

**2.3 Product Rule**

<p><b>A Product Rule</b> If <math>f</math> and <math>g</math> are differentiable at <math>x</math> then so is <math>fg</math> and:</p> $(fg)'(x) = f'(x)g(x) + f(x)g'(x)$ $(fg)' = f'g + fg'$ $\frac{d}{dx}[f(x)g(x)] = g(x)\frac{d}{dx}f(x) + f(x)\frac{d}{dx}g(x)$ $\frac{d}{dx}(uv) = v\frac{d}{dx}u + u\frac{d}{dx}v$	<p>Ex 1. Use the product rule to prove the rule <math>(cf)' = cf'</math> where <math>c</math> is a constant and <math>f</math> is a function.</p> <p>Ex 2. Differentiate using the product rule: <math>y = (x^2 + 1)(2x^3 - x)</math>.</p>
<p>Ex 3. Find the equation of the tangent line to the curve <math>y = (x + \sqrt{x})\left(x^2 + \frac{1}{x}\right)</math> at the point <math>P(1,4)</math>.</p>	<p>Ex 4. Consider the function <math>y = f(x) = \sqrt[3]{x}(x-1)</math>.</p> <p>a) Differentiate and simplify.</p> <p>b) Find the numbers where this function is not differentiable. Explain.</p> <p>c) Find the numbers <math>x</math> where the tangent line is horizontal.</p>

<p><b>B Product of three functions</b> If <math>f</math>, <math>g</math> and <math>h</math> are differentiable at <math>x</math> then so is <math>fgh</math> and:</p> $(fgh)' = f'gh + fg'h + fgh'$ <p>Proof:  <math>(fgh)' = (fg)'h + fgh' = (f'g + fg')h + fgh'</math>  <math>= f'gh + fg'h + fgh'</math></p>	<p>Ex 5. Differentiate. Do not simplify.</p> $y = (\sqrt{x} + 1)(x - x^2) \left(1 - \frac{1}{x}\right)$
<p><b>C Generalized Power Rule</b> If <math>f</math> is differentiable at <math>x</math>, then so is <math>f^n</math> and:</p> $([f(x)]^n)' = n[f(x)]^{n-1} f'(x)$ $(f^n)' = n f^{n-1} f'$ $\frac{d}{dx} [f(x)]^n = n[f(x)]^{n-1} \frac{d}{dx} f(x)$ <p>Proof for <math>n = 3</math>.</p> $(fgh)' = f'gh + fg'h + fgh'$ <p>If <math>f \equiv g \equiv h</math> then:  <math>(fff)' = f'ff + ff'f + fff'</math> or <math>(f^3)' = 3f^2 f'</math>.</p>	<p>Ex 6. Differentiate <math>y = (x^2 - 3x)^{15}</math>.</p> <p>Ex 7. Given <math>s(t) = t^2(2 - 3t)^3</math> find the velocity at <math>t = 1</math>.</p>

**Reading:** Nelson Textbook, Pages 85-90

**Homework:** Nelson Textbook: Page 91 #1b, 2d, 5e, 8b, 10, 12, 13