1. Find $f'(x)$, $f''(x)$, $f'''(x)$, and $f^{(4)}$ for the following function: $f(x) = \dfrac{2x+3}{-x-2}$

2. Find $f'(x)$, $f''(x)$, $f'''(x)$, and $f^{(4)}$ for the following function: $f(x) = \dfrac{x-2}{x+2}$

3. Find $f'(x)$, $f''(x)$, $f'''(x)$, and $f^{(4)}$ for the following function: $f(x) = \dfrac{3x-3}{x+2}$

4. Find $f'(x)$, $f''(x)$, $f'''(x)$, and $f^{(4)}$ for the following function: $f(x) = \dfrac{-2x-5}{2x+3}$

5. Find $f'(x)$, $f''(x)$, $f'''(x)$, and $f^{(4)}$ for the following function: $f(x) = \dfrac{-5x+3}{-x-3}$

6. Find $f'(x)$, $f''(x)$, $f'''(x)$, and $f^{(4)}$ for the following function: $f(x) = \dfrac{3x+4}{-x-4}$

7. Find $f'(x)$, $f''(x)$, $f'''(x)$, and $f^{(4)}$ for the following function: $f(x) = \dfrac{x+5}{x-3}$

8. Find $f'(x)$, $f''(x)$, $f'''(x)$, and $f^{(4)}$ for the following function: $f(x) = \dfrac{-3x+1}{-x-1}$

9. Find $f'(x)$, $f''(x)$, $f'''(x)$, and $f^{(4)}$ for the following function: $f(x) = \dfrac{-4x-5}{-x+3}$

10. Find $f'(x)$, $f''(x)$, $f'''(x)$, and $f^{(4)}$ for the following function: $f(x) = \dfrac{x-5}{-x-3}$

Answers:

1. $f'(x) = \dfrac{-1}{(-x-2)^2}$ $f''(x) = \dfrac{-2}{(-x-2)^3}$ $f'''(x) = \dfrac{-6}{(-x-2)^4}$ $f^{(4)}(x) = \dfrac{-24}{(-x-2)^5}$

2. $f'(x) = \dfrac{4}{(x+2)^2}$ $f''(x) = \dfrac{-8}{(x+2)^3}$ $f'''(x) = \dfrac{24}{(x+2)^4}$ $f^{(4)}(x) = \dfrac{-96}{(x+2)^5}$

3. $f'(x) = \dfrac{9}{(x+2)^2}$ $f''(x) = \dfrac{-18}{(x+2)^3}$ $f'''(x) = \dfrac{54}{(x+2)^4}$ $f^{(4)}(x) = \dfrac{-216}{(x+2)^5}$

4. $f'(x) = \dfrac{4}{(2x+3)^2}$ $f''(x) = \dfrac{-16}{(2x+3)^3}$ $f'''(x) = \dfrac{96}{(2x+3)^4}$ $f^{(4)}(x) = \dfrac{-768}{(2x+3)^5}$

5. $f'(x) = \dfrac{18}{(-x-3)^2}$ $f''(x) = \dfrac{36}{(-x-3)^3}$ $f'''(x) = \dfrac{108}{(-x-3)^4}$ $f^{(4)}(x) = \dfrac{432}{(-x-3)^5}$

6. $f'(x) = \dfrac{-8}{(-x-4)^2}$ $f''(x) = \dfrac{-16}{(-x-4)^3}$ $f'''(x) = \dfrac{-48}{(-x-4)^4}$ $f^{(4)}(x) = \dfrac{-192}{(-x-4)^5}$

7. $f'(x) = \dfrac{-8}{(x-3)^2}$ $f''(x) = \dfrac{16}{(x-3)^3}$ $f'''(x) = \dfrac{-48}{(x-3)^4}$ $f^{(4)}(x) = \dfrac{192}{(x-3)^5}$

8. $f'(x) = \dfrac{4}{(-x-1)^2}$ $f''(x) = \dfrac{8}{(-x-1)^3}$ $f'''(x) = \dfrac{24}{(-x-1)^4}$ $f^{(4)}(x) = \dfrac{96}{(-x-1)^5}$

9. $f'(x) = \dfrac{-17}{(-x+3)^2}$ $f''(x) = \dfrac{-34}{(-x+3)^3}$ $f'''(x) = \dfrac{-102}{(-x+3)^4}$ $f^{(4)}(x) = \dfrac{-408}{(-x+3)^5}$

10. $f'(x) = \dfrac{-8}{(-x-3)^2}$ $f''(x) = \dfrac{-16}{(-x-3)^3}$ $f'''(x) = \dfrac{-48}{(-x-3)^4}$ $f^{(4)}(x) = \dfrac{-192}{(-x-3)^5}$

Solutions:

1. $f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{2x+3}{-x-2}$ ▶ Apply the Quotient Rule: $\dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{f(x)}{g(x)} = \dfrac{g(x)\frac{\mathrm{d}}{\mathrm{d}x}f(x) - f(x)\frac{\mathrm{d}}{\mathrm{d}x}g(x)}{g^2(x)}$

$= \dfrac{(-x-2)\frac{\mathrm{d}}{\mathrm{d}x}(2x+3) - (2x+3)\frac{\mathrm{d}}{\mathrm{d}x}(-x-2)}{(-x-2)^2} = \dfrac{(-x-2)(2) - (2x+3)(-1)}{(-x-2)^2} = \dfrac{-1}{(-x-2)^2}$

$f''(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} - 1(-x-2)^{-2}$ ▶ Apply the Power and Chain Rule: $\dfrac{\mathrm{d}}{\mathrm{d}x}(f(x))^n = n(f(x))^{n-1}\dfrac{\mathrm{d}}{\mathrm{d}x}f(x)$

$= -1(-2)(-x-2)^{-3}(-1) = \dfrac{-2}{(-x-2)^3}$

$f'''(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} - 2(-x-2)^{-3} = -2(-3)(-x-2)^{-4}(-1) = \dfrac{-6}{(-x-2)^4}$

$f^{(4)}(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} - 6(-x-2)^{-4} = -6(-4)(-x-2)^{-5}(-1) = \dfrac{-24}{(-x-2)^5}$

2. $f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{x-2}{x+2}$ ▶ Apply the Quotient Rule: $\dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{f(x)}{g(x)} = \dfrac{g(x)\frac{\mathrm{d}}{\mathrm{d}x}f(x) - f(x)\frac{\mathrm{d}}{\mathrm{d}x}g(x)}{g^2(x)}$

$= \dfrac{(x+2)\frac{\mathrm{d}}{\mathrm{d}x}(x-2) - (x-2)\frac{\mathrm{d}}{\mathrm{d}x}(x+2)}{(x+2)^2} = \dfrac{(x+2)(1) - (x-2)(1)}{(x+2)^2} = \dfrac{4}{(x+2)^2}$

$f''(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}4(x+2)^{-2}$ ▶ Apply the Power and Chain Rule: $\dfrac{\mathrm{d}}{\mathrm{d}x}(f(x))^n = n(f(x))^{n-1}\dfrac{\mathrm{d}}{\mathrm{d}x}f(x)$

$= 4(-2)(x+2)^{-3}(1) = \dfrac{-8}{(x+2)^3}$

$f'''(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} - 8(x+2)^{-3} = -8(-3)(x+2)^{-4}(1) = \dfrac{24}{(x+2)^4}$

$f^{(4)}(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}24(x+2)^{-4} = 24(-4)(x+2)^{-5}(1) = \dfrac{-96}{(x+2)^5}$

3. $f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{3x-3}{x+2}$ ▶ Apply the Quotient Rule: $\dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{f(x)}{g(x)} = \dfrac{g(x)\frac{\mathrm{d}}{\mathrm{d}x}f(x) - f(x)\frac{\mathrm{d}}{\mathrm{d}x}g(x)}{g^2(x)}$

$= \dfrac{(x+2)\frac{\mathrm{d}}{\mathrm{d}x}(3x-3) - (3x-3)\frac{\mathrm{d}}{\mathrm{d}x}(x+2)}{(x+2)^2} = \dfrac{(x+2)(3) - (3x-3)(1)}{(x+2)^2} = \dfrac{9}{(x+2)^2}$

$f''(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}9(x+2)^{-2}$ ▶ Apply the Power and Chain Rule: $\dfrac{\mathrm{d}}{\mathrm{d}x}(f(x))^n = n(f(x))^{n-1}\dfrac{\mathrm{d}}{\mathrm{d}x}f(x)$

$= 9(-2)(x+2)^{-3}(1) = \dfrac{-18}{(x+2)^3}$

$f'''(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} - 18(x+2)^{-3} = -18(-3)(x+2)^{-4}(1) = \dfrac{54}{(x+2)^4}$

$f^{(4)}(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}54(x+2)^{-4} = 54(-4)(x+2)^{-5}(1) = \dfrac{-216}{(x+2)^5}$

4. $f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{-2x-5}{2x+3}$ ▶ Apply the Quotient Rule: $\dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{f(x)}{g(x)} = \dfrac{g(x)\frac{\mathrm{d}}{\mathrm{d}x}f(x) - f(x)\frac{\mathrm{d}}{\mathrm{d}x}g(x)}{g^2(x)}$

$= \dfrac{(2x+3)\frac{\mathrm{d}}{\mathrm{d}x}(-2x-5) - (-2x-5)\frac{\mathrm{d}}{\mathrm{d}x}(2x+3)}{(2x+3)^2} = \dfrac{(2x+3)(-2) - (-2x-5)(2)}{(2x+3)^2} = \dfrac{4}{(2x+3)^2}$

$f''(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}4(2x+3)^{-2}$ ▶ Apply the Power and Chain Rule: $\dfrac{\mathrm{d}}{\mathrm{d}x}(f(x))^n = n(f(x))^{n-1}\dfrac{\mathrm{d}}{\mathrm{d}x}f(x)$

$= 4(-2)(2x+3)^{-3}(2) = \dfrac{-16}{(2x+3)^3}$

$f'''(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} - 16(2x+3)^{-3} = -16(-3)(2x+3)^{-4}(2) = \dfrac{96}{(2x+3)^4}$

$f^{(4)}(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}96(2x+3)^{-4} = 96(-4)(2x+3)^{-5}(2) = \dfrac{-768}{(2x+3)^5}$

5. $f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{-5x+3}{-x-3}$ ► Apply the Quotient Rule: $\dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{f(x)}{g(x)} = \dfrac{g(x)\frac{\mathrm{d}}{\mathrm{d}x}f(x) - f(x)\frac{\mathrm{d}}{\mathrm{d}x}g(x)}{g^2(x)}$

$= \dfrac{(-x-3)\frac{\mathrm{d}}{\mathrm{d}x}(-5x+3) - (-5x+3)\frac{\mathrm{d}}{\mathrm{d}x}(-x-3)}{(-x-3)^2} = \dfrac{(-x-3)(-5) - (-5x+3)(-1)}{(-x-3)^2} = \dfrac{18}{(-x-3)^2}$

$f''(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}18(-x-3)^{-2}$ ► Apply the Power and Chain Rule: $\dfrac{\mathrm{d}}{\mathrm{d}x}(f(x))^n = n(f(x))^{n-1}\dfrac{\mathrm{d}}{\mathrm{d}x}f(x)$

$= 18(-2)(-x-3)^{-3}(-1) = \dfrac{36}{(-x-3)^3}$

$f'''(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}36(-x-3)^{-3} = 36(-3)(-x-3)^{-4}(-1) = \dfrac{108}{(-x-3)^4}$

$f^{(4)}(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}108(-x-3)^{-4} = 108(-4)(-x-3)^{-5}(-1) = \dfrac{432}{(-x-3)^5}$

6. $f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{3x+4}{-x-4}$ ► Apply the Quotient Rule: $\dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{f(x)}{g(x)} = \dfrac{g(x)\frac{\mathrm{d}}{\mathrm{d}x}f(x) - f(x)\frac{\mathrm{d}}{\mathrm{d}x}g(x)}{g^2(x)}$

$= \dfrac{(-x-4)\frac{\mathrm{d}}{\mathrm{d}x}(3x+4) - (3x+4)\frac{\mathrm{d}}{\mathrm{d}x}(-x-4)}{(-x-4)^2} = \dfrac{(-x-4)(3) - (3x+4)(-1)}{(-x-4)^2} = \dfrac{-8}{(-x-4)^2}$

$f''(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} - 8(-x-4)^{-2}$ ► Apply the Power and Chain Rule: $\dfrac{\mathrm{d}}{\mathrm{d}x}(f(x))^n = n(f(x))^{n-1}\dfrac{\mathrm{d}}{\mathrm{d}x}f(x)$

$= -8(-2)(-x-4)^{-3}(-1) = \dfrac{-16}{(-x-4)^3}$

$f'''(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} - 16(-x-4)^{-3} = -16(-3)(-x-4)^{-4}(-1) = \dfrac{-48}{(-x-4)^4}$

$f^{(4)}(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} - 48(-x-4)^{-4} = -48(-4)(-x-4)^{-5}(-1) = \dfrac{-192}{(-x-4)^5}$

7. $f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{x+5}{x-3}$ ► Apply the Quotient Rule: $\dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{f(x)}{g(x)} = \dfrac{g(x)\frac{\mathrm{d}}{\mathrm{d}x}f(x) - f(x)\frac{\mathrm{d}}{\mathrm{d}x}g(x)}{g^2(x)}$

$= \dfrac{(x-3)\frac{\mathrm{d}}{\mathrm{d}x}(x+5) - (x+5)\frac{\mathrm{d}}{\mathrm{d}x}(x-3)}{(x-3)^2} = \dfrac{(x-3)(1) - (x+5)(1)}{(x-3)^2} = \dfrac{-8}{(x-3)^2}$

$f''(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} - 8(x-3)^{-2}$ ► Apply the Power and Chain Rule: $\dfrac{\mathrm{d}}{\mathrm{d}x}(f(x))^n = n(f(x))^{n-1}\dfrac{\mathrm{d}}{\mathrm{d}x}f(x)$

$= -8(-2)(x-3)^{-3}(1) = \dfrac{16}{(x-3)^3}$

$f'''(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}16(x-3)^{-3} = 16(-3)(x-3)^{-4}(1) = \dfrac{-48}{(x-3)^4}$

$f^{(4)}(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} - 48(x-3)^{-4} = -48(-4)(x-3)^{-5}(1) = \dfrac{192}{(x-3)^5}$

8. $f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{-3x+1}{-x-1}$ ► Apply the Quotient Rule: $\dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{f(x)}{g(x)} = \dfrac{g(x)\frac{\mathrm{d}}{\mathrm{d}x}f(x) - f(x)\frac{\mathrm{d}}{\mathrm{d}x}g(x)}{g^2(x)}$

$= \dfrac{(-x-1)\frac{\mathrm{d}}{\mathrm{d}x}(-3x+1) - (-3x+1)\frac{\mathrm{d}}{\mathrm{d}x}(-x-1)}{(-x-1)^2} = \dfrac{(-x-1)(-3) - (-3x+1)(-1)}{(-x-1)^2} = \dfrac{4}{(-x-1)^2}$

$f''(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}4(-x-1)^{-2}$ ► Apply the Power and Chain Rule: $\dfrac{\mathrm{d}}{\mathrm{d}x}(f(x))^n = n(f(x))^{n-1}\dfrac{\mathrm{d}}{\mathrm{d}x}f(x)$

$= 4(-2)(-x-1)^{-3}(-1) = \dfrac{8}{(-x-1)^3}$

$f'''(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}8(-x-1)^{-3} = 8(-3)(-x-1)^{-4}(-1) = \dfrac{24}{(-x-1)^4}$

$f^{(4)}(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}24(-x-1)^{-4} = 24(-4)(-x-1)^{-5}(-1) = \dfrac{96}{(-x-1)^5}$

9. $f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{-4x-5}{-x+3}$ ► Apply the Quotient Rule: $\dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{f(x)}{g(x)} = \dfrac{g(x)\frac{\mathrm{d}}{\mathrm{d}x}f(x) - f(x)\frac{\mathrm{d}}{\mathrm{d}x}g(x)}{g^2(x)}$

$= \dfrac{(-x+3)\frac{\mathrm{d}}{\mathrm{d}x}(-4x-5) - (-4x-5)\frac{\mathrm{d}}{\mathrm{d}x}(-x+3)}{(-x+3)^2} = \dfrac{(-x+3)(-4) - (-4x-5)(-1)}{(-x+3)^2} = \dfrac{-17}{(-x+3)^2}$

$f''(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} - 17(-x+3)^{-2}$ ► Apply the Power and Chain Rule: $\dfrac{\mathrm{d}}{\mathrm{d}x}(f(x))^n = n(f(x))^{n-1}\dfrac{\mathrm{d}}{\mathrm{d}x}f(x)$

$= -17(-2)(-x+3)^{-3}(-1) = \dfrac{-34}{(-x+3)^3}$

$f'''(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} - 34(-x+3)^{-3} = -34(-3)(-x+3)^{-4}(-1) = \dfrac{-102}{(-x+3)^4}$

$f^{(4)}(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} - 102(-x+3)^{-4} = -102(-4)(-x+3)^{-5}(-1) = \dfrac{-408}{(-x+3)^5}$

10. $f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{x-5}{-x-3}$ ► Apply the Quotient Rule: $\dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{f(x)}{g(x)} = \dfrac{g(x)\frac{\mathrm{d}}{\mathrm{d}x}f(x) - f(x)\frac{\mathrm{d}}{\mathrm{d}x}g(x)}{g^2(x)}$

$= \dfrac{(-x-3)\frac{\mathrm{d}}{\mathrm{d}x}(x-5) - (x-5)\frac{\mathrm{d}}{\mathrm{d}x}(-x-3)}{(-x-3)^2} = \dfrac{(-x-3)(1) - (x-5)(-1)}{(-x-3)^2} = \dfrac{-8}{(-x-3)^2}$

$f''(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} - 8(-x-3)^{-2}$ ► Apply the Power and Chain Rule: $\dfrac{\mathrm{d}}{\mathrm{d}x}(f(x))^n = n(f(x))^{n-1}\dfrac{\mathrm{d}}{\mathrm{d}x}f(x)$

$= -8(-2)(-x-3)^{-3}(-1) = \dfrac{-16}{(-x-3)^3}$

$f'''(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} - 16(-x-3)^{-3} = -16(-3)(-x-3)^{-4}(-1) = \dfrac{-48}{(-x-3)^4}$

$f^{(4)}(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} - 48(-x-3)^{-4} = -48(-4)(-x-3)^{-5}(-1) = \dfrac{-192}{(-x-3)^5}$