1. Consider the following piece-wise defined function: $f(x) = \begin{cases} -3x - 3x^2 & \text{if } x < -1 \\ 2 + x - x^2 & \text{if } x \geqslant -1 \end{cases}$
Analyze the differentiability of the function $f(x)$ at $x = -1$.

2. Consider the following piece-wise defined function: $f(x) = \begin{cases} 1 - 2x^2 & \text{if } x < 1 \\ 2 - 2x - x^2 & \text{if } x \geqslant 1 \end{cases}$
Analyze the differentiability of the function $f(x)$ at $x = 1$.

3. Consider the following piece-wise defined function: $f(x) = \begin{cases} -2 - 3x - 3x^2 & \text{if } x < -2 \\ 9 + 6x - x^2 & \text{if } x \geqslant -2 \end{cases}$
Analyze the differentiability of the function $f(x)$ at $x = -2$.

4. Consider the following piece-wise defined function: $f(x) = \begin{cases} 3 + x + x^2 & \text{if } x < 1 \\ 2 + 5x - x^2 & \text{if } x \geqslant 1 \end{cases}$
Analyze the differentiability of the function $f(x)$ at $x = 1$.

5. Consider the following piece-wise defined function: $f(x) = \begin{cases} 2 - x^2 & \text{if } x < 2 \\ 9 - 7x + x^2 & \text{if } x \geqslant 2 \end{cases}$
Analyze the differentiability of the function $f(x)$ at $x = 2$.

6. Consider the following piece-wise defined function: $f(x) = \begin{cases} 2 + 3x - x^2 & \text{if } x < -1 \\ 5 + 7x + x^2 & \text{if } x \geqslant -1 \end{cases}$
Analyze the differentiability of the function $f(x)$ at $x = -1$.

7. Consider the following piece-wise defined function: $f(x) = \begin{cases} -x - x^2 & \text{if } x < 2 \\ 6 - 8x + x^2 & \text{if } x \geqslant 2 \end{cases}$
Analyze the differentiability of the function $f(x)$ at $x = 2$.

8. Consider the following piece-wise defined function: $f(x) = \begin{cases} -2 - 2x - 2x^2 & \text{if } x < -2 \\ 2 + 2x - x^2 & \text{if } x \geqslant -2 \end{cases}$
Analyze the differentiability of the function $f(x)$ at $x = -2$.

9. Consider the following piece-wise defined function: $f(x) = \begin{cases} 3 + 2x + 3x^2 & \text{if } x < -2 \\ -3 - 5x + x^2 & \text{if } x \geqslant -2 \end{cases}$
Analyze the differentiability of the function $f(x)$ at $x = -2$.

10. Consider the following piece-wise defined function: $f(x) = \begin{cases} x + 2x^2 & \text{if } x < 1 \\ -1 + 3x + x^2 & \text{if } x \geqslant 1 \end{cases}$
Analyze the differentiability of the function $f(x)$ at $x = 1$.

Answers:

1. differentiable
2. differentiable
3. not differentiable (jump discontinuity)
4. not differentiable (jump discontinuity)
5. not differentiable (jump discontinuity)
6. not differentiable (jump discontinuity)
7. not differentiable (corner point)
8. differentiable
9. not differentiable (corner point)
10. differentiable

Solutions:

1. $\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} -3x - 3x^2 = 0$ $\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} 2 + x - x^2 = 0$

$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} f(x)$ ► The funtion is continuous at $x = -1$.

$$f'(x) = \begin{cases} -3 - 6x & \text{if } x < -1 \\ 1 - 2x & \text{if } x > -1 \end{cases}$$

$f'_-(-1) = \lim_{x\to -1^-} -3 - 6x = 3$ $f'_+(-1) = \lim_{x\to -1^+} 1 - 2x = 3$

$f'_-(-1) = f'_+(-1)$ ► The left- and right-hand derivatives are equal.

∴ Therefore the function is differentiable at $x = -1$ and $f'(-1) = 3$.

2. $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 1 - 2x^2 = -1$ $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 2 - 2x - x^2 = -1$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$ ► The funtion is continuous at $x = 1$.

$$f'(x) = \begin{cases} -4x & \text{if } x < 1 \\ -2 - 2x & \text{if } x > 1 \end{cases}$$

$f'_-(1) = \lim_{x\to 1^-} -4x = -4$ $f'_+(1) = \lim_{x\to 1^+} -2 - 2x = -4$

$f'_-(1) = f'_+(1)$ ► The left- and right-hand derivatives are equal.

∴ Therefore the function is differentiable at $x = 1$ and $f'(1) = -4$.

3. $\lim_{x\to -2^-} f(x) = \lim_{x\to -2^-} -2 - 3x - 3x^2 = -8$ $\lim_{x\to -2^+} f(x) = \lim_{x\to -2^+} 9 + 6x - x^2 = -7$

$\lim_{x\to -2^-} f(x) \neq \lim_{x\to -2^+} f(x)$ ► The function is not continuous at $x = -2$.

∴ Therefore the function is not differentiable at $x = -2$. There is a jump discontinuity at $x = -2$.

4. $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 3 + x + x^2 = 5$ $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 2 + 5x - x^2 = 6$

$\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$ ► The function is not continuous at $x = 1$.

∴ Therefore the function is not differentiable at $x = 1$. There is a jump discontinuity at $x = 1$.

5. $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} 2 - x^2 = -2$ $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} 9 - 7x + x^2 = -1$

$\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)$ ► The function is not continuous at $x = 2$.

∴ Therefore the function is not differentiable at $x = 2$. There is a jump discontinuity at $x = 2$.

6. $\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} 2 + 3x - x^2 = -2$ $\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} 5 + 7x + x^2 = -1$

$\lim_{x\to -1^-} f(x) \neq \lim_{x\to -1^+} f(x)$ ► The function is not continuous at $x = -1$.

∴ Therefore the function is not differentiable at $x = -1$. There is a jump discontinuity at $x = -1$.

7. $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} -x - x^2 = -6$ $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} 6 - 8x + x^2 = -6$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$ ► The funtion is continuous at $x = 2$.

$$f'(x) = \begin{cases} -1 - 2x & \text{if } x < 2 \\ -8 + 2x & \text{if } x > 2 \end{cases}$$

$f'_-(2) = \lim_{x\to 2^-} -1 - 2x = -5$ $f'_+(2) = \lim_{x\to 2^+} -8 + 2x = -4$

$f'_{-}(2) \neq f'_{+}(2)$ ► The left- and right-hand derivatives are not equal.

$\therefore$ Therefore the function is not differentiable at $x = 2$. The point $P(2, -6)$ is a corner point.

8. $\lim\limits_{x \to -2^-} f(x) = \lim\limits_{x \to -2^-} -2 - 2x - 2x^2 = -6$ $\lim\limits_{x \to -2^+} f(x) = \lim\limits_{x \to -2^+} 2 + 2x - x^2 = -6$

$\lim\limits_{x \to -2^-} f(x) = \lim\limits_{x \to -2^+} f(x)$ ► The funtion is continuous at $x = -2$.

$$f'(x) = \begin{cases} -2 - 4x & \text{if } x < -2 \\ 2 - 2x & \text{if } x > -2 \end{cases}$$

$f'_{-}(-2) = \lim\limits_{x \to -2^-} -2 - 4x = 6$ $f'_{+}(-2) = \lim\limits_{x \to -2^+} 2 - 2x = 6$

$f'_{-}(-2) = f'_{+}(-2)$ ► The left- and right-hand derivatives are equal.

$\therefore$ Therefore the function is differentiable at $x = -2$ and $f'(-2) = 6$.

9. $\lim\limits_{x \to -2^-} f(x) = \lim\limits_{x \to -2^-} 3 + 2x + 3x^2 = 11$ $\lim\limits_{x \to -2^+} f(x) = \lim\limits_{x \to -2^+} -3 - 5x + x^2 = 11$

$\lim\limits_{x \to -2^-} f(x) = \lim\limits_{x \to -2^+} f(x)$ ► The funtion is continuous at $x = -2$.

$$f'(x) = \begin{cases} 2 + 6x & \text{if } x < -2 \\ -5 + 2x & \text{if } x > -2 \end{cases}$$

$f'_{-}(-2) = \lim\limits_{x \to -2^-} 2 + 6x = -10$ $f'_{+}(-2) = \lim\limits_{x \to -2^+} -5 + 2x = -9$

$f'_{-}(-2) \neq f'_{+}(-2)$ ► The left- and right-hand derivatives are not equal.

$\therefore$ Therefore the function is not differentiable at $x = -2$. The point $P(-2, 11)$ is a corner point.

10. $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} x + 2x^2 = 3$ $\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} -1 + 3x + x^2 = 3$

$\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^+} f(x)$ ► The funtion is continuous at $x = 1$.

$$f'(x) = \begin{cases} 1 + 4x & \text{if } x < 1 \\ 3 + 2x & \text{if } x > 1 \end{cases}$$

$f'_{-}(1) = \lim\limits_{x \to 1^-} 1 + 4x = 5$ $f'_{+}(1) = \lim\limits_{x \to 1^+} 3 + 2x = 5$

$f'_{-}(1) = f'_{+}(1)$ ► The left- and right-hand derivatives are equal.

$\therefore$ Therefore the function is differentiable at $x = 1$ and $f'(1) = 5$.