1. Find the equation of the tangent line of the slope $m = \dfrac{1}{9}$ to the graph of the function: $f(x) = \dfrac{-2x-1}{3x+1}$. Find also the point of tangency Q.

2. Find the equation of the tangent line of the slope $m = \dfrac{-1}{2}$ to the graph of the function: $f(x) = \dfrac{-2x+3}{2x-2}$. Find also the point of tangency Q.

3. Find the equation of the tangent line of the slope $m = 3$ to the graph of the function: $f(x) = \dfrac{-3x-3}{-x-2}$. Find also the point of tangency Q.

4. Find the equation of the tangent line of the slope $m = \dfrac{1}{3}$ to the graph of the function: $f(x) = \dfrac{-x-1}{2x-1}$. Find also the point of tangency Q.

5. Find the equation of the tangent line of the slope $m = \dfrac{3}{4}$ to the graph of the function: $f(x) = \dfrac{-3x+3}{-x}$. Find also the point of tangency Q.

6. Find the equation of the tangent line of the slope $m = -1$ to the graph of the function: $f(x) = \dfrac{2x+1}{-x-1}$. Find also the point of tangency Q.

7. Find the equation of the tangent line of the slope $m = \dfrac{9}{4}$ to the graph of the function: $f(x) = \dfrac{2x-1}{3x+3}$. Find also the point of tangency Q.

8. Find the equation of the tangent line of the slope $m = \dfrac{1}{2}$ to the graph of the function: $f(x) = \dfrac{2x}{-x+1}$. Find also the point of tangency Q.

9. Find the equation of the tangent line of the slope $m = -3$ to the graph of the function: $f(x) = \dfrac{x+1}{2x-1}$. Find also the point of tangency Q.

10. Find the equation of the tangent line of the slope $m = \dfrac{-2}{3}$ to the graph of the function: $f(x) = \dfrac{2x}{-3x-3}$. Find also the point of tangency Q.

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Answers:

1. $y = \frac{1}{9}x + \frac{-11}{27}$ $\quad Q_1 = \left(\frac{-4}{3}, \frac{-5}{9}\right)$ $\quad$ or $\quad y = \frac{1}{9}x + \frac{-23}{27}$ $\quad Q_2 = \left(\frac{2}{3}, \frac{-7}{9}\right)$
2. $y = \frac{-1}{2}x + \frac{-3}{2}$ $\quad Q_1 = \left(0, \frac{-3}{2}\right)$ $\quad$ or $\quad y = \frac{-1}{2}x + \frac{1}{2}$ $\quad Q_2 = \left(2, \frac{-1}{2}\right)$
3. $y = 3x + 3$ $\quad Q_1 = (-1, 0)$ $\quad$ or $\quad y = 3x + 15$ $\quad Q_2 = (-3, 6)$
4. $y = \frac{1}{3}x + \frac{1}{3}$ $\quad Q_1 = (-1, 0)$ $\quad$ or $\quad y = \frac{1}{3}x + \frac{-5}{3}$ $\quad Q_2 = (2, -1)$
5. $y = \frac{3}{4}x + 0$ $\quad Q_1 = \left(2, \frac{3}{2}\right)$ $\quad$ or $\quad y = \frac{3}{4}x + 6$ $\quad Q_2 = \left(-2, \frac{9}{2}\right)$
6. $y = -1x - 1$ $\quad Q_1 = (0, -1)$ $\quad$ or $\quad y = -1x - 5$ $\quad Q_2 = (-2, -3)$
7. $y = \frac{9}{4}x + \frac{71}{12}$ $\quad Q_1 = \left(\frac{-5}{3}, \frac{13}{6}\right)$ $\quad$ or $\quad y = \frac{9}{4}x + \frac{-1}{12}$ $\quad Q_2 = \left(\frac{-1}{3}, \frac{-5}{6}\right)$
8. $y = \frac{1}{2}x + \frac{-9}{2}$ $\quad Q_1 = (3, -3)$ $\quad$ or $\quad y = \frac{1}{2}x + \frac{-1}{2}$ $\quad Q_2 = (-1, -1)$
9. $y = -3x - 1$ $\quad Q_1 = (0, -1)$ $\quad$ or $\quad y = -3x + 5$ $\quad Q_2 = (1, 2)$
10. $y = \frac{-2}{3}x + 0$ $\quad Q_1 = (0, 0)$ $\quad$ or $\quad y = \frac{-2}{3}x + \frac{-8}{3}$ $\quad Q_2 = \left(-2, \frac{-4}{3}\right)$

Solutions:

1. $f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{-2x-1}{3x+1} = \dfrac{1}{(3x+1)^2}$ ◀ Find the first derivative of the function.

$\dfrac{1}{9} = \dfrac{1}{(3x+1)^2}$ ◀ Use: $m = f'(x)$

$(3x+1)^2 = 9 \qquad 3x+1 = \pm 3 \qquad x = \dfrac{-1 \pm 3}{3}$ ◀ Solve for x. ▶ There are two solutions:

Solution 1: $x_1 = \dfrac{-4}{3} \qquad y_1 = \dfrac{(-2)\left(\frac{-4}{3}\right) - 1}{(3)\left(\frac{-4}{3}\right) + 1} = \dfrac{-5}{9}$ Point of tangency: $Q_1 = \left(\frac{-4}{3}, \frac{-5}{9}\right)$

Equation of tangent line: $y - \left(\dfrac{-5}{9}\right) = \dfrac{1}{9}\left[x - \left(\dfrac{-4}{3}\right)\right]$ or $y = \frac{1}{9}x + \frac{-11}{27}$

Solution 2: $x_2 = \dfrac{2}{3} \qquad y_2 = \dfrac{(-2)\left(\frac{2}{3}\right) - 1}{(3)\left(\frac{2}{3}\right) + 1} = \dfrac{-7}{9}$ Point of tangency: $Q_2 = \left(\frac{2}{3}, \frac{-7}{9}\right)$

Equation of tangent line: $y - \left(\dfrac{-7}{9}\right) = \dfrac{1}{9}\left[x - \left(\dfrac{2}{3}\right)\right]$ or $y = \frac{1}{9}x + \frac{-23}{27}$

2. $f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{-2x+3}{2x-2} = \dfrac{-2}{(2x-2)^2}$ ◀ Find the first derivative of the function.

$\dfrac{-1}{2} = \dfrac{-2}{(2x-2)^2}$ ◀ Use: $m = f'(x)$

$(2x-2)^2 = 4 \qquad 2x-2 = \pm 2 \qquad x = \dfrac{--2 \pm 2}{2}$ ◀ Solve for x. ▶ There are two solutions:

Solution 1: $x_1 = 0 \qquad y_1 = \dfrac{(-2)(0) + 3}{(2)(0) - 2} = \dfrac{-3}{2}$ Point of tangency: $Q_1 = \left(0, \frac{-3}{2}\right)$

Equation of tangent line: $y - \left(\dfrac{-3}{2}\right) = \dfrac{-1}{2}\left[x - (0)\right]$ or $y = \frac{-1}{2}x + \frac{-3}{2}$

Solution 2: $x_2 = 2 \qquad y_2 = \dfrac{(-2)(2) + 3}{(2)(2) - 2} = \dfrac{-1}{2}$ Point of tangency: $Q_2 = \left(2, \frac{-1}{2}\right)$

Equation of tangent line: $y - \left(\dfrac{-1}{2}\right) = \dfrac{-1}{2}\left[x - (2)\right]$ or $y = \frac{-1}{2}x + \frac{1}{2}$

3. $f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{-3x-3}{-x-2} = \dfrac{3}{(-x-2)^2}$ ◀ Find the first derivative of the function.

$3 = \dfrac{3}{(-x-2)^2}$ ◀ Use: $m = f'(x)$

$(-x-2)^2 = 1 \qquad -x-2 = \pm 1 \qquad x = \dfrac{--2 \pm 1}{-1}$ ◀ Solve for x. ▶ There are two solutions:

Solution 1: $x_1 = -1 \qquad y_1 = \dfrac{(-3)(-1) - 3}{(-1)(-1) - 2} = 0$ Point of tangency: $Q_1 = (-1, 0)$

Equation of tangent line: $y - (0) = 3\left[x - (-1)\right]$ or $y = 3x + 3$

Solution 2: $x_2 = -3 \qquad y_2 = \dfrac{(-3)(-3) - 3}{(-1)(-3) - 2} = 6$ Point of tangency: $Q_2 = (-3, 6)$

Equation of tangent line: $y - (6) = 3\left[x - (-3)\right]$ or $y = 3x + 15$

4. $f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{-x-1}{2x-1} = \dfrac{3}{(2x-1)^2}$ ◀ Find the first derivative of the function.

$\frac{1}{3} = \frac{3}{(2x-1)^2}$ ◀ Use: $m = f'(x)$

$(2x-1)^2 = 9 \qquad 2x - 1 = \pm 3 \qquad x = \frac{--1 \pm 3}{2}$ ◀ Solve for x. ▶ There are two solutions:

Solution 1: $x_1 = -1 \qquad y_1 = \frac{(-1)\,(-1) - 1}{(2)\,(-1) - 1} = 0$ Point of tangency: $Q_1 = (-1, 0)$

Equation of tangent line: $y - (0) = \frac{1}{3}\,[x - (-1)]$ or $y = \frac{1}{3}x + \frac{1}{3}$

Solution 2: $x_2 = 2 \qquad y_2 = \frac{(-1)\,(2) - 1}{(2)\,(2) - 1} = -1$ Point of tangency: $Q_2 = (2, -1)$

Equation of tangent line: $y - (-1) = \frac{1}{3}\,[x - (2)]$ or $y = \frac{1}{3}x + \frac{-5}{3}$

5. $f'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\frac{-3x+3}{-x} = \frac{3}{(-x)^2}$ ◀ Find the first derivative of the function.

$\frac{3}{4} = \frac{3}{(-x)^2}$ ◀ Use: $m = f'(x)$

$(-x)^2 = 4 \qquad -x = \pm 2 \qquad x = \frac{-0 \pm 2}{-1}$ ◀ Solve for x. ▶ There are two solutions:

Solution 1: $x_1 = 2 \qquad y_1 = \frac{(-3)\,(2) + 3}{(-1)\,(2) + 0} = \frac{3}{2}$ Point of tangency: $Q_1 = \left(2, \frac{3}{2}\right)$

Equation of tangent line: $y - \left(\frac{3}{2}\right) = \frac{3}{4}\,[x - (2)]$ or $y = \frac{3}{4}x + 0$

Solution 2: $x_2 = -2 \qquad y_2 = \frac{(-3)\,(-2) + 3}{(-1)\,(-2) + 0} = \frac{9}{2}$ Point of tangency: $Q_2 = \left(-2, \frac{9}{2}\right)$

Equation of tangent line: $y - \left(\frac{9}{2}\right) = \frac{3}{4}\,[x - (-2)]$ or $y = \frac{3}{4}x + 6$

6. $f'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\frac{2x+1}{-x-1} = \frac{-1}{(-x-1)^2}$ ◀ Find the first derivative of the function.

$-1 = \frac{-1}{(-x-1)^2}$ ◀ Use: $m = f'(x)$

$(-x-1)^2 = 1 \qquad -x - 1 = \pm 1 \qquad x = \frac{--1 \pm 1}{-1}$ ◀ Solve for x. ▶ There are two solutions:

Solution 1: $x_1 = 0 \qquad y_1 = \frac{(2)\,(0) + 1}{(-1)\,(0) - 1} = -1$ Point of tangency: $Q_1 = (0, -1)$

Equation of tangent line: $y - (-1) = -1\,[x - (0)]$ or $y = -x - 1$

Solution 2: $x_2 = -2 \qquad y_2 = \frac{(2)\,(-2) + 1}{(-1)\,(-2) - 1} = -3$ Point of tangency: $Q_2 = (-2, -3)$

Equation of tangent line: $y - (-3) = -1\,[x - (-2)]$ or $y = -x - 5$

7. $f'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\frac{2x-1}{3x+3} = \frac{9}{(3x+3)^2}$ ◀ Find the first derivative of the function.

$\frac{9}{4} = \frac{9}{(3x+3)^2}$ ◀ Use: $m = f'(x)$

$(3x+3)^2 = 4 \qquad 3x + 3 = \pm 2 \qquad x = \frac{-3 \pm 2}{3}$ ◀ Solve for x. ▶ There are two solutions:

Solution 1: $x_1 = \frac{-5}{3}$ $y_1 = \frac{(2)\left(\frac{-5}{3}\right) - 1}{(3)\left(\frac{-5}{3}\right) + 3} = \frac{13}{6}$ Point of tangency: $Q_1 = \left(\frac{-5}{3}, \frac{13}{6}\right)$

Equation of tangent line: $y - \left(\frac{13}{6}\right) = \frac{9}{4}\left[x - \left(\frac{-5}{3}\right)\right]$ or $y = \frac{9}{4}x + \frac{71}{12}$

Solution 2: $x_2 = \frac{-1}{3}$ $y_2 = \frac{(2)\left(\frac{-1}{3}\right) - 1}{(3)\left(\frac{-1}{3}\right) + 3} = \frac{-5}{6}$ Point of tangency: $Q_2 = \left(\frac{-1}{3}, \frac{-5}{6}\right)$

Equation of tangent line: $y - \left(\frac{-5}{6}\right) = \frac{9}{4}\left[x - \left(\frac{-1}{3}\right)\right]$ or $y = \frac{9}{4}x + \frac{-1}{12}$

8. $f'(x) = \frac{d}{dx}\frac{2x}{-x+1} = \frac{2}{(-x+1)^2}$ ◀ Find the first derivative of the function.

$\frac{1}{2} = \frac{2}{(-x+1)^2}$ ◀ Use: $m = f'(x)$

$(-x+1)^2 = 4$ $\quad -x+1 = \pm 2$ $\quad x = \frac{-1 \pm 2}{-1}$ ◀ Solve for x. ▶ There are two solutions:

Solution 1: $x_1 = 3$ $y_1 = \frac{(2)(3) + 0}{(-1)(3) + 1} = -3$ Point of tangency: $Q_1 = (3, -3)$

Equation of tangent line: $y - (-3) = \frac{1}{2}[x - (3)]$ or $y = \frac{1}{2}x + \frac{-9}{2}$

Solution 2: $x_2 = -1$ $y_2 = \frac{(2)(-1) + 0}{(-1)(-1) + 1} = -1$ Point of tangency: $Q_2 = (-1, -1)$

Equation of tangent line: $y - (-1) = \frac{1}{2}[x - (-1)]$ or $y = \frac{1}{2}x + \frac{-1}{2}$

9. $f'(x) = \frac{d}{dx}\frac{x+1}{2x-1} = \frac{-3}{(2x-1)^2}$ ◀ Find the first derivative of the function.

$-3 = \frac{-3}{(2x-1)^2}$ ◀ Use: $m = f'(x)$

$(2x-1)^2 = 1$ $\quad 2x-1 = \pm 1$ $\quad x = \frac{--1 \pm 1}{2}$ ◀ Solve for x. ▶ There are two solutions:

Solution 1: $x_1 = 0$ $y_1 = \frac{(1)(0) + 1}{(2)(0) - 1} = -1$ Point of tangency: $Q_1 = (0, -1)$

Equation of tangent line: $y - (-1) = -3[x - (0)]$ or $y = -3x - 1$

Solution 2: $x_2 = 1$ $y_2 = \frac{(1)(1) + 1}{(2)(1) - 1} = 2$ Point of tangency: $Q_2 = (1, 2)$

Equation of tangent line: $y - (2) = -3[x - (1)]$ or $y = -3x + 5$

10. $f'(x) = \frac{d}{dx}\frac{2x}{-3x-3} = \frac{-6}{(-3x-3)^2}$ ◀ Find the first derivative of the function.

$\frac{-2}{3} = \frac{-6}{(-3x-3)^2}$ ◀ Use: $m = f'(x)$

$(-3x-3)^2 = 9$ $\quad -3x-3 = \pm 3$ $\quad x = \frac{--3 \pm 3}{-3}$ ◀ Solve for x. ▶ There are two solutions:

Solution 1: $x_1 = 0$ $y_1 = \frac{(2)(0) + 0}{(-3)(0) - 3} = 0$ Point of tangency: $Q_1 = (0, 0)$

Equation of tangent line: $y - (0) = \frac{-2}{3}[x - (0)]$ or $y = \frac{-2}{3}x + 0$

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Solution 2: $x_2 = -2$ $y_2 = \dfrac{(2)(-2)+0}{(-3)(-2)-3} = \dfrac{-4}{3}$ Point of tangency: $Q_2 = \left(-2, \frac{-4}{3}\right)$

Equation of tangent line: $y - \left(\dfrac{-4}{3}\right) = \dfrac{-2}{3}\left[x - (-2)\right]$ or $y = \frac{-2}{3}x + \frac{-8}{3}$