1. Consider the following piece-wise defined function: \( f(x) = \begin{cases} -3x - 3x^2 & \text{if } x < -1 \\ 2 + x - x^2 & \text{if } x \geq -1 \end{cases} \)

Analyze the differentiability of the function \( f(x) \) at \( x = -1 \).

2. Consider the following piece-wise defined function: \( f(x) = \begin{cases} 1 - 2x^2 & \text{if } x < 1 \\ 2 - 2x - x^2 & \text{if } x \geq 1 \end{cases} \)

Analyze the differentiability of the function \( f(x) \) at \( x = 1 \).

3. Consider the following piece-wise defined function: \( f(x) = \begin{cases} -2 - 3x - 3x^2 & \text{if } x < -2 \\ 9 + 6x - x^2 & \text{if } x \geq -2 \end{cases} \)

Analyze the differentiability of the function \( f(x) \) at \( x = -2 \).

4. Consider the following piece-wise defined function: \( f(x) = \begin{cases} 3 + x + x^2 & \text{if } x < 1 \\ 2 + 5x - x^2 & \text{if } x \geq 1 \end{cases} \)

Analyze the differentiability of the function \( f(x) \) at \( x = 1 \).

5. Consider the following piece-wise defined function: \( f(x) = \begin{cases} 2 - x^2 & \text{if } x < 2 \\ 9 - 7x + x^2 & \text{if } x \geq 2 \end{cases} \)

Analyze the differentiability of the function \( f(x) \) at \( x = 2 \).

6. Consider the following piece-wise defined function: \( f(x) = \begin{cases} 2 + 3x - x^2 & \text{if } x < -1 \\ 5 + 7x + x^2 & \text{if } x \geq -1 \end{cases} \)

Analyze the differentiability of the function \( f(x) \) at \( x = -1 \).

7. Consider the following piece-wise defined function: \( f(x) = \begin{cases} -x - x^2 & \text{if } x < 2 \\ 6 - 8x + x^2 & \text{if } x \geq 2 \end{cases} \)

Analyze the differentiability of the function \( f(x) \) at \( x = 2 \).

8. Consider the following piece-wise defined function: \( f(x) = \begin{cases} -2 - 2x - 2x^2 & \text{if } x < -2 \\ 2 + 2x - x^2 & \text{if } x \geq -2 \end{cases} \)

Analyze the differentiability of the function \( f(x) \) at \( x = -2 \).

9. Consider the following piece-wise defined function: \( f(x) = \begin{cases} 3 + 2x + 3x^2 & \text{if } x < -2 \\ -3 - 5x + x^2 & \text{if } x \geq -2 \end{cases} \)

Analyze the differentiability of the function \( f(x) \) at \( x = -2 \).

10. Consider the following piece-wise defined function: \( f(x) = \begin{cases} x + 2x^2 & \text{if } x < 1 \\ -1 + 3x + x^2 & \text{if } x \geq 1 \end{cases} \)

Analyze the differentiability of the function \( f(x) \) at \( x = 1 \).

Answers:

1. Differentiable
2. Differentiable
3. Not differentiable (jump discontinuity)
4. Not differentiable (jump discontinuity)
5. Not differentiable (jump discontinuity)
6. Not differentiable (corner point)
7. Not differentiable (corner point)
8. Not differentiable (corner point)
9. Not differentiable (corner point)
10. Differentiable
CALCULUS

Differentiability (I)

Solutions:

1. \( \lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} -3x - 3x^2 = 0 \quad \lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} 2 + x - x^2 = 0 \)
   \( \lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x) \quad \text{Therefore the function is continuous at } x = -1. \)

2. \( f'(x) = \begin{cases} -3 - 6x & \text{if } x < -1 \\ 1 - 2x & \text{if } x > -1 \end{cases} \)
   \( f'(-1) = -3 - 6(-1) = 3 \quad f'(-1) = \lim_{x \to -1^+} f(x) \quad \text{Therefore the function is differentiable at } x = -1 \)

3. \( \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} -2 - 3x - 3x^2 = -8 \quad \lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 9 + 6x - x^2 = -7 \)
   \( \lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x) \quad \text{The function is not continuous at } x = 2. \)

4. \( \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} 3 + x + x^2 = 5 \quad \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 2 + 5x - x^2 = 6 \)
   \( \lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x) \quad \text{The function is not continuous at } x = 1. \)

5. \( \lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} 2 - x^2 = -2 \quad \lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} 9 - 7x + x^2 = -1 \)
   \( \lim_{x \to -1^-} f(x) \neq \lim_{x \to -1^+} f(x) \quad \text{The function is not continuous at } x = -1. \)

6. \( \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} 2 + 3x - x^2 = -2 \quad \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 5 + 7x + x^2 = -1 \)
   \( \lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x) \quad \text{The function is not continuous at } x = 1. \)

7. \( \lim_{x \to -2^-} f(x) = \lim_{x \to -2^-} -x - x^2 = -6 \quad \lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} 6 - 8x + x^2 = -6 \)
   \( \lim_{x \to -2^-} f(x) = \lim_{x \to -2^+} f(x) \quad \text{The function is continuous at } x = -2. \)

8. \( f'(x) = \begin{cases} -1 - 2x & \text{if } x < 2 \\ -8 + 2x & \text{if } x > 2 \end{cases} \)
   \( f'(-2) = \lim_{x \to -2^-} -1 - 2x = -5 \quad f'(-2) = \lim_{x \to -2^+} -8 + 2x = -4 \)
CALCULUS

Differentiability (I)

The left- and right-hand derivatives are not equal.

∴ Therefore the function is not differentiable at \( x = 2 \). The point \( P(2, -6) \) is a corner point.

8. \[
\lim_{x \to -2^-} f(x) = \lim_{x \to -2^-} (-2 - 2x - 2x^2) = -6 \quad \lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} (2 + 2x - x^2) = -6
\]
\[
\lim_{x \to -2^-} f(x) = \lim_{x \to -2^+} f(x) \quad \text{▸ The function is continuous at } x = -2.
\]
\[
f'(x) = \begin{cases} 
-2 - 4x & \text{if } x < -2 \\
2 - 2x & \text{if } x > -2
\end{cases}
\]
\[
f'(-2) = \lim_{x \to -2^-} (-2 - 4x) = 6 \quad f'_+(2) = \lim_{x \to -2^+} (2 - 2x) = 6
\]
\[
f'(-2) = f'_+(2) \quad \text{▸ The left- and right-hand derivatives are equal.}
\]
∴ Therefore the function is differentiable at \( x = -2 \) and \( f'(-2) = 6 \).

9. \[
\lim_{x \to -2^-} f(x) = \lim_{x \to -2^-} (3 + 2x + 3x^2) = 11 \quad \lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} (-3 - 5x + x^2) = 11
\]
\[
\lim_{x \to -2^-} f(x) = \lim_{x \to -2^+} f(x) \quad \text{▸ The function is continuous at } x = -2.
\]
\[
f'(x) = \begin{cases} 
2 + 6x & \text{if } x < -2 \\
-5 + 2x & \text{if } x > -2
\end{cases}
\]
\[
f'(-2) = \lim_{x \to -2^-} (2 + 6x) = -10 \quad f'_+(2) = \lim_{x \to -2^+} (-5 + 2x) = -9
\]
\[
f'(-2) \neq f'_+(2) \quad \text{▸ The left- and right-hand derivatives are not equal.}
\]
∴ Therefore the function is not differentiable at \( x = -2 \). The point \( P(-2, 11) \) is a corner point.

10. \[
\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x + 2x^2) = 3 \quad \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (-1 + 3x + x^2) = 3
\]
\[
\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) \quad \text{▸ The function is continuous at } x = 1.
\]
\[
f'(x) = \begin{cases} 
1 + 4x & \text{if } x < 1 \\
3 + 2x & \text{if } x > 1
\end{cases}
\]
\[
f'_-(1) = \lim_{x \to 1^-} (1 + 4x) = 5 \quad f'_+(1) = \lim_{x \to 1^+} (3 + 2x) = 5
\]
\[
f'_-(1) = f'_+(1) \quad \text{▸ The left- and right-hand derivatives are equal.}
\]
∴ Therefore the function is differentiable at \( x = 1 \) and \( f'(1) = 5 \).