1. Find the point of tangency and the equation of the tangent line passing through the point $P(-1,-23)$ to the graph of the function: $f(x) = 3 + 2x + 3x^2$.

2. Find the point of tangency and the equation of the tangent line passing through the point $P(-2,-17)$ to the graph of the function: $f(x) = 5 + x + 4x^2$.

3. Find the point of tangency and the equation of the tangent line passing through the point $P(-3,-2)$ to the graph of the function: $f(x) = 1 + x + 5x^2$.

4. Find the point of tangency and the equation of the tangent line passing through the point $P(-3,17)$ to the graph of the function: $f(x) = 5 - 4x - 5x^2$.

5. Find the point of tangency and the equation of the tangent line passing through the point $P(-5,-50)$ to the graph of the function: $f(x) = -2 - 3x^2$.

6. Find the point of tangency and the equation of the tangent line passing through the point $P(0,-23)$ to the graph of the function: $f(x) = 4 + 4x + 3x^2$.

7. Find the point of tangency and the equation of the tangent line passing through the point $P(-1,-2)$ to the graph of the function: $f(x) = -2 + 2x^2$.

8. Find the point of tangency and the equation of the tangent line passing through the point $P(-2,13)$ to the graph of the function: $f(x) = 2 - x + 3x^2$.

9. Find the point of tangency and the equation of the tangent line passing through the point $P(-3,11)$ to the graph of the function: $f(x) = -3 + 2x + 4x^2$.

10. Find the point of tangency and the equation of the tangent line passing through the point $P(-4,-2)$ to the graph of the function: $f(x) = -2 - 3x - x^2$.

Answers:

1. $y = -22x - 45$ $Q(-4,43)$ or $y = 14x - 9$ $Q(2,19)$
2. $y = -39x - 95$ $Q(-5,100)$ or $y = 9x + 1$ $Q(1,10)$
3. $y = -59x - 179$ $Q(-6,175)$ or $y = x + 1$ $Q(0,1)$
4. $y = 56x + 185$ $Q(-6,-151)$ or $y = -4x + 5$ $Q(0,5)$
5. $y = 48x + 190$ $Q(-8,-194)$ or $y = 12x + 10$ $Q(-2,-14)$
6. $y = -14x - 23$ $Q(-3,19)$ or $y = 22x - 23$ $Q(3,43)$
7. $y = -8x - 10$ $Q(-2,6)$ or $y = -2$ $Q(0,-2)$
8. $y = -19x - 25$ $Q(-3,32)$ or $y = -7x - 1$ $Q(-1,6)$
9. $y = -38x - 103$ $Q(-5,87)$ or $y = -6x - 7$ $Q(-1,-1)$
10. $y = 9x + 34$ $Q(-6,-20)$ or $y = x + 2$ $Q(-2,0)$

Solutions:

1. The slope of the line segment passing through the given point $P(-1,-23)$ and the point of tangency $Q(x,y)$ is given by $m = \dfrac{y-(-23)}{x-(-1)}$ (1) where $y = 3+2x+3x^2$ (2)

The slope of the tangent line at the point of tangency $Q(x,y)$ is given by the first derivative:

$m = f'(x) = 2+6x$ (3)

Let use (1) and (2) and (3) : $\dfrac{3+2x+3x^2-(-23)}{x-(-1)} = 2+6x$ (4)

Cross multiply (4) and simplify: $x^2+(2)x-8=0$ (5)

Use quadratic formula to solve (5): $x = \dfrac{-(2) \pm \sqrt{(2)^2-4\times(-8)}}{2} = -1 \pm 3$

Solution 1: $x=-4$; $y = 3+2(-4)+3(-4)^2 = 43$; $m = 2+6(-4) = -22$

Point of tangency: $Q(-4,43)$

Equation of the tangent line: $y-(43) = -22(x-(-4))$ or $y = -22x-45$

Solution 2: $x=2$; $y = 3+2(2)+3(2)^2 = 19$; $m = 2+6(2) = 14$

Point of tangency: $Q(2,19)$

Equation of the tangent line: $y-(19) = 14(x-(2))$ or $y = 14x-9$

2. The slope of the line segment passing through the given point $P(-2,-17)$ and the point of tangency $Q(x,y)$ is given by $m = \dfrac{y-(-17)}{x-(-2)}$ (1) where $y = 5+x+4x^2$ (2)

The slope of the tangent line at the point of tangency $Q(x,y)$ is given by the first derivative:

$m = f'(x) = 1+8x$ (3)

Let use (1) and (2) and (3) : $\dfrac{5+x+4x^2-(-17)}{x-(-2)} = 1+8x$ (4)

Cross multiply (4) and simplify: $x^2+(4)x-5=0$ (5)

Use quadratic formula to solve (5): $x = \dfrac{-(4) \pm \sqrt{(4)^2-4\times(-5)}}{2} = -2 \pm 3$

Solution 1: $x=-5$; $y = 5+(-5)+4(-5)^2 = 100$; $m = 1+8(-5) = -39$

Point of tangency: $Q(-5,100)$

Equation of the tangent line: $y-(100) = -39(x-(-5))$ or $y = -39x-95$

Solution 2: $x=1$; $y = 5+(1)+4(1)^2 = 10$; $m = 1+8(1) = 9$

Point of tangency: $Q(1,10)$

Equation of the tangent line: $y-(10) = 9(x-(1))$ or $y = 9x+1$

3. The slope of the line segment passing through the given point $P(-3,-2)$ and the point of tangency $Q(x,y)$ is given by $m = \dfrac{y-(-2)}{x-(-3)}$ (1) where $y = 1+x+5x^2$ (2)

The slope of the tangent line at the point of tangency $Q(x,y)$ is given by the first derivative:

$m = f'(x) = 1+10x$ (3)

Let use (1) and (2) and (3) : $\dfrac{1+x+5x^2-(-2)}{x-(-3)} = 1+10x$ (4)

Cross multiply (4) and simplify: $x^2+(6)x+0=0$ (5)

Use quadratic formula to solve (5): $x = \dfrac{-(6) \pm \sqrt{(6)^2 - 4 \times (0)}}{2} = -3 \pm 3$

Solution 1: $x = -6$; $y = 1 + (-6) + 5(-6)^2 = 175$; $m = 1 + 10(-6) = -59$

Point of tangency: $Q(-6, 175)$

Equation of the tangent line: $y - (175) = -59(x - (-6))$ or $y = -59x - 179$

Solution 2: $x = 0$; $y = 1 + (0) + 5(0)^2 = 1$; $m = 1 + 10(0) = 1$

Point of tangency: $Q(0, 1)$

Equation of the tangent line: $y - (1) = 1(x - (0))$ or $y = 1x + 1$

4. The slope of the line segment passing through the given point $P(-3, 17)$ and the point of tangency $Q(x, y)$ is given by $m = \dfrac{y - (17)}{x - (-3)}$ (1) where $y = 5 - 4x - 5x^2$ (2)

The slope of the tangent line at the point of tangency $Q(x, y)$ is given by the first derivative:
$m = f'(x) = -4 - 10x$ (3)

Let use (1) and (2) and (3) : $\dfrac{5 - 4x - 5x^2 - (17)}{x - (-3)} = -4 - 10x$ (4)

Cross multiply (4) and simplify: $x^2 + (6)x + 0 = 0$ (5)

Use quadratic formula to solve (5): $x = \dfrac{-(6) \pm \sqrt{(6)^2 - 4 \times (0)}}{2} = -3 \pm 3$

Solution 1: $x = -6$; $y = 5 - 4(-6) - 5(-6)^2 = -151$; $m = -4 - 10(-6) = 56$

Point of tangency: $Q(-6, -151)$

Equation of the tangent line: $y - (-151) = 56(x - (-6))$ or $y = 56x + 185$

Solution 2: $x = 0$; $y = 5 - 4(0) - 5(0)^2 = 5$; $m = -4 - 10(0) = -4$

Point of tangency: $Q(0, 5)$

Equation of the tangent line: $y - (5) = -4(x - (0))$ or $y = -4x + 5$

5. The slope of the line segment passing through the given point $P(-5, -50)$ and the point of tangency $Q(x, y)$ is given by $m = \dfrac{y - (-50)}{x - (-5)}$ (1) where $y = -2 - 3x^2$ (2)

The slope of the tangent line at the point of tangency $Q(x, y)$ is given by the first derivative:
$m = f'(x) = -6x$ (3)

Let use (1) and (2) and (3) : $\dfrac{-2 - 3x^2 - (-50)}{x - (-5)} = -6x$ (4)

Cross multiply (4) and simplify: $x^2 + (10)x + 16 = 0$ (5)

Use quadratic formula to solve (5): $x = \dfrac{-(10) \pm \sqrt{(10)^2 - 4 \times (16)}}{2} = -5 \pm 3$

Solution 1: $x = -8$; $y = -2 - 3(-8)^2 = -194$; $m = -6(-8) = 48$

Point of tangency: $Q(-8, -194)$

Equation of the tangent line: $y - (-194) = 48(x - (-8))$ or $y = 48x + 190$

Solution 2: $x = -2$; $y = -2 - 3(-2)^2 = -14$; $m = -6(-2) = 12$

Point of tangency: $Q(-2, -14)$

Equation of the tangent line: $y - (-14) = 12(x - (-2))$ or $y = 12x + 10$

6. The slope of the line segment passing through the given point $P(0, -23)$ and the point of tangency $Q(x, y)$ is given by $m = \dfrac{y - (-23)}{x - (0)}$ (1) where $y = 4 + 4x + 3x^2$ (2)

The slope of the tangent line at the point of tangency $Q(x, y)$ is given by the first derivative:

$m = f'(x) = 4 + 6x$ (3)

Let use (1) and (2) and (3) : $\dfrac{4 + 4x + 3x^2 - (-23)}{x - (0)} = 4 + 6x$ (4)

Cross multiply (4) and simplify: $x^2 + (0)x - 9 = 0$ (5)

Use quadratic formula to solve (5): $x = \dfrac{-(0) \pm \sqrt{(0)^2 - 4 \times (-9)}}{2} = 0 \pm 3$

Solution 1: $x = -3$; $y = 4 + 4(-3) + 3(-3)^2 = 19$; $m = 4 + 6(-3) = -14$

Point of tangency: $Q(-3, 19)$

Equation of the tangent line: $y - (19) = -14(x - (-3))$ or $y = -14x - 23$

Solution 2: $x = 3$; $y = 4 + 4(3) + 3(3)^2 = 43$; $m = 4 + 6(3) = 22$

Point of tangency: $Q(3, 43)$

Equation of the tangent line: $y - (43) = 22(x - (3))$ or $y = 22x - 23$

7. The slope of the line segment passing through the given point $P(-1, -2)$ and the point of tangency $Q(x, y)$ is given by $m = \dfrac{y - (-2)}{x - (-1)}$ (1) where $y = -2 + 2x^2$ (2)

The slope of the tangent line at the point of tangency $Q(x, y)$ is given by the first derivative:

$m = f'(x) = 4x$ (3)

Let use (1) and (2) and (3) : $\dfrac{-2 + 2x^2 - (-2)}{x - (-1)} = 4x$ (4)

Cross multiply (4) and simplify: $x^2 + (2)x + 0 = 0$ (5)

Use quadratic formula to solve (5): $x = \dfrac{-(2) \pm \sqrt{(2)^2 - 4 \times (0)}}{2} = -1 \pm 1$

Solution 1: $x = -2$; $y = -2 + 2(-2)^2 = 6$; $m = 4(-2) = -8$

Point of tangency: $Q(-2, 6)$

Equation of the tangent line: $y - (6) = -8(x - (-2))$ or $y = -8x - 10$

Solution 2: $x = 0$; $y = -2 + 2(0)^2 = -2$; $m = 4(0) = 0$

Point of tangency: $Q(0, -2)$

Equation of the tangent line: $y - (-2) = 0(x - (0))$ or $y = 0x - 2$

8. The slope of the line segment passing through the given point $P(-2, 13)$ and the point of tangency $Q(x, y)$ is given by $m = \dfrac{y - (13)}{x - (-2)}$ (1) where $y = 2 - x + 3x^2$ (2)

The slope of the tangent line at the point of tangency $Q(x, y)$ is given by the first derivative:

$m = f'(x) = -1 + 6x$ (3)

Let use (1) and (2) and (3) : $\dfrac{2 - x + 3x^2 - (13)}{x - (-2)} = -1 + 6x$ (4)

Cross multiply (4) and simplify: $x^2 + (4)x + 3 = 0$ (5)

Use quadratic formula to solve (5): $x = \dfrac{-(4) \pm \sqrt{(4)^2 - 4 \times (3)}}{2} = -2 \pm 1$

Solution 1: $x = -3$; $y = 2 - (-3) + 3(-3)^2 = 32$; $m = -1 + 6(-3) = -19$

Point of tangency: $Q(-3, 32)$

Equation of the tangent line: $y - (32) = -19(x - (-3))$ or $y = -19x - 25$

Solution 2: $x = -1$; $y = 2 - (-1) + 3(-1)^2 = 6$; $m = -1 + 6(-1) = -7$

Point of tangency: $Q(-1, 6)$

Equation of the tangent line: $y - (6) = -7(x - (-1))$ or $y = -7x - 1$

9. The slope of the line segment passing through the given point $P(-3, 11)$ and the point of tangency $Q(x, y)$ is given by $m = \dfrac{y - (11)}{x - (-3)}$ (1) where $y = -3 + 2x + 4x^2$ (2)

The slope of the tangent line at the point of tangency $Q(x, y)$ is given by the first derivative:
$m = f'(x) = 2 + 8x$ (3)

Let use (1) and (2) and (3) : $\dfrac{-3 + 2x + 4x^2 - (11)}{x - (-3)} = 2 + 8x$ (4)

Cross multiply (4) and simplify: $x^2 + (6)x + 5 = 0$ (5)

Use quadratic formula to solve (5): $x = \dfrac{-(6) \pm \sqrt{(6)^2 - 4 \times (5)}}{2} = -3 \pm 2$

Solution 1: $x = -5$; $y = -3 + 2(-5) + 4(-5)^2 = 87$; $m = 2 + 8(-5) = -38$

Point of tangency: $Q(-5, 87)$

Equation of the tangent line: $y - (87) = -38(x - (-5))$ or $y = -38x - 103$

Solution 2: $x = -1$; $y = -3 + 2(-1) + 4(-1)^2 = -1$; $m = 2 + 8(-1) = -6$

Point of tangency: $Q(-1, -1)$

Equation of the tangent line: $y - (-1) = -6(x - (-1))$ or $y = -6x - 7$

10. The slope of the line segment passing through the given point $P(-4, -2)$ and the point of tangency $Q(x, y)$ is given by $m = \dfrac{y - (-2)}{x - (-4)}$ (1) where $y = -2 - 3x - x^2$ (2)

The slope of the tangent line at the point of tangency $Q(x, y)$ is given by the first derivative:
$m = f'(x) = -3 - 2x$ (3)

Let use (1) and (2) and (3) : $\dfrac{-2 - 3x - x^2 - (-2)}{x - (-4)} = -3 - 2x$ (4)

Cross multiply (4) and simplify: $x^2 + (8)x + 12 = 0$ (5)

Use quadratic formula to solve (5): $x = \dfrac{-(8) \pm \sqrt{(8)^2 - 4 \times (12)}}{2} = -4 \pm 2$

Solution 1: $x = -6$; $y = -2 - 3(-6) - (-6)^2 = -20$; $m = -3 - 2(-6) = 9$

Point of tangency: $Q(-6, -20)$

Equation of the tangent line: $y - (-20) = 9(x - (-6))$ or $y = 9x + 34$

Solution 2: $x = -2$; $y = -2 - 3(-2) - (-2)^2 = 0$; $m = -3 - 2(-2) = 1$

Point of tangency: $Q(-2, 0)$

Equation of the tangent line: $y - (0) = 1(x - (-2))$ or $y = 1x + 2$