1. Find the point of tangency and the equation of the tangent line passing through the point $P(-7, -3)$ to the graph of the function: $f(x) = \sqrt{x}$.

2. Find the point of tangency and the equation of the tangent line passing through the point $P(0, -3)$ to the graph of the function: $f(x) = \sqrt{x}$.

3. Find the point of tangency and the equation of the tangent line passing through the point $P(12, 4)$ to the graph of the function: $f(x) = \sqrt{x}$.

4. Find the point of tangency and the equation of the tangent line passing through the point $P(5, 3)$ to the graph of the function: $f(x) = \sqrt{x}$.

5. Find the point of tangency and the equation of the tangent line passing through the point $P(3, 2)$ to the graph of the function: $f(x) = \sqrt{x}$.

6. Find the point of tangency and the equation of the tangent line passing through the point $P(8, -3)$ to the graph of the function: $f(x) = \sqrt{x}$.

7. Find the point of tangency and the equation of the tangent line passing through the point $P(24, -5)$ to the graph of the function: $f(x) = \sqrt{x}$.

8. Find the point of tangency and the equation of the tangent line passing through the point $P(-9, 0)$ to the graph of the function: $f(x) = \sqrt{x}$.

9. Find the point of tangency and the equation of the tangent line passing through the point $P(-12, -2)$ to the graph of the function: $f(x) = \sqrt{x}$.

10. Find the point of tangency and the equation of the tangent line passing through the point $P(24, 5)$ to the graph of the function: $f(x) = \sqrt{x}$.

Answers:

1. $y = \frac{1}{2}x + \frac{1}{2}$
2. No solution.
3. $y = \frac{1}{4}x + 1$ $Q(4, 2)$ or $y = \frac{1}{12}x + 3$ $Q(36, 6)$
4. $y = \frac{1}{2}x + \frac{1}{2}$ $Q(1, 1)$ or $y = \frac{1}{10}x + \frac{5}{2}$ $Q(25, 5)$
5. $y = \frac{1}{2}x + \frac{1}{2}$ $Q(1, 1)$ or $y = \frac{1}{6}x + \frac{3}{2}$ $Q(9, 3)$
6. No solution.
7. No solution.
8. $y = \frac{1}{6}x + \frac{3}{2}$
9. $y = \frac{1}{4}x + 1$
10. $y = \frac{1}{8}x + 2$ $Q(16, 4)$ or $y = \frac{1}{12}x + 3$ $Q(36, 6)$

Solutions:

1. The slope of the line segment passing through the given point $P\,(-7,-3)$ and the point of tangency $Q(x,y)$ is given by $m=\dfrac{y-(-3)}{x-(-7)}$ (1) where $y=\sqrt{x}$ (2)

The slope of the tangent line at the point of tangency $Q(x,y)$ is given by the first derivative:

$m=f'(x)=\dfrac{1}{2\sqrt{x}}$ (3)

Let use (1) and (2) and (3): $\dfrac{\sqrt{x}-(-3)}{x-(-7)}=\dfrac{1}{2\sqrt{x}}$ (4)

Let square (2) to get: $x=y^2$ and change (4) to: $\dfrac{y-(-3)}{y^2-(-7)}=\dfrac{1}{2y}$ or by cross-multiplication: $2y^2-2(-3)y=y^2-(-7)$ and finally: $y^2+6y-7=0$ (5)

Use quadratic formula to solve (5): $x=\dfrac{-6\pm\sqrt{(-6)^2-4\times(-7)}}{2}=-3\pm 4$

According to (2) both x and y are positive or zero. So, there is one solution.

$y=-3+4=1$; $x=(1)^2=1$; $m=\frac{1}{2\sqrt{1}}=\frac{1}{2}$; Point of tangency: $Q(1,1)$

Tangent line: $y-(1)=\frac{1}{2}(x-(1))$ or $y=\frac{1}{2}x+\frac{1}{2}$

2. The slope of the line segment passing through the given point $P\,(0,-3)$ and the point of tangency $Q(x,y)$ is given by $m=\dfrac{y-(-3)}{x-(0)}$ (1) where $y=\sqrt{x}$ (2)

The slope of the tangent line at the point of tangency $Q(x,y)$ is given by the first derivative:

$m=f'(x)=\dfrac{1}{2\sqrt{x}}$ (3)

Let use (1) and (2) and (3): $\dfrac{\sqrt{x}-(-3)}{x-(0)}=\dfrac{1}{2\sqrt{x}}$ (4)

Let square (2) to get: $x=y^2$ and change (4) to: $\dfrac{y-(-3)}{y^2-(0)}=\dfrac{1}{2y}$ or by cross-multiplication: $2y^2-2(-3)y=y^2-(0)$ and finally: $y^2+6y+0=0$ (5)

Use quadratic formula to solve (5): $x=\dfrac{-6\pm\sqrt{(-6)^2-4\times(0)}}{2}=-3\pm 3$

According to (2) both x and y are positive or zero. So, there is no solution.

3. The slope of the line segment passing through the given point $P\,(12,4)$ and the point of tangency $Q(x,y)$ is given by $m=\dfrac{y-(4)}{x-(12)}$ (1) where $y=\sqrt{x}$ (2)

The slope of the tangent line at the point of tangency $Q(x,y)$ is given by the first derivative:

$m=f'(x)=\dfrac{1}{2\sqrt{x}}$ (3)

Let use (1) and (2) and (3): $\dfrac{\sqrt{x}-(4)}{x-(12)}=\dfrac{1}{2\sqrt{x}}$ (4)

Let square (2) to get: $x=y^2$ and change (4) to: $\dfrac{y-(4)}{y^2-(12)}=\dfrac{1}{2y}$ or by cross-multiplication: $2y^2-2(4)y=y^2-(12)$ and finally: $y^2-8y+12=0$ (5)

Use quadratic formula to solve (5): $x=\dfrac{8\pm\sqrt{(8)^2-4\times(12)}}{2}=4\pm 2$

According to (2) both x and y are positive or zero. So, there are two solutions.

Solution 1 $y = 4 - 2 = 2$; $x = (2)^2 = 4$; $m = \frac{1}{2\sqrt{4}} = \frac{1}{4}$; Point of tangency: $Q(4, 2)$

Tangent line: $y - (2) = \frac{1}{4}(x - (4))$ or $y = \frac{1}{4}x + 1$

Solution 2 $y = 4 + 2 = 6$; $x = (6)^2 = 36$; $m = \frac{1}{2\sqrt{36}} = \frac{1}{12}$; Point of tangency: $Q(36, 6)$

Tangent line: $y - (6) = \frac{1}{12}(x - (36))$ or $y = \frac{1}{12}x + 3$

4. The slope of the line segment passing through the given point $P\,(5, 3)$ and the point of tangency $Q(x, y)$ is given by $m = \dfrac{y - (3)}{x - (5)}$ (1) where $y = \sqrt{x}$ (2)

The slope of the tangent line at the point of tangency $Q(x, y)$ is given by the first derivative:

$m = f'(x) = \dfrac{1}{2\sqrt{x}}$ (3)

Let use (1) and (2) and (3): $\dfrac{\sqrt{x} - (3)}{x - (5)} = \dfrac{1}{2\sqrt{x}}$ (4)

Let square (2) to get: $x = y^2$ and change (4) to: $\dfrac{y - (3)}{y^2 - (5)} = \dfrac{1}{2y}$ or by cross-multiplication: $2y^2 - 2(3)y = y^2 - (5)$ and finally: $y^2 - 6y + 5 = 0$ (5)

Use quadratic formula to solve (5): $x = \dfrac{6 \pm \sqrt{(6)^2 - 4 \times (5)}}{2} = 3 \pm 2$

According to (2) both x and y are positive or zero. So, there are two solutions.

Solution 1 $y = 3 - 2 = 1$; $x = (1)^2 = 1$; $m = \frac{1}{2\sqrt{1}} = \frac{1}{2}$; Point of tangency: $Q(1, 1)$

Tangent line: $y - (1) = \frac{1}{2}(x - (1))$ or $y = \frac{1}{2}x + \frac{1}{2}$

Solution 2 $y = 3 + 2 = 5$; $x = (5)^2 = 25$; $m = \frac{1}{2\sqrt{25}} = \frac{1}{10}$; Point of tangency: $Q(25, 5)$

Tangent line: $y - (5) = \frac{1}{10}(x - (25))$ or $y = \frac{1}{10}x + \frac{5}{2}$

5. The slope of the line segment passing through the given point $P\,(3, 2)$ and the point of tangency $Q(x, y)$ is given by $m = \dfrac{y - (2)}{x - (3)}$ (1) where $y = \sqrt{x}$ (2)

The slope of the tangent line at the point of tangency $Q(x, y)$ is given by the first derivative:

$m = f'(x) = \dfrac{1}{2\sqrt{x}}$ (3)

Let use (1) and (2) and (3): $\dfrac{\sqrt{x} - (2)}{x - (3)} = \dfrac{1}{2\sqrt{x}}$ (4)

Let square (2) to get: $x = y^2$ and change (4) to: $\dfrac{y - (2)}{y^2 - (3)} = \dfrac{1}{2y}$ or by cross-multiplication: $2y^2 - 2(2)y = y^2 - (3)$ and finally: $y^2 - 4y + 3 = 0$ (5)

Use quadratic formula to solve (5): $x = \dfrac{4 \pm \sqrt{(4)^2 - 4 \times (3)}}{2} = 2 \pm 1$

According to (2) both x and y are positive or zero. So, there are two solutions.

Solution 1 $y = 2 - 1 = 1$; $x = (1)^2 = 1$; $m = \frac{1}{2\sqrt{1}} = \frac{1}{2}$; Point of tangency: $Q(1, 1)$

Tangent line: $y - (1) = \frac{1}{2}(x - (1))$ or $y = \frac{1}{2}x + \frac{1}{2}$

Solution 2 $y = 2 + 1 = 3$; $x = (3)^2 = 9$; $m = \frac{1}{2\sqrt{9}} = \frac{1}{6}$; Point of tangency: $Q(9, 3)$

Tangent line: $y - (3) = \frac{1}{6}(x - (9))$ or $y = \frac{1}{6}x + \frac{3}{2}$

6. The slope of the line segment passing through the given point $P\,(8,-3)$ and the point of tangency $Q(x,y)$ is given by $m=\dfrac{y-(-3)}{x-(8)}$ (1) where $y=\sqrt{x}$ (2)

The slope of the tangent line at the point of tangency $Q(x,y)$ is given by the first derivative:

$m=f'(x)=\dfrac{1}{2\sqrt{x}}$ (3)

Let use (1) and (2) and (3): $\dfrac{\sqrt{x}-(-3)}{x-(8)}=\dfrac{1}{2\sqrt{x}}$ (4)

Let square (2) to get: $x=y^2$ and change (4) to: $\dfrac{y-(-3)}{y^2-(8)}=\dfrac{1}{2y}$ or by cross-multiplication: $2y^2-2(-3)y=y^2-(8)$ and finally: $y^2+6y+8=0$ (5)

Use quadratic formula to solve (5): $x=\dfrac{-6\pm\sqrt{(-6)^2-4\times(8)}}{2}=-3\pm 1$

According to (2) both x and y are positive or zero. So, there is no solution.

7. The slope of the line segment passing through the given point $P\,(24,-5)$ and the point of tangency $Q(x,y)$ is given by $m=\dfrac{y-(-5)}{x-(24)}$ (1) where $y=\sqrt{x}$ (2)

The slope of the tangent line at the point of tangency $Q(x,y)$ is given by the first derivative:

$m=f'(x)=\dfrac{1}{2\sqrt{x}}$ (3)

Let use (1) and (2) and (3): $\dfrac{\sqrt{x}-(-5)}{x-(24)}=\dfrac{1}{2\sqrt{x}}$ (4)

Let square (2) to get: $x=y^2$ and change (4) to: $\dfrac{y-(-5)}{y^2-(24)}=\dfrac{1}{2y}$ or by cross-multiplication: $2y^2-2(-5)y=y^2-(24)$ and finally: $y^2+10y+24=0$ (5)

Use quadratic formula to solve (5): $x=\dfrac{-10\pm\sqrt{(-10)^2-4\times(24)}}{2}=-5\pm 1$

According to (2) both x and y are positive or zero. So, there is no solution.

8. The slope of the line segment passing through the given point $P\,(-9,0)$ and the point of tangency $Q(x,y)$ is given by $m=\dfrac{y-(0)}{x-(-9)}$ (1) where $y=\sqrt{x}$ (2)

The slope of the tangent line at the point of tangency $Q(x,y)$ is given by the first derivative:

$m=f'(x)=\dfrac{1}{2\sqrt{x}}$ (3)

Let use (1) and (2) and (3): $\dfrac{\sqrt{x}-(0)}{x-(-9)}=\dfrac{1}{2\sqrt{x}}$ (4)

Let square (2) to get: $x=y^2$ and change (4) to: $\dfrac{y-(0)}{y^2-(-9)}=\dfrac{1}{2y}$ or by cross-multiplication: $2y^2-2(0)y=y^2-(-9)$ and finally: $y^2+0y-9=0$ (5)

Use quadratic formula to solve (5): $x=\dfrac{0\pm\sqrt{(0)^2-4\times(-9)}}{2}=0\pm 3$

According to (2) both x and y are positive or zero. So, there is one solution.

$y=0+3=3$; $x=(3)^2=9$; $m=\frac{1}{2\sqrt{9}}=\frac{1}{6}$; Point of tangency: $Q(9,3)$

Tangent line: $y-(3)=\frac{1}{6}(x-(9))$ or $y=\frac{1}{6}x+\frac{3}{2}$

9. The slope of the line segment passing through the given point $P(-12,-2)$ and the point of tangency $Q(x,y)$ is given by $m = \frac{y-(-2)}{x-(-12)}$ (1) where $y = \sqrt{x}$ (2)

The slope of the tangent line at the point of tangency $Q(x,y)$ is given by the first derivative:

$m = f'(x) = \frac{1}{2\sqrt{x}}$ (3)

Let use (1) and (2) and (3): $\frac{\sqrt{x}-(-2)}{x-(-12)} = \frac{1}{2\sqrt{x}}$ (4)

Let square (2) to get: $x = y^2$ and change (4) to: $\frac{y-(-2)}{y^2-(-12)} = \frac{1}{2y}$ or by cross-multiplication: $2y^2 - 2(-2)y = y^2 - (-12)$ and finally: $y^2 + 4y - 12 = 0$ (5)

Use quadratic formula to solve (5): $x = \frac{-4 \pm \sqrt{(-4)^2 - 4 \times (-12)}}{2} = -2 \pm 4$

According to (2) both x and y are positive or zero. So, there is one solution.

$y = -2 + 4 = 2$; $x = (2)^2 = 4$; $m = \frac{1}{2\sqrt{4}} = \frac{1}{4}$; Point of tangency: $Q(4,2)$

Tangent line: $y - (2) = \frac{1}{4}(x - (4))$ or $y = \frac{1}{4}x + 1$

10. The slope of the line segment passing through the given point $P(24,5)$ and the point of tangency $Q(x,y)$ is given by $m = \frac{y-(5)}{x-(24)}$ (1) where $y = \sqrt{x}$ (2)

The slope of the tangent line at the point of tangency $Q(x,y)$ is given by the first derivative:

$m = f'(x) = \frac{1}{2\sqrt{x}}$ (3)

Let use (1) and (2) and (3): $\frac{\sqrt{x}-(5)}{x-(24)} = \frac{1}{2\sqrt{x}}$ (4)

Let square (2) to get: $x = y^2$ and change (4) to: $\frac{y-(5)}{y^2-(24)} = \frac{1}{2y}$ or by cross-multiplication: $2y^2 - 2(5)y = y^2 - (24)$ and finally: $y^2 - 10y + 24 = 0$ (5)

Use quadratic formula to solve (5): $x = \frac{10 \pm \sqrt{(10)^2 - 4 \times (24)}}{2} = 5 \pm 1$

According to (2) both x and y are positive or zero. So, there are two solutions.

Solution 1 $y = 5 - 1 = 4$; $x = (4)^2 = 16$; $m = \frac{1}{2\sqrt{16}} = \frac{1}{8}$; Point of tangency: $Q(16,4)$

Tangent line: $y - (4) = \frac{1}{8}(x - (16))$ or $y = \frac{1}{8}x + 2$

Solution 2 $y = 5 + 1 = 6$; $x = (6)^2 = 36$; $m = \frac{1}{2\sqrt{36}} = \frac{1}{12}$; Point of tangency: $Q(36,6)$

Tangent line: $y - (6) = \frac{1}{12}(x - (36))$ or $y = \frac{1}{12}x + 3$