left( \frac{f}{g} \right)' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}
\]

\[
\left( \frac{f}{g} \right)' = \frac{f'g - fg'}{g^2}
\]

\[
\frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{g(x) \frac{d}{dx} f(x) - f(x) \frac{d}{dx} g(x)}{[g(x)]^2}
\]

\[
\frac{d}{dx} u = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}
\]

**Proof:**

\[
\left( \frac{f}{g} \right)' = (fg^{-1})' = f'g^{-1} + f(-1)g^{-2}g'
\]

\[
= \frac{f'g - fg'}{g^2}
\]

**Ex 1.** Differentiate. Simplify the answer.

\[ f(x) = \frac{x^2 - 1}{x^3 + 1} \]

**Ex 2.** Given that \( f(2) = 1 \), \( f'(2) = -1 \), \( g(2) = 2 \), and \( g'(2) = -2 \) find \( \left( \frac{g}{f + g} \right)'(2) \).

**Ex 3.** Let \( f(x) = \frac{x^3}{(1+x)^2} \). Find the point(s) on the graph of \( y = f(x) \) where the tangent line is horizontal.
Ex 4. Consider the position function \( s(t) = \frac{\sqrt{t}}{t^2 + 1}, t \geq 0 \).
Find the moment(s) of time when the particle is at rest.

Ex 5. Let \( y = f(x) = \frac{\sqrt{x^2}}{x^2 + 1} \).

a) Differentiate. Simplify the answer.

b) Find the points where the function is not differentiable.

c) Find the numbers \( x \) where the tangent line is horizontal.

d) Use technology to graph the function.

**Reading:** Nelson Textbook, Pages 94-95

**Homework:** Nelson Textbook: Page 95 #4f, 5b, 8, 9a, 12, 14, 15, 16