

1.5 Properties of Limits

<p>A Limits Properties We assume that $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ exist. Then:</p> <ol style="list-style-type: none"> $\lim_{x \rightarrow a} k = k$ $\lim_{x \rightarrow a} x = a$ $\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$ $\lim_{x \rightarrow a} [cf(x)] = c \lim_{x \rightarrow a} f(x)$ $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}, \quad \lim_{x \rightarrow a} g(x) \neq 0$ $\lim_{x \rightarrow a} [f(x)g(x)] = [\lim_{x \rightarrow a} f(x)][\lim_{x \rightarrow a} g(x)]$ $\lim_{x \rightarrow a} [f(x)]^n = [\lim_{x \rightarrow a} f(x)]^n$ If $P(x)$ is a polynomial function, then $\lim_{x \rightarrow a} P(x) = P(a)$ 	<p>Ex 1. Given $\lim_{x \rightarrow 3} f(x) = -2$ and $\lim_{x \rightarrow 3} g(x) = 1$, use the limits properties to find $\lim_{x \rightarrow 3} \frac{2f(x) + g(x)}{-4\sqrt{g(x)}}$.</p>
<p>B Substitution Substitution is the best strategy to find a limit. Note: Substitution does not work if (by substitution) you get one of the following 7 <i>indeterminate cases</i>:</p> $\infty - \infty \quad 0 \times \infty \quad \frac{0}{0} \quad \frac{\infty}{\infty} \quad 1^\infty \quad \infty^0 \quad 0^0$ <p>Specific strategies are available to avoid an indeterminate case to appear.</p>	<p>Ex 2. Compute $\lim_{x \rightarrow 2} \frac{x^2 - 2x + 1}{x - 1}$.</p>
<p>C Factoring The indeterminate form $\frac{0}{0}$ may be eliminated by factoring and <i>canceling out the common factor</i> that generates zeros:</p> $\lim_{x \rightarrow a} \frac{F(x)}{G(x)} = \lim_{x \rightarrow a} \frac{(x-a)f(x)}{(x-a)g(x)} = \lim_{x \rightarrow a} \frac{f(x)}{g(x)}$ <p>Note: Canceling out the common factor $(x-a)$ is a correct operation because $\lim_{x \rightarrow a}$ means that x approaches a but is not equal to a.</p>	<p>Ex 3. Compute $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x - 1}$.</p>
<p>D Conjugate Radicals When dealing with the indeterminate form $\frac{0}{0}$ you may use the <i>conjugate radicals</i> to cancel out the common factor that generates zeros.</p>	<p>Ex 4. Compute $\lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x}$.</p>

<p>E Change of Variables By changing the variable, the process of canceling the common factor may be simplified.</p> <p>Note. If the change of variable is $u = g(x)$ then: as $x \rightarrow a$, $u \rightarrow g(a)$</p>	<p>Ex 5. Change the variable to compute $\lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{\sqrt[3]{x} - 1}$.</p>
<p>F Absolute Value Function When dealing with an absolute value function, rewrite it as piece-wise defined function according to:</p> $ f(x) = \begin{cases} f(x), & f(x) \geq 0 \\ -f(x), & f(x) < 0 \end{cases}$	<p>Ex 6. Compute each limit. Draw a diagram to illustrate.</p> <p>a) $\lim_{x \rightarrow 0} \frac{ x }{x}$</p>
<p>b) $\lim_{x \rightarrow 0} x x$</p>	<p>c) $\lim_{x \rightarrow 0} \frac{x^2}{ x }$</p>

Reading: Nelson Textbook, Pages 40-45

Homework: Nelson Textbook: Page 45 #4f, 7, 8, 9, 10, 13b, 14b, 16, 17