MCV4U, CALCULUS AND VECTORS **TEST Chapter 6** **PLANES** Teacher: Teodoru Gugoiu	Date FALL Name 2009	$\frac{}{25}$ =

1. Find the Cartesian equation of a plane passing through the points $A(-1,0,1)$ and $B(0,1,2)$ and $C(1,-1,0)$.

[K/U 4 marks]

$\vec{u} = \vec{AB} = (1,1,1)$

$\vec{v} = \vec{AC} = (2,-1,-1)$

$$\vec{n} = \vec{u} \times \vec{v} = \begin{matrix} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & -1 & -1 & 2 & -1 & -1 \end{matrix} = (-1+1, 2+1, -1-2)$$

$\vec{n} = (0,3,-3)$ or $\vec{n} = (0,1,-1)$

$\pi: y - z + D = 0$

$A(-1,0,1) \in \pi$

$-1 + D = 0$

$D = 1$

$\therefore \pi: y - z + 1 = 0$

2. Determine the acute angle between the planes:

[K/U 2 marks]

$\pi_1 : x + y - z + 1 = 0$, $\pi_2 : -3x - 3y + 3z - 4 = 0$

$\vec{n_1} = (1,1,-1)$

$\vec{n_2} = (-3,-3,3)$ or

$\vec{n_2} = (-1,-1,1)$

$\vec{n_1} \cdot \vec{n_2} = -1-1-1 = -3$

$\theta = \cos^{-1} \frac{\vec{n_1} \cdot \vec{n_2}}{|\vec{n_1}||\vec{n_2}|}$

$\theta = \cos^{-1} \frac{-3}{\sqrt{3}\sqrt{3}} = \cos^{-1} -1$

$\theta = 180°$

$\therefore$ The acute angle is $0°$

$\pi_1 \parallel \pi_2$

3. Find the distance between the point $P(0,2,-3)$ and the plane $\pi : 2x - 3y + 4z + 12 - 0$.

[K/U 2 marks]

$$d = \frac{|Ax + By + Cz + D|}{\sqrt{A^2 + B^2 + C^2}}$$

$$= \frac{|\cancel{2(0)} - 3(2) \cancel{+4(-3)} \cancel{+12}|}{\sqrt{4+9+16}}$$

$$\therefore d = \frac{6}{\sqrt{29}} = \frac{6\sqrt{29}}{29} \simeq 1.114$$

4. Find the vector equation of a plane containing the line $L_1 : \vec{r} = \underbrace{(-2,-1,0)}_{P_0} + t\underbrace{(-1,0,1)}_{\vec{u}}$ and parallel to the line $L_2 : \vec{r} = (0,1,2) + s\underbrace{(1,-2,3)}_{\vec{v}}$. [A 3 marks]

$P_0 = (-2,-1,0)$

$\vec{u} = (-1,0,1)$

$\vec{v} = (1,-2,3)$

$\therefore \vec{\pi} = (-2,-1,0) + t(-1,0,1) + s(1,-2,3)$

5. Find the Cartesian equation of a plane having the following intersections with the coordinate axes: $x-\text{int} = -1$, $y-\text{int} = 2$, and $z-\text{int} = 4$. [A 3 marks]

$Ax + By + Cz + D = 0$

$x\text{-int} = -\frac{D}{A} = -1 \Rightarrow A = D$

$y\text{-int} = -\frac{D}{B} = 2 \Rightarrow B = -\frac{D}{2}$

$z\text{-int} = -\frac{D}{C} = 4 \Rightarrow C = -\frac{D}{4}$

$\cancel{D}x - \frac{\cancel{D}}{2}y - \frac{\cancel{D}}{4}z + \cancel{D} = 0$

$\therefore 4x - 2y - z + 4 = 0$

6. Find the intersection between the given line and the given plane. [A 4 marks]

$\pi : 3x - 2y + z + 8 = 0$, $L : \vec{r} = (0,-1,2) + t(0,1,-2)$

$L = \begin{cases} x = 0 \\ y = -1 + t \\ z = 2 - 2t \end{cases}$

$3\cancel{(0)} - 2(-1+t) + (2-2t) + 8 = 0$

$2 - 2t + 2 - 2t + 8 = 0$

$-4t = -12$

$t = 3$

$P = \begin{cases} x = 0 \\ y = -1 + 3 = 2 \\ z = 2 - 2(3) = -4 \end{cases}$

$P = \pi \cap L$

$\therefore P(0, 2, -4)$

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7. Find the equation of the line of intersection for each pair of planes (if it exists). [A 3 marks]

$\pi_1 : x+y+z=0$, $\pi_2 : x-y+z-1=0$

$$\begin{cases} x+y+z=0 & ① \\ x-y+z-1=0 & ② \end{cases}$$

$① \Rightarrow z = -x-y$

$$\cancel{x}-y-\cancel{x}-y-1=0$$

$$-2y=1$$

$$\boxed{y=-\frac{1}{2}}$$

$L = \Pi_1 \cap \Pi_2$

$$L : \begin{cases} x = t \\ y = -\frac{1}{2} \\ z = -t+\frac{1}{2} \end{cases}$$

or $\vec{r} = (0, -\frac{1}{2}, \frac{1}{2}) + t(1, 0, -1)$

8. Solve the following system of equations. Give a geometric interpretation of the result. [A 4 marks]

$$\begin{cases} x+y+z-1=0 & ① \\ x-y+z+2=0 & ② \\ -x+y+z+1=0 & ③ \end{cases}$$

$③ \Rightarrow z = x-y-1$

$$\begin{cases} x+\cancel{y}+x-\cancel{y}-1-1=0 \\ x-y+x-y-1+2=0 \end{cases}$$

$$\begin{cases} z = x-y-1 \\ 2x=2 \Rightarrow \boxed{x=1} \\ 2x-2y=-1 \end{cases}$$

$\downarrow$

$$-2y=-1-2$$

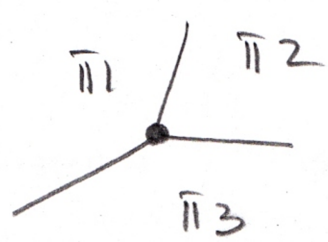
$$\boxed{y=\frac{3}{2}}$$

$$z = 1-\frac{3}{2}-1$$

$$\boxed{z=-\frac{3}{2}}$$

$P = \Pi_1 \cap \Pi_2 \cap \Pi_3$

$\therefore$ $\underline{P}\ (1, \frac{3}{2}, -\frac{3}{2})$



$\vec{n_1} \cdot (\vec{n_2} \times \vec{n_3}) \neq 0$

normal vectors are not coplanar