1.3 Rate of Change

A Average Rate of Change

Let \( y = f(x) \), \( y_1 = f(x_1) \), \( y_2 = f(x_2) \)

\[ \Delta x = x_2 - x_1 \quad (\text{change in variable } x) \]
\[ \Delta y = y_2 - y_1 \quad (\text{change in variable } y) \]

The **Average Rate of Change** (ARC) in \( y \) variable over the interval \([x_1, x_2]\) is given by:

\[ ARC = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \]

Note: The Average Rate of Change is the same as the slope of the secant line passing through the points \( P(x_1, y_1) \) and \( Q(x_2, y_2) \).

If \( x_1 = a \) and \( x_2 = a + h \) then:

\[ ARC = \frac{f(a+h) - f(a)}{h} \]

Ex 1. Consider \( y = f(x) = (x+1)^2 \). Find the rate of change in the \( y \) variable over the interval \([-1, 2]\).

B Average Velocity

Let \( s = s(t) \) be the position function, where \( s \) is position in meters and \( t \) is the time in seconds.

\[ s = s(t), \quad s_1 = s(t_1), \quad s_2 = s(t_2) \]
\[ \Delta t = t_2 - t_1 \quad (\text{time duration}) \]
\[ \Delta s = s_2 - s_1 \quad (\text{displacement}) \]

The **Average Velocity** (AV) over the time interval \([t_1, t_2]\) is given by:

\[ AV = \frac{\Delta s}{\Delta t} = \frac{s_2 - s_1}{t_2 - t_1} = \frac{s(t_2) - s(t_1)}{t_2 - t_1} \]

Note: The unit of measurement for velocity is \( \text{m/s} \).

Ex 2. A rock is launched vertically upward. The height of the rock is given by \( s(t) = 100t - 10t^2 \). Find the average velocity over the time interval \([1, 2]\).

C Instantaneous Rate of Change

As \( h \to 0 \) the Average Rate of Change approaches to the **Instantaneous Rate of Change** (IRC):

\[ IRC = RC = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \]

Note: The Instantaneous Rate of Change (IRC) is the same as the slope of the tangent line at the point \( P(a, f(a)) \).

Similarly, the Average Velocity (AV) approaches **Instantaneous Velocity** (IV):

\[ IV = v = \lim_{h \to 0} \frac{s(a+h) - s(a)}{h} \]

Ex 3. Consider the following position function: \( s(t) = t^2 - 4t \).

a) Find the instantaneous velocity at \( t = 3 \).

b) Find the instantaneous velocity at the generic moment \( t = a \).
c) Use the formula at part b) to compute the velocity at time $t = 5s$.

d) Find the moment(s) of time at which the velocity is zero.

Ex 4. A spherical balloon is inflated. Find the instantaneous rate of change in volume of the balloon with respect to its radius when the radius is 10m.

**Reading:** Nelson Textbook, Pages 22-28  
**Homework:** Nelson Textbook: Page 28 #2a, 7, 12, 14, 15b, 20, 22