1. Analyze the differentiablility of the function: $f(x) = \sqrt[4]{x^5}$
2. Analyze the differentiablility of the function: $f(x) = \dfrac{1}{\sqrt[3]{x}}$
3. Analyze the differentiablility of the function: $f(x) = \dfrac{1}{\sqrt[5]{x^4}}$
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5. Analyze the differentiablility of the function: $f(x) = \sqrt[3]{x^2}$
6. Analyze the differentiablility of the function: $f(x) = \dfrac{1}{\sqrt{x}}$
7. Analyze the differentiablility of the function: $f(x) = x^3$
8. Analyze the differentiablility of the function: $f(x) = \sqrt[5]{x}$
9. Analyze the differentiablility of the function: $f(x) = \dfrac{1}{\sqrt[3]{x^2}}$
10. Analyze the differentiablility of the function: $f(x) = \sqrt[5]{x^2}$

Answers:

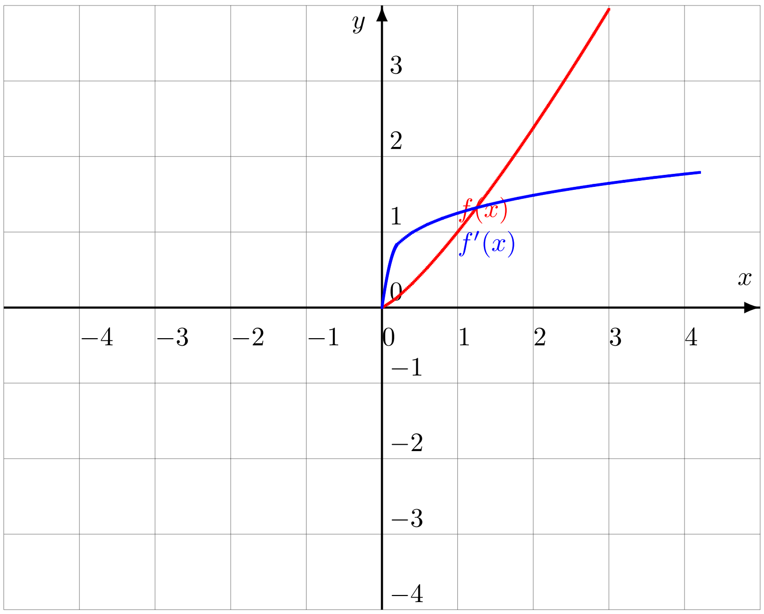
1. $f'(x) = \frac{5}{4}\sqrt[4]{x}$ The function f is differentiable over $[0, \infty)$.
2. $f'(x) = \frac{-1}{3}\frac{1}{\sqrt[3]{x^4}}$ The function f is differentiable over $\mathbb{R}\backslash\{0\}$.
3. $f'(x) = \frac{-4}{5}\frac{1}{\sqrt[5]{x^9}}$ The function f is differentiable over $\mathbb{R}\backslash\{0\}$.
4. $f'(x) = \frac{-4}{1}\frac{1}{x^5}$ The function f is differentiable over $\mathbb{R}\backslash\{0\}$.
5. $f'(x) = \frac{2}{3}\frac{1}{\sqrt[3]{x}}$ The function f is differentiable over $\mathbb{R}\backslash\{0\}$. The point $P(0,0)$ is a cusp point.
6. $f'(x) = \frac{-1}{2}\frac{1}{\sqrt{x^3}}$ The function f is differentiable over $(0, \infty)$.
7. $f'(x) = \frac{3}{1}x^2$ The function f is differentiable over $\mathbb{R}$.
8. $f'(x) = \frac{1}{5}\frac{1}{\sqrt[5]{x^4}}$ The function f is differentiable over $\mathbb{R}\backslash\{0\}$. The point $P(0,0)$ is an infinite slope point.
9. $f'(x) = \frac{-2}{3}\frac{1}{\sqrt[3]{x^5}}$ The function f is differentiable over $\mathbb{R}\backslash\{0\}$.
10. $f'(x) = \frac{2}{5}\frac{1}{\sqrt[5]{x^3}}$ The function f is differentiable over $\mathbb{R}\backslash\{0\}$. The point $P(0,0)$ is a cusp point.

Solutions:

1. $f(x) = \sqrt[4]{x^5}$ The domain of f is $D_f = [0, \infty)$

$f'(x) = \frac{5}{4}\sqrt[4]{x}$ The domain of f' is $D_{f'} = [0, \infty)$

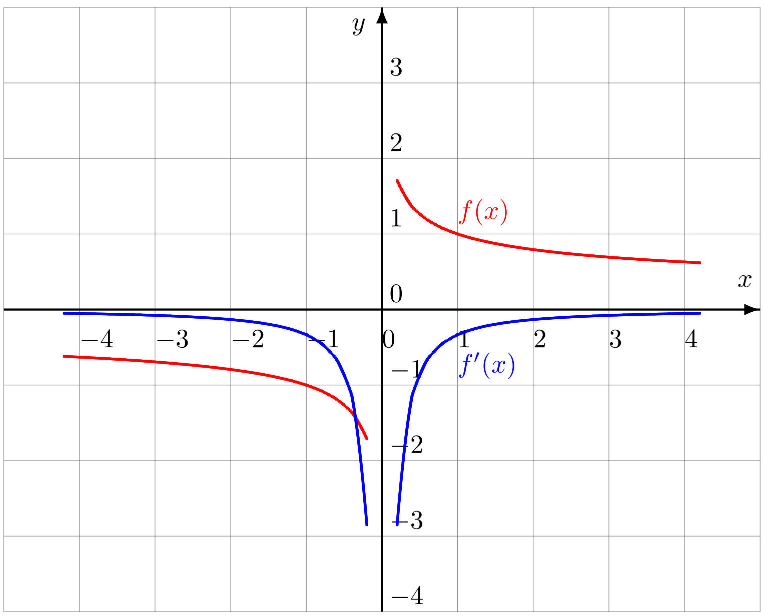
$\therefore$ The function f is differentiable over $[0, \infty)$.



2. $f(x) = \frac{1}{\sqrt[3]{x}}$ The domain of f is $D_f = \mathbb{R}\backslash\{0\}$

$f'(x) = \frac{-1}{3}\frac{1}{\sqrt[3]{x^4}}$ The domain of f' is $D_{f'} = \mathbb{R}\backslash\{0\}$

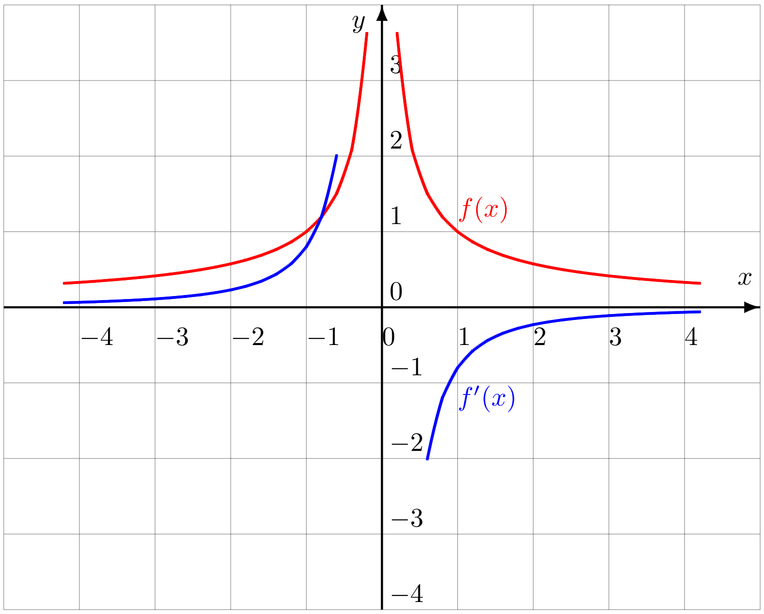
$\therefore$ The function f is differentiable over $\mathbb{R}\backslash\{0\}$.



3. $f(x) = \dfrac{1}{\sqrt[5]{x^4}}$ The domain of f is $D_f = \mathbb{R}\backslash\{0\}$

$f'(x) = \dfrac{-4}{5}\dfrac{1}{\sqrt[5]{x^9}}$ The domain of f' is $D_{f'} = \mathbb{R}\backslash\{0\}$

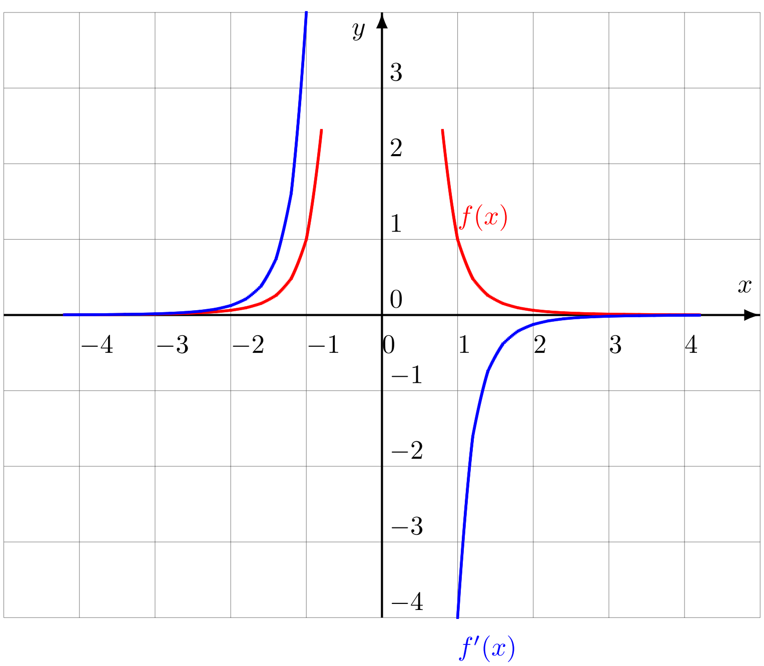
$\therefore$ The function f is differentiable over $\mathbb{R}\backslash\{0\}$.



4. $f(x) = \dfrac{1}{x^4}$ The domain of f is $D_f = \mathbb{R}\backslash\{0\}$

$f'(x) = \dfrac{-4}{1}\dfrac{1}{x^5}$ The domain of f' is $D_{f'} = \mathbb{R}\backslash\{0\}$

$\therefore$ The function f is differentiable over $\mathbb{R}\backslash\{0\}$.

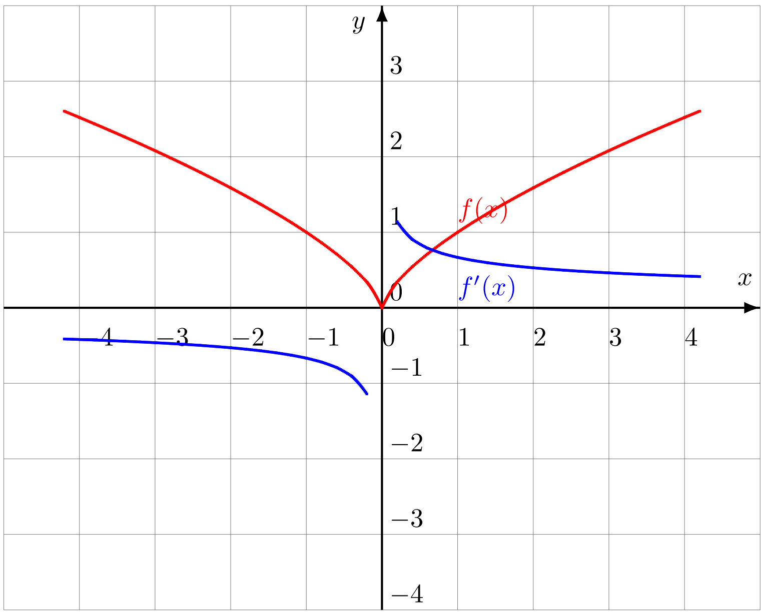


5. $f(x) = \sqrt[3]{x^2}$ The domain of f is $D_f = \mathbb{R}$

$f'(x) = \frac{2}{3}\frac{1}{\sqrt[3]{x}}$ The domain of f' is $D_{f'} = \mathbb{R}\backslash\{0\}$

$\therefore$ The function f is differentiable over $\mathbb{R}\backslash\{0\}$.

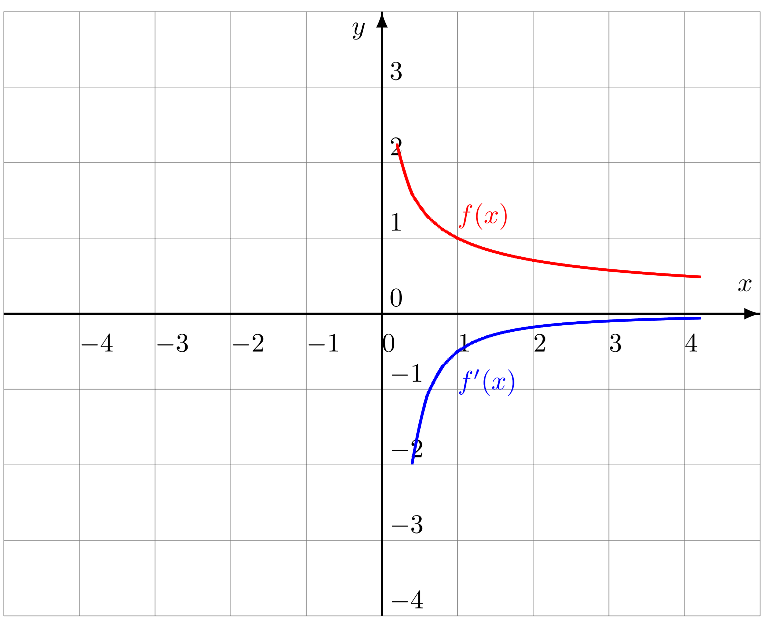
$\lim_{x\to 0^-} f'(x) = -\infty$ $\lim_{x\to 0^+} f'(x) = +\infty$ $\therefore$ The point $P(0,0)$ is a cusp point.



6. $f(x) = \frac{1}{\sqrt{x}}$ The domain of f is $D_f = (0, \infty)$

$f'(x) = \frac{-1}{2}\frac{1}{\sqrt{x^3}}$ The domain of f' is $D_{f'} = (0, \infty)$

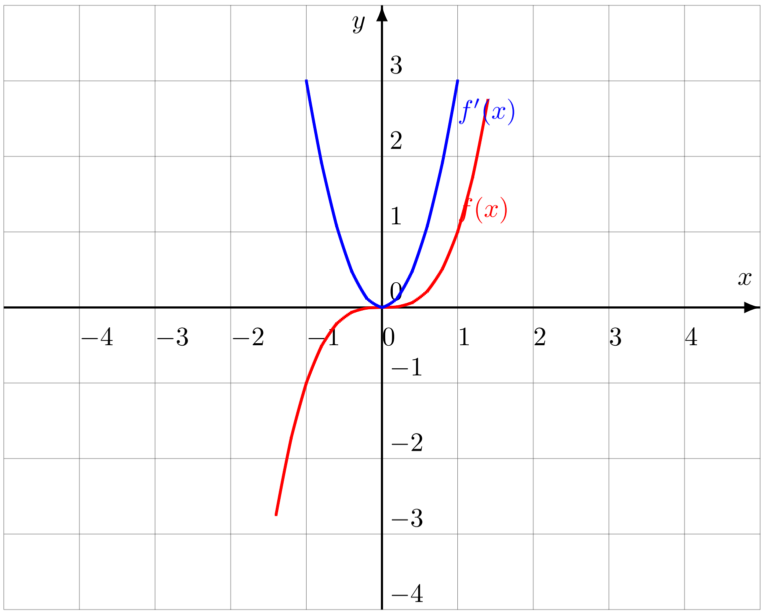
$\therefore$ The function f is differentiable over $(0, \infty)$.



7. $f(x) = x^3$ The domain of f is $D_f = \mathbb{R}$

$f'(x) = \frac{3}{1}x^2$ The domain of f' is $D_{f'} = \mathbb{R}$

$\therefore$ The function f is differentiable over $\mathbb{R}$.

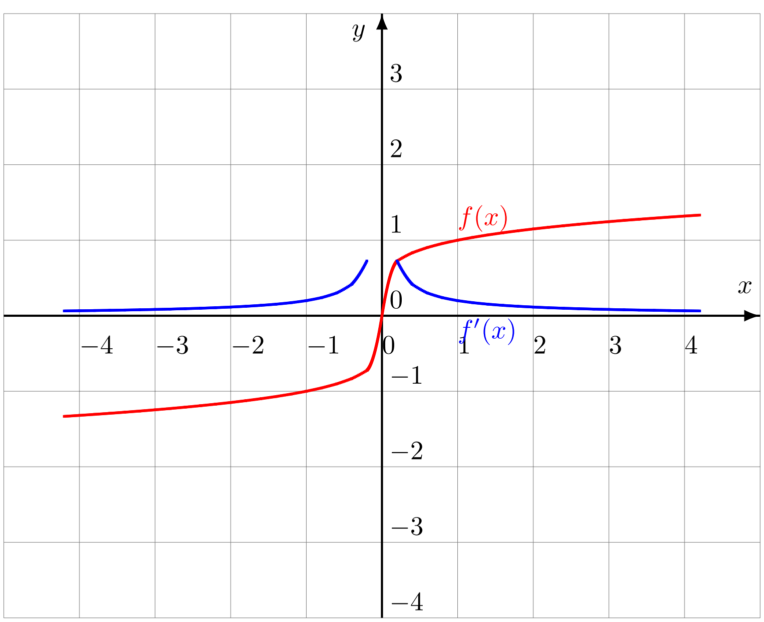


8. $f(x) = \sqrt[5]{x}$ The domain of f is $D_f = \mathbb{R}$

$f'(x) = \frac{1}{5}\frac{1}{\sqrt[5]{x^4}}$ The domain of f' is $D_{f'} = \mathbb{R}\backslash\{0\}$

$\therefore$ The function f is differentiable over $\mathbb{R}\backslash\{0\}$.

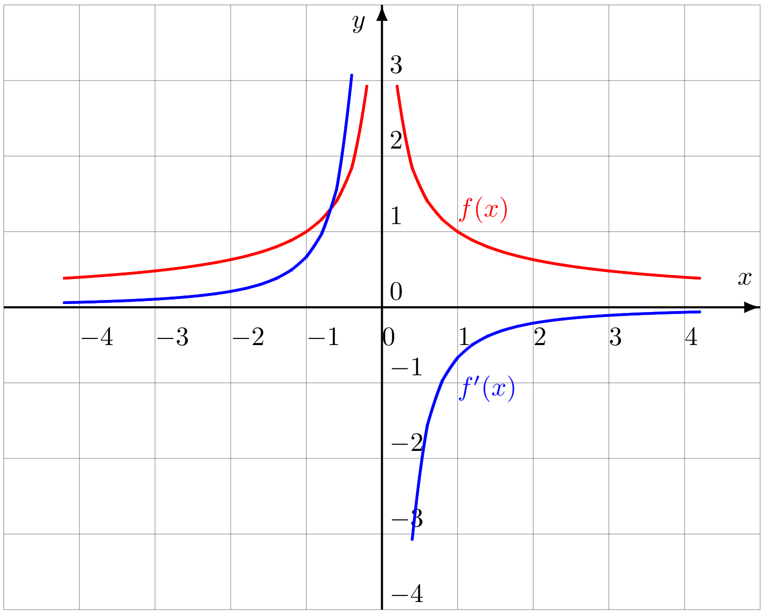
$\lim_{x\to 0^-} f'(x) = +\infty$ $\lim_{x\to 0^+} f'(x) = +\infty$ $\therefore$ The point $P(0, 0)$ is an infinite slope point.



9. $f(x) = \frac{1}{\sqrt[3]{x^2}}$ The domain of f is $D_f = \mathbb{R}\backslash\{0\}$

$f'(x) = \frac{-2}{3}\frac{1}{\sqrt[3]{x^5}}$ The domain of f' is $D_{f'} = \mathbb{R}\backslash\{0\}$

$\therefore$ The function f is differentiable over $\mathbb{R}\backslash\{0\}$.



10. $f(x) = \sqrt[5]{x^2}$ The domain of f is $D_f = \mathbb{R}$

$f'(x) = \frac{2}{5}\frac{1}{\sqrt[5]{x^3}}$ The domain of f' is $D_{f'} = \mathbb{R}\backslash\{0\}$

$\therefore$ The function f is differentiable over $\mathbb{R}\backslash\{0\}$.

$\lim_{x\to 0^-} f'(x) = -\infty$ $\lim_{x\to 0^+} f'(x) = +\infty$ $\therefore$ The point $P(0,0)$ is a cusp point.