1.3 Rate of Change

A Average Rate of Change

$y = f(x), \quad y_1 = f(x_1), \quad y_2 = f(x_2)$

$\Delta x = x_2 - x_1$ (change in variable x)

$\Delta y = y_2 - y_1$ (change in variable y)

The *Average Rate of Change* (ARC) in y variable over the interval $[x_1, x_2]$ is given by:

$$ARC = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Note: The *Average Rate of Change* is the same as the *slope of the secant line* passing through the points $P(x_1, y_1)$ and $Q(x_2, y_2)$.

If $x_1 = a$ and $x_2 = a + h$ then:

$$ARC = \frac{f(a+h) - f(a)}{h}$$

Ex 1. Consider $y = f(x) = (x+1)^2$. Find the rate of change in the y variable over the interval $[-1,2]$.

B Average Velocity

Let $s = s(t)$ be the position function, where s is position in meters and t is the time in seconds.

$s = s(t), \quad s_1 = s(t_1), \quad s_2 = s(t_2)$

$\Delta t = t_2 - t_1$ (time duration)

$\Delta s = s_2 - s_1$ (displacement)

The *Average Velocity* (AV) over the time interval $[t_1, t_2]$ is given by:

$$AV = \frac{rise}{run} = \frac{\Delta s}{\Delta t} = \frac{s_2 - s_1}{t_2 - t_1} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

Note: The *unit* of measurement for velocity is m/s.

Ex 2. A rock is launched vertically upward. The height of the rock is given by $s(t) = 100t - 10t^2$. Find the average velocity over the time interval $[1,2]$.

C Instantaneous Rate of Change

As $h \to 0$ the Average Rate of Change approaches to the *Instantaneous Rate of Change* (IRC):

$$IRC = RC = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Note: The *Instantaneous Rate of Change* (IRC) is the same as the *slope of the tangent line* at the point $P(a, f(a))$.

Similarly, the *Average Velocity* (AV) approaches *Instantaneous Velocity* (IV):

$$IV = v = \lim_{h \to 0} \frac{s(a+h) - s(a)}{h}$$

Ex 3. Consider the following position function: $s(t) = t^2 - 4t$.

a) Find the instantaneous velocity at $t = 3s$.

b) Find the instantaneous velocity at the generic moment $t = a$

	c) Use the formula at part b) to compute the velocity at time $t = 5s$. d) Find the moment(s) of time at which the velocity is zero. Ex 4. A spherical balloon is inflated. Find the instantaneous rate of change in volume of the balloon with respect to its radius when the radius is $10m$.

Reading: Nelson Textbook, Pages 22-28
Homework: Nelson Textbook: Page 28 #2a, 7, 12, 14, 15b, 20, 22

