

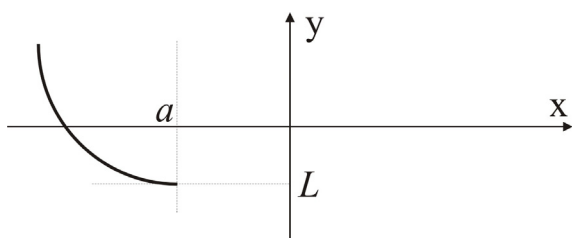
1.4 Limit of a Function

A Left-Hand Limit

If the values of $y = f(x)$ can be made arbitrarily close to L by taking x sufficiently close to a with $x < a$, then:

$$\lim_{x \rightarrow a^-} f(x) = L$$

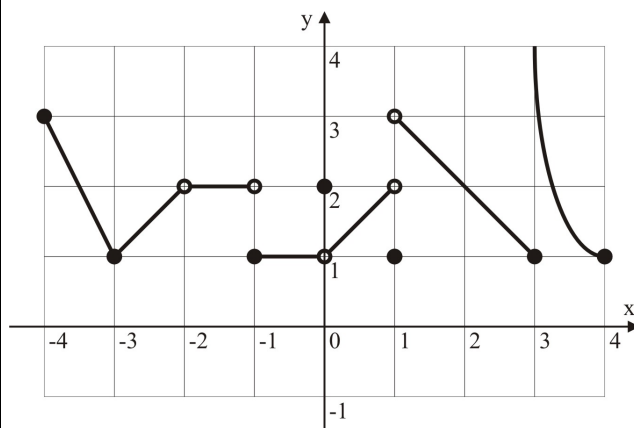
Read: The limit of the function $f(x)$ as x approaches a from the left is L .



Notes:

1. The function *may be or not defined* at a .
2. DNE stands for *Does Not Exist*.
3. L must be a number.
4. ∞ is not a number.

Ex 1. Use the function $y = f(x)$ defined by the following graph to find each limit.



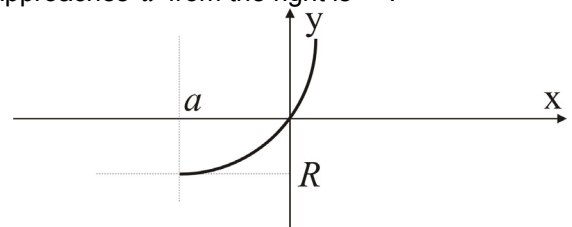
- a) $\lim_{x \rightarrow -4^-} f(x)$
- b) $\lim_{x \rightarrow -2^-} f(x)$
- c) $\lim_{x \rightarrow -1^-} f(x)$
- d) $\lim_{x \rightarrow 3^-} f(x)$

B Right-Hand Limit

If the values of $y = f(x)$ can be made arbitrarily close to R by taking x sufficiently close to a with $x > a$, then:

$$\lim_{x \rightarrow a^+} f(x) = R$$

Read: The limit of the function $f(x)$ as x approaches a from the right is R .



Notes:

1. R must be a number. ∞ is not a number.
2. The function *may be or not defined* at a .

Ex 2. Use the function $y = f(x)$ defined at Ex 1. to find each limit.

- a) $\lim_{x \rightarrow -1^+} f(x)$
- b) $\lim_{x \rightarrow 3^+} f(x)$
- c) $\lim_{x \rightarrow 1^+} f(x)$
- d) $\lim_{x \rightarrow -2^+} f(x)$

C Limit

If the values of $y = f(x)$ can be made arbitrarily close to l by taking x sufficiently close to a (*from both sides*), then:

$$\lim_{x \rightarrow a} f(x) = l$$

Read: The limit of the function $f(x)$ as x approaches a is l .

Ex 3. Use the function $y = f(x)$ defined at Ex 1. to find each limit.

- a) $\lim_{x \rightarrow -4} f(x)$
- b) $\lim_{x \rightarrow -1} f(x)$
- c) $\lim_{x \rightarrow 3} f(x)$
- d) $\lim_{x \rightarrow -3} f(x)$
- e) $\lim_{x \rightarrow -2} f(x)$

<p>Notes:</p> <ol style="list-style-type: none"> If $\lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a^-} f(x)$ then $\lim_{x \rightarrow a} f(x)$ does exist and $L = R = l$. If $\lim_{x \rightarrow a^+} f(x) \neq \lim_{x \rightarrow a^-} f(x)$ then $\lim_{x \rightarrow a} f(x)$ Does Not Exist (DNE). l must be a number. ∞ is not a number. The function <i>may be or not defined</i> at a. 	<p>f) $\lim_{x \rightarrow 0} f(x)$</p>
<p>D Substitution</p> <p>If the function is defined by a <i>formula</i> (algebraic expression) then the limit of the function at a point a may be determined by <i>substitution</i>:</p> $\lim_{x \rightarrow a} f(x) = f(a)$ <p>Notes:</p> <ol style="list-style-type: none"> In order to use substitution, the function must be defined <i>on both sides</i> of the number a. Substitution does not work if you get one of the following 7 <i>indeterminate cases</i>: $\infty - \infty \quad 0 \times \infty \quad \frac{0}{0} \quad \frac{\infty}{\infty} \quad 1^\infty \quad \infty^0 \quad 0^0$	<p>Ex 4. Compute each limit.</p> <ol style="list-style-type: none"> $\lim_{x \rightarrow 1^-} \frac{x^2}{x+1}$ $\lim_{x \rightarrow 1^+} \frac{x^2}{x+1}$ $\lim_{x \rightarrow 1} \frac{x^2}{x+1}$ $\lim_{x \rightarrow 2^-} \sqrt{x-2}$ $\lim_{x \rightarrow 2^+} \sqrt{x-2}$ $\lim_{x \rightarrow 2} \sqrt{x-2}$
<p>E Piece-wise defined functions</p> <p>If the function changes formula at a then:</p> <ol style="list-style-type: none"> Use the appropriate formula to find first the <i>left-side</i> and the <i>right-side</i> limits. Compare the left-side and the right-side limits to conclude about the limit of the function at a. <p>Example:</p> $f(x) = \begin{cases} f_1(x), & x < a \\ f_2(x), & x > a \end{cases}$ $L = f_1(a), \quad R = f_2(a) \quad (\text{if exist})$	<p>Ex 5. Consider $f(x) = \begin{cases} 2x-3, & x < 2 \\ 0, & x = 2 \\ x^2-1, & x > 2 \end{cases}$</p> <ol style="list-style-type: none"> Find $\lim_{x \rightarrow 2} f(x)$. Find $\lim_{x \rightarrow 0} f(x)$. Draw a diagram to illustrate the situation.

Reading: Nelson Textbook, Pages 34-37

Homework: Nelson Textbook: Page 37 #4d, 5, 6, 7, 10cef, 11c, 15