1. Analyze the differentiablility of the function: $f(x) = \dfrac{1}{\sqrt{x+1}}$

2. Analyze the differentiablility of the function: $f(x) = \dfrac{3}{-2x-4}$

3. Analyze the differentiablility of the function: $f(x) = 3\sqrt[5]{(x+3)^4}$

4. Analyze the differentiablility of the function: $f(x) = -\sqrt[5]{3x+3}$

5. Analyze the differentiablility of the function: $f(x) = \dfrac{1}{\sqrt[5]{(-2x-2)^2}}$

6. Analyze the differentiablility of the function: $f(x) = \dfrac{1}{2x-1}$

7. Analyze the differentiablility of the function: $f(x) = 2\sqrt[3]{(x+4)^4}$

8. Analyze the differentiablility of the function: $f(x) = 3\sqrt[3]{(2x+2)^5}$

9. Analyze the differentiablility of the function: $f(x) = \dfrac{-3}{\sqrt[4]{(2x+4)^5}}$

10. Analyze the differentiablility of the function: $f(x) = 3\sqrt[5]{(-2x-4)^3}$

Answers:

1. $f(x) = \dfrac{1}{\sqrt{x+1}}$ $\quad f'(x) = \dfrac{-1}{2}\dfrac{1}{\sqrt{(x+1)^3}}$ $\quad$ The function f is differentiable over $(-1, \infty)$.

2. $f(x) = \dfrac{3}{-2x-4}$ $\quad f'(x) = 6\dfrac{1}{(-2x-4)^2}$ $\quad$ The function f is differentiable over $\mathbb{R}\backslash\{-2\}$.

3. $f(x) = 3\sqrt[5]{(x+3)^4}$ $\quad f'(x) = \dfrac{12}{5}\dfrac{1}{\sqrt[5]{x+3}}$ $\quad$ The function f is differentiable over $\mathbb{R}\backslash\{-3\}$.

The point $P(-3, 0)$ is a cusp point.

4. $f(x) = -\sqrt[5]{3x+3}$ $\quad f'(x) = \dfrac{-3}{5}\dfrac{1}{\sqrt[5]{(3x+3)^4}}$ $\quad$ The function f is differentiable over $\mathbb{R}\backslash\{-1\}$.

The point $P(-1, 0)$ is an infinite slope point.

5. $f(x) = \dfrac{1}{\sqrt[5]{(-2x-2)^2}}$ $\quad f'(x) = \dfrac{4}{5}\dfrac{1}{\sqrt[5]{(-2x-2)^7}}$ $\quad$ The function f is differentiable over $\mathbb{R}\backslash\{-1\}$.

6. $f(x) = \dfrac{1}{2x-1}$ $\quad f'(x) = -2\dfrac{1}{(2x-1)^2}$ $\quad$ The function f is differentiable over $\mathbb{R}\backslash\{\frac{1}{2}\}$.

7. $f(x) = 2\sqrt[3]{(x+4)^4}$ $\quad f'(x) = \dfrac{8}{3}\sqrt[3]{x+4}$ $\quad$ The function f is differentiable over $\mathbb{R}$.

8. $f(x) = 3\sqrt[3]{(2x+2)^5}$ $\quad f'(x) = 10\sqrt[3]{(2x+2)^2}$ $\quad$ The function f is differentiable over $\mathbb{R}$.

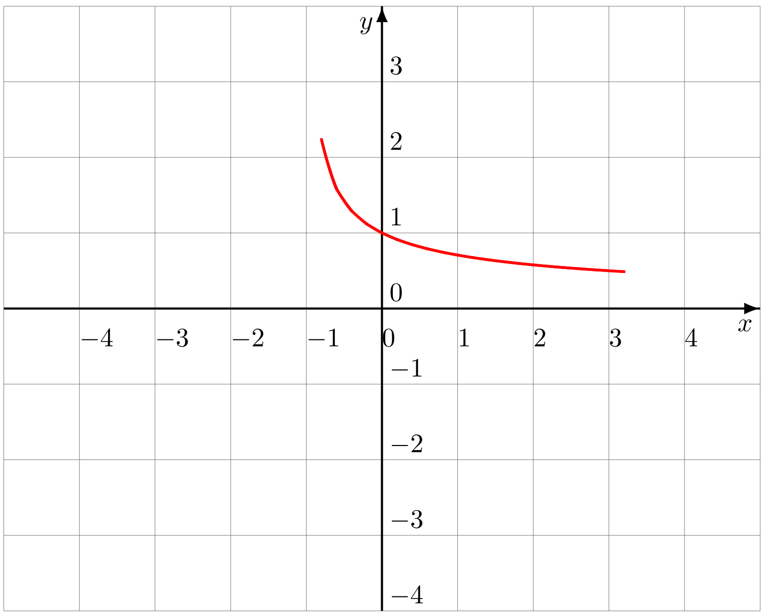
9. $f(x) = \dfrac{-3}{\sqrt[4]{(2x+4)^5}}$ $\quad f'(x) = \dfrac{15}{2}\dfrac{1}{\sqrt[4]{(2x+4)^9}}$ $\quad$ The function f is differentiable over $(-2, \infty)$.

10. $f(x) = 3\sqrt[5]{(-2x-4)^3}$ $\quad f'(x) = \dfrac{-18}{5}\dfrac{1}{\sqrt[5]{(-2x-4)^2}}$ $\quad$ The function f is differentiable over $\mathbb{R}\backslash\{-2\}$.

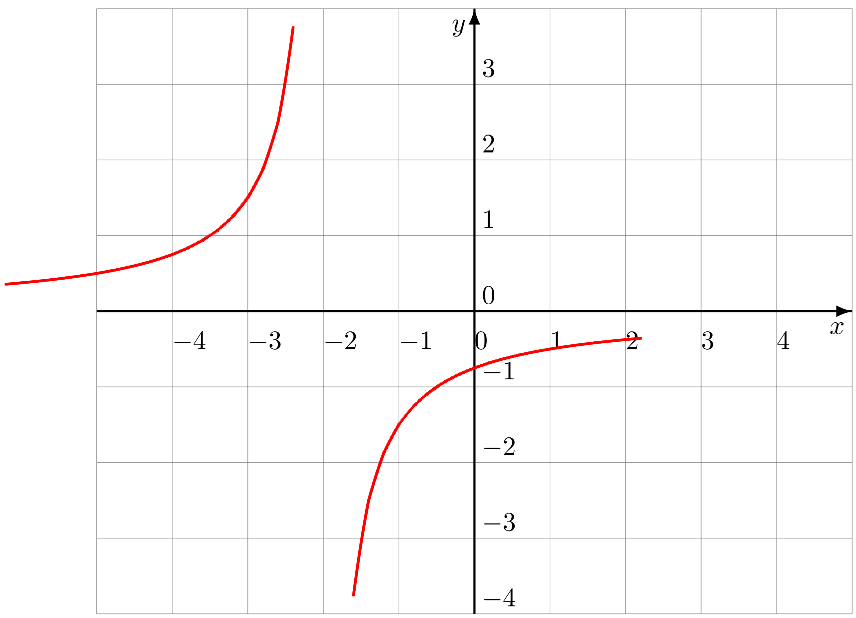
The point $P(-2, 0)$ is an infinite slope point.

Solutions:

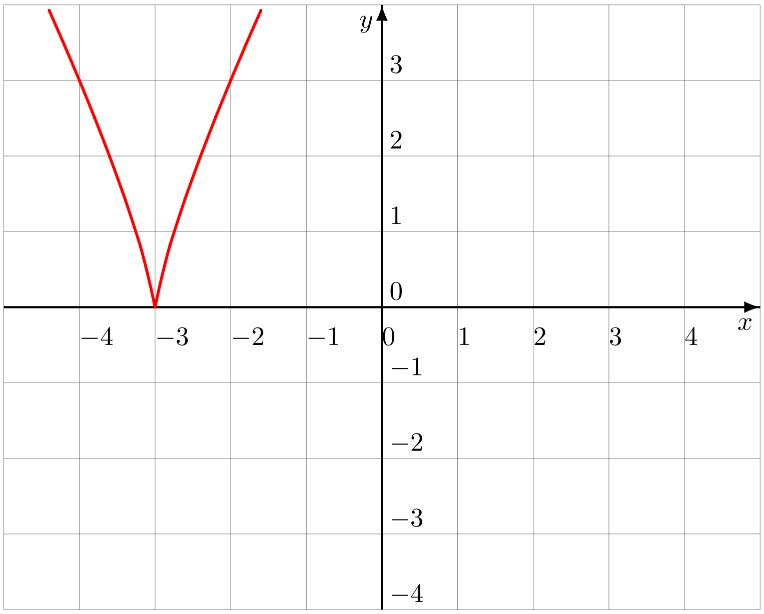
1. $f(x) = \dfrac{1}{\sqrt{x+1}}$ The domain of f is $D_f = (-1, \infty)$ $f'(x) = \dfrac{-1}{2}\dfrac{1}{\sqrt{(x+1)^3}}$ The domain of f' is $D_{f'} = (-1, \infty)$ $\therefore$ The function f is differentiable over $(-1, \infty)$.



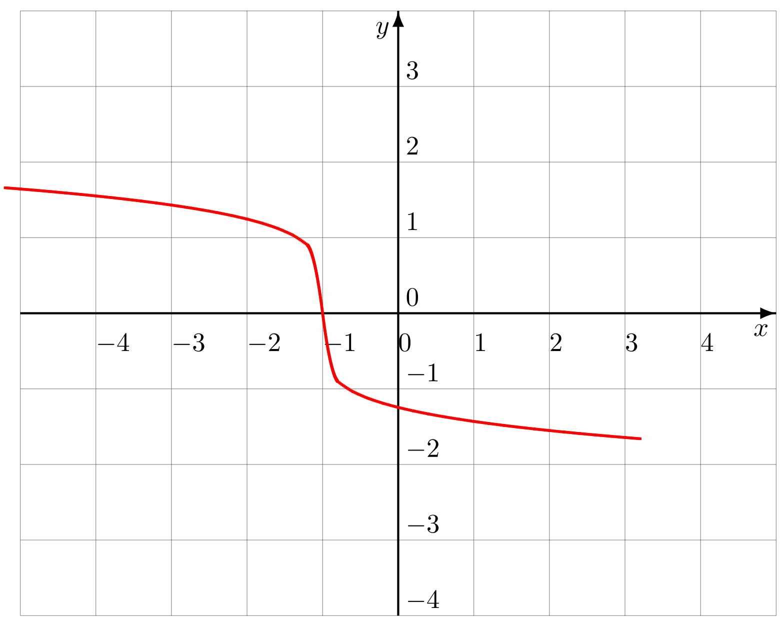
2. $f(x) = \dfrac{3}{-2x-4}$ The domain of f is $D_f = \mathbb{R}\backslash\{-2\}$ $f'(x) = 6\dfrac{1}{(-2x-4)^2}$ The domain of f' is $D_{f'} = \mathbb{R}\backslash\{-2\}$ $\therefore$ The function f is differentiable over $\mathbb{R}\backslash\{-2\}$.



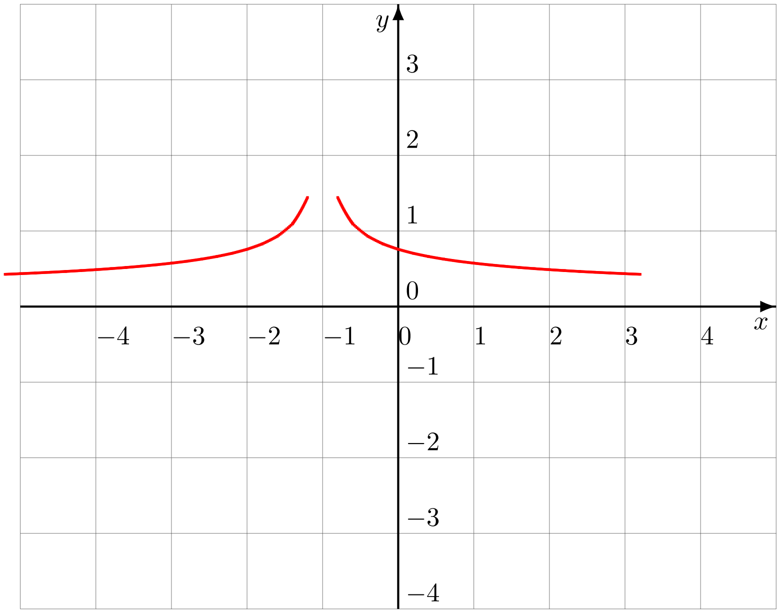
3. $f(x) = 3\sqrt[5]{(x+3)^4}$ The domain of f is $D_f = \mathbb{R}$ $f'(x) = \frac{12}{5}\frac{1}{\sqrt[5]{x+3}}$ The domain of f' is $D_{f'} = \mathbb{R}\backslash\{-3\}$ $\therefore$ The function f is differentiable over $\mathbb{R}\backslash\{-3\}$. $\lim_{x\to-3^-} f'(x) = -\infty$ $\lim_{x\to-3^+} f'(x) = +\infty$ $\therefore$ The point $P(-3,0)$ is a cusp point.



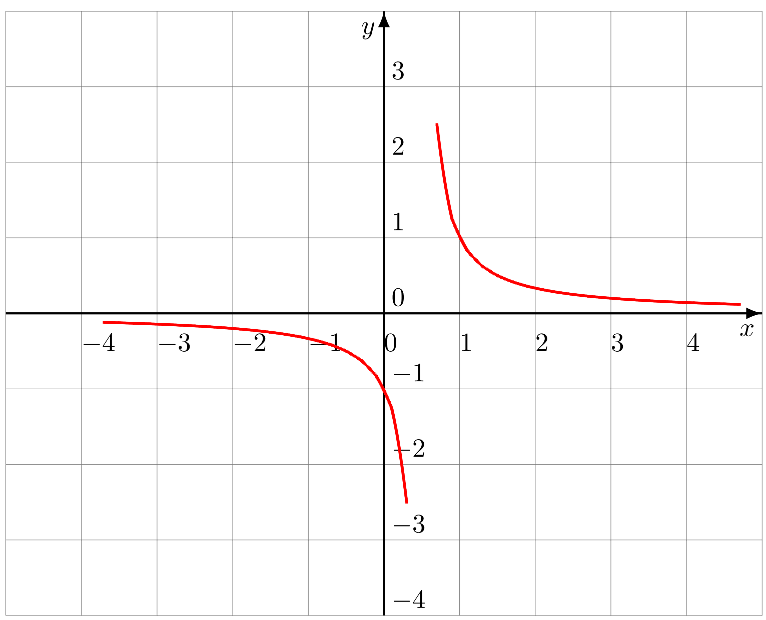
4. $f(x) = -\sqrt[5]{3x+3}$ The domain of f is $D_f = \mathbb{R}$ $f'(x) = \frac{-3}{5}\frac{1}{\sqrt[5]{(3x+3)^4}}$ The domain of f' is $D_{f'} = \mathbb{R}\backslash\{-1\}$ $\therefore$ The function f is differentiable over $\mathbb{R}\backslash\{-1\}$. $\lim_{x\to-1^-} f'(x) = -\infty$ $\lim_{x\to-1^+} f'(x) = -\infty$ $\therefore$ The point $P(-1,0)$ is an infinite slope point.



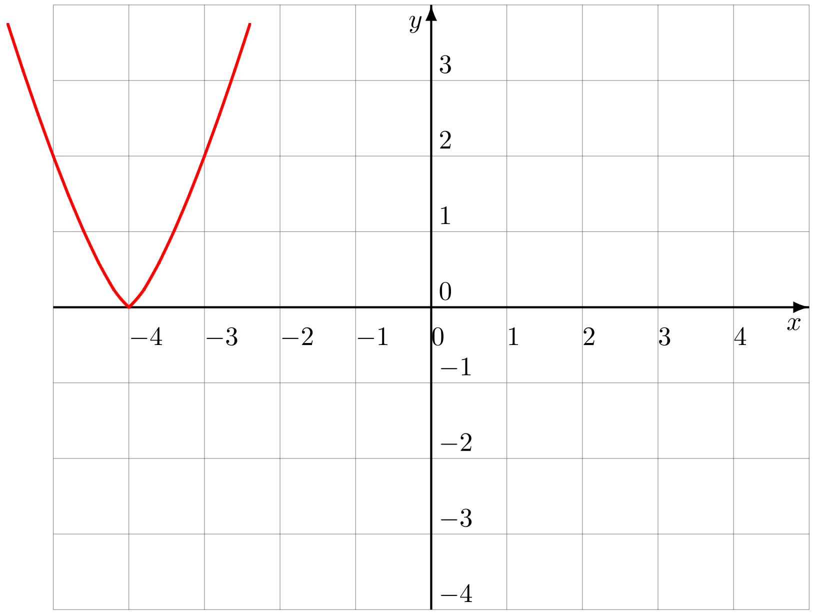
5. $f(x) = \dfrac{1}{\sqrt[5]{(-2x-2)^2}}$ The domain of f is $D_f = \mathbb{R}\backslash\{-1\}$ $f'(x) = \dfrac{4}{5}\dfrac{1}{\sqrt[5]{(-2x-2)^7}}$ The domain of f' is $D_{f'} = \mathbb{R}\backslash\{-1\}$ $\therefore$ The function f is differentiable over $\mathbb{R}\backslash\{-1\}$.



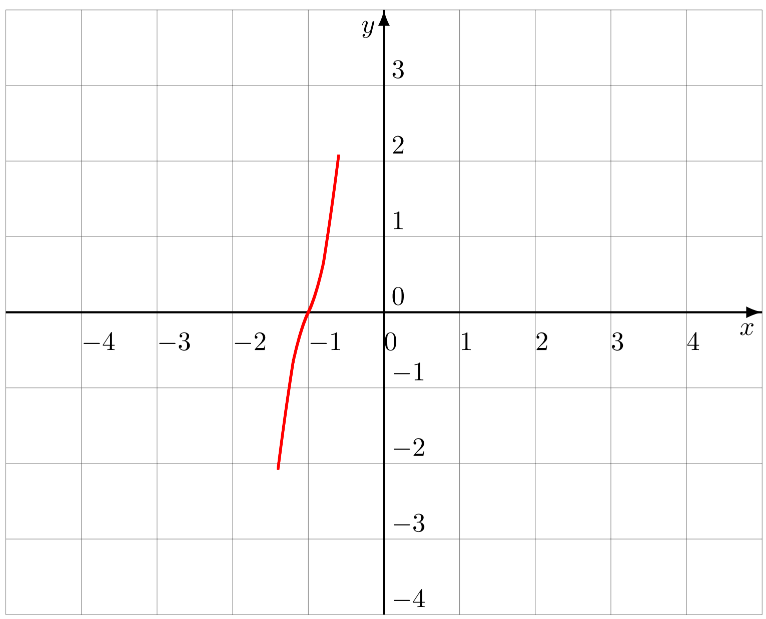
6. $f(x) = \dfrac{1}{2x-1}$ The domain of f is $D_f = \mathbb{R}\backslash\{\frac{1}{2}\}$ $f'(x) = -2\dfrac{1}{(2x-1)^2}$ The domain of f' is $D_{f'} = \mathbb{R}\backslash\{\frac{1}{2}\}$ $\therefore$ The function f is differentiable over $\mathbb{R}\backslash\{\frac{1}{2}\}$.



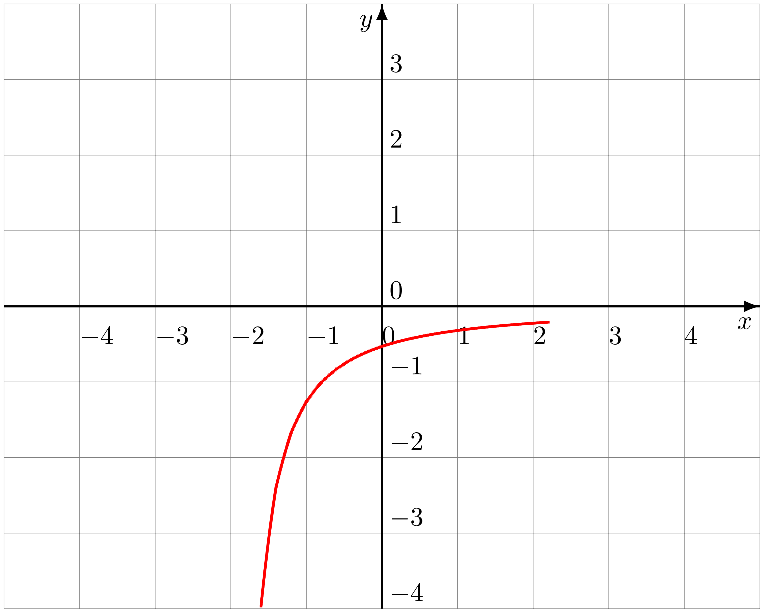
7. $f(x) = 2\sqrt[3]{(x+4)^4}$ The domain of f is $D_f = \mathbb{R}$ $f'(x) = \frac{8}{3}\sqrt[3]{x+4}$ The domain of f' is $D_{f'} = \mathbb{R}$ $\therefore$ The function f is differentiable over $\mathbb{R}$.



8. $f(x) = 3\sqrt[3]{(2x+2)^5}$ The domain of f is $D_f = \mathbb{R}$ $f'(x) = 10\sqrt[3]{(2x+2)^2}$ The domain of f' is $D_{f'} = \mathbb{R}$ $\therefore$ The function f is differentiable over $\mathbb{R}$.



9. $f(x) = \dfrac{-3}{\sqrt[4]{(2x+4)^5}}$ The domain of f is $D_f = (-2, \infty)$ $f'(x) = \dfrac{15}{2}\dfrac{1}{\sqrt[4]{(2x+4)^9}}$ The domain of f' is $D_{f'} = (-2, \infty)$ $\therefore$ The function f is differentiable over $(-2, \infty)$.



10. $f(x) = 3\sqrt[5]{(-2x-4)^3}$ The domain of f is $D_f = \mathbb{R}$ $f'(x) = \dfrac{-18}{5}\dfrac{1}{\sqrt[5]{(-2x-4)^2}}$ The domain of f' is $D_{f'} = \mathbb{R}\backslash\{-2\}$ $\therefore$ The function f is differentiable over $\mathbb{R}\backslash\{-2\}$. $\lim\limits_{x\to -2^-} f'(x) = -\infty$ $\lim\limits_{x\to -2^+} f'(x) = -\infty$ $\therefore$ The point $P(-2, 0)$ is an infinite slope point.