

1. Find  $f'(x)$ ,  $f''(x)$ ,  $f'''(x)$ , and  $f^{(4)}$  for the following function:  $f(x) = \frac{2}{x^3} + \frac{-1}{x^2} - 1$
2. Find  $f'(x)$ ,  $f''(x)$ ,  $f'''(x)$ , and  $f^{(4)}$  for the following function:  $f(x) = \frac{5}{x^3} + \frac{5}{x^2} + \frac{2}{x} + 1 - 4x$
3. Find  $f'(x)$ ,  $f''(x)$ ,  $f'''(x)$ , and  $f^{(4)}$  for the following function:  $f(x) = \frac{-2}{x} + 4 + 5x^2 + 5x^3 + 2x^4$
4. Find  $f'(x)$ ,  $f''(x)$ ,  $f'''(x)$ , and  $f^{(4)}$  for the following function:  $f(x) = -4 + x - 3x^3$
5. Find  $f'(x)$ ,  $f''(x)$ ,  $f'''(x)$ , and  $f^{(4)}$  for the following function:  $f(x) = \frac{1}{x^3} + \frac{-4}{x^2} + \frac{-4}{x} + 5$
6. Find  $f'(x)$ ,  $f''(x)$ ,  $f'''(x)$ , and  $f^{(4)}$  for the following function:  $f(x) = \frac{-3}{x^2} + \frac{4}{x} + 1 - 5x - 2x^2 + 2x^3 - 3x^4$
7. Find  $f'(x)$ ,  $f''(x)$ ,  $f'''(x)$ , and  $f^{(4)}$  for the following function:  $f(x) = \frac{5}{x} + 5 - x - 2x^2 + x^3 - 2x^4 + 5x^5$
8. Find  $f'(x)$ ,  $f''(x)$ ,  $f'''(x)$ , and  $f^{(4)}$  for the following function:  $f(x) = \frac{5}{x^2} + \frac{4}{x} - 2 - 5x$
9. Find  $f'(x)$ ,  $f''(x)$ ,  $f'''(x)$ , and  $f^{(4)}$  for the following function:  $f(x) = \frac{4}{x^3} + \frac{5}{x^2} + \frac{-1}{x} + 3 + x + 5x^2 + 3x^3$
10. Find  $f'(x)$ ,  $f''(x)$ ,  $f'''(x)$ , and  $f^{(4)}$  for the following function:  $f(x) = \frac{-5}{x^3} + \frac{3}{x} - 2 + 4x$

- ANSWERS: 1.  $f'(x) = -\frac{6}{2}x^{\frac{3}{2}} + \frac{24}{6}x^{\frac{5}{2}} - \frac{x^{\frac{4}{2}}}{6} = f''(x) = \frac{24}{2}x^{\frac{3}{2}} - \frac{x^{\frac{4}{2}}}{6} = f'''(x) = -\frac{x^{\frac{6}{2}}}{120} + \frac{x^{\frac{7}{2}}}{24} = f^{(4)}(x) = \frac{x^{\frac{5}{2}}}{24} - \frac{x^{\frac{6}{2}}}{120}$
2.  $f'(x) = -\frac{15}{1800}x^{\frac{4}{3}} + \frac{x^{\frac{3}{3}}}{10} - \frac{x^{\frac{2}{3}}}{2} = f''(x) = \frac{2}{60}x^{\frac{1}{3}} + \frac{x^{\frac{2}{3}}}{30} + \frac{x^{\frac{3}{3}}}{4} = f'''(x) = -\frac{x^{\frac{2}{3}}}{300} - \frac{x^{\frac{1}{3}}}{120} + \frac{x^{\frac{4}{3}}}{12} = f^{(4)}(x) = \frac{x^{\frac{1}{3}}}{12} + \frac{x^{\frac{2}{3}}}{96} + \frac{x^{\frac{3}{3}}}{24}$
3.  $f'(x) = \frac{2}{2}x^{\frac{2}{2}} + 10x + 15x^{\frac{2}{2}} + 8x^{\frac{3}{2}} = f''(x) = -\frac{4}{4}x^{\frac{3}{2}} + 10 + 30x + 24x^{\frac{1}{2}} = f'''(x) = \frac{12}{12}x^{\frac{1}{2}} + 30 + 48x = f^{(4)}(x) = \frac{x}{48} + 48$
4.  $f'(x) = 1 - 9x^{\frac{2}{2}} = f''(x) = -18x = f'''(x) = -18 = f^{(4)}(x) = 0$
5.  $f'(x) = -\frac{3}{8}x^{\frac{3}{4}} + \frac{x^{\frac{2}{4}}}{4} + \frac{x^{\frac{1}{4}}}{8} = f''(x) = \frac{x^{\frac{2}{4}}}{12} - \frac{x^{\frac{1}{4}}}{24} + \frac{x^{\frac{3}{4}}}{8} = f'''(x) = -\frac{x^{\frac{3}{4}}}{60} + \frac{x^{\frac{2}{4}}}{30} + \frac{x^{\frac{4}{4}}}{4} = f^{(4)}(x) = \frac{x^{\frac{3}{4}}}{24} + \frac{x^{\frac{4}{4}}}{96} + \frac{x^{\frac{5}{4}}}{24}$
6.  $f'(x) = \frac{6}{-4}x^{\frac{3}{2}} + \frac{x^{\frac{2}{2}}}{-4} - 5 - 4x + 6x^{\frac{2}{2}} - 12x^{\frac{3}{2}} = f''(x) = -\frac{3}{8}x^{\frac{1}{2}} + \frac{x^{\frac{1}{2}}}{8} - 4 + 12x - 36x^{\frac{1}{2}} = f'''(x) = \frac{x}{8} - \frac{x^{\frac{3}{2}}}{8} + 12x - 36x^{\frac{1}{2}}$
7.  $f'(x) = \frac{x^{\frac{2}{5}}}{-5} - 1 - 4x + 3x^{\frac{2}{2}} - 8x^{\frac{3}{2}} + 25x^{\frac{4}{2}} = f''(x) = \frac{x^{\frac{1}{5}}}{10} - 4 + 6x - 24x^{\frac{1}{2}} + 100x^{\frac{3}{2}} = f'''(x) = \frac{x^{\frac{4}{5}}}{30} - 48 + 600x^{\frac{1}{2}}$
8.  $f'(x) = \frac{x^{\frac{3}{-10}}}{-10} + \frac{x^{\frac{2}{-4}}}{-4} - 5 = f''(x) = \frac{x^{\frac{2}{-10}}}{30} + \frac{x^{\frac{1}{-4}}}{8} = f'''(x) = -\frac{x^{\frac{1}{-10}}}{120} - \frac{x^{\frac{0}{-4}}}{-24} = f^{(4)}(x) = \frac{x^{\frac{0}{-10}}}{96} + \frac{x^{\frac{1}{-4}}}{600}$
9.  $f'(x) = \frac{x^{\frac{6}{-240}}}{-240} + \frac{x^{\frac{5}{-120}}}{-120} + \frac{x^{\frac{4}{-10}}}{-10} + \frac{x^{\frac{3}{-1}}}{-1} + 1 + 10x + 9x^{\frac{2}{2}} = f''(x) = \frac{x^{\frac{5}{-24}}}{24} + \frac{x^{\frac{4}{-30}}}{30} + \frac{x^{\frac{3}{-4}}}{4} + \frac{x^{\frac{2}{-2}}}{2} + 10 + 18x = f'''(x) = \frac{x^{\frac{4}{-24}}}{24} + \frac{x^{\frac{3}{-30}}}{30} + \frac{x^{\frac{2}{-4}}}{4} + \frac{x^{\frac{1}{-2}}}{2} + 18x = f^{(4)}(x) = \frac{x^{\frac{3}{-24}}}{72} + \frac{x^{\frac{2}{-30}}}{360} - \frac{x^{\frac{1}{-4}}}{72} + \frac{x^{\frac{0}{-2}}}{180}$
10.  $f'(x) = \frac{x^{\frac{4}{15}}}{15} + \frac{x^{\frac{3}{-3}}}{-3} + 4 = f''(x) = \frac{x^{\frac{3}{15}}}{9} + \frac{x^{\frac{2}{-3}}}{-6} = f'''(x) = \frac{x^{\frac{2}{15}}}{30} + \frac{x^{\frac{1}{-3}}}{-9} = f^{(4)}(x) = \frac{x^{\frac{1}{15}}}{180} + \frac{x^{\frac{0}{-3}}}{-180}$

Solutions:

$$1. f'(x) = \frac{d}{dx}(2x^{-3} + -x^{-2} - 1) = -6x^{-4} + 2x^{-3} = \frac{-6}{x^4} + \frac{2}{x^3}$$

$$f''(x) = \frac{d}{dx}(-6x^{-4} + 2x^{-3}) = 24x^{-5} + -6x^{-4} = \frac{24}{x^5} + \frac{-6}{x^4}$$

$$f'''(x) = \frac{d}{dx}(24x^{-5} + -6x^{-4}) = -120x^{-6} + 24x^{-5} = \frac{-120}{x^6} + \frac{24}{x^5}$$

$$f^{(4)}(x) = \frac{d}{dx}(-120x^{-6} + 24x^{-5}) = 720x^{-7} + -120x^{-6} = \frac{720}{x^7} + \frac{-120}{x^6}$$

$$2. f'(x) = \frac{d}{dx}(5x^{-3} + 5x^{-2} + 2x^{-1} + 1 - 4x) = -15x^{-4} + -10x^{-3} + -2x^{-2} - 4 = \frac{-15}{x^4} + \frac{-10}{x^3} + \frac{-2}{x^2} - 4$$

$$f''(x) = \frac{d}{dx}(-15x^{-4} + -10x^{-3} + -2x^{-2} - 4) = 60x^{-5} + 30x^{-4} + 4x^{-3} = \frac{60}{x^5} + \frac{30}{x^4} + \frac{4}{x^3}$$

$$f'''(x) = \frac{d}{dx}(60x^{-5} + 30x^{-4} + 4x^{-3}) = -300x^{-6} + -120x^{-5} + -12x^{-4} = \frac{-300}{x^6} + \frac{-120}{x^5} + \frac{-12}{x^4}$$

$$f^{(4)}(x) = \frac{d}{dx}(-300x^{-6} + -120x^{-5} + -12x^{-4}) = 1800x^{-7} + 600x^{-6} + 48x^{-5} = \frac{1800}{x^7} + \frac{600}{x^6} + \frac{48}{x^5}$$

$$3. f'(x) = \frac{d}{dx}(-2x^{-1} + 4 + 5x^2 + 5x^3 + 2x^4) = 2x^{-2} + 10x + 15x^2 + 8x^3 = \frac{2}{x^2} + 10x + 15x^2 + 8x^3$$

$$f''(x) = \frac{d}{dx}(2x^{-2} + 10x + 15x^2 + 8x^3) = -4x^{-3} + 10 + 30x + 24x^2 = \frac{-4}{x^3} + 10 + 30x + 24x^2$$

$$f'''(x) = \frac{d}{dx}(-4x^{-3} + 10 + 30x + 24x^2) = 12x^{-4} + 30 + 48x = \frac{12}{x^4} + 30 + 48x$$

$$f^{(4)}(x) = \frac{d}{dx}(12x^{-4} + 30 + 48x) = -48x^{-5} + 48 = \frac{-48}{x^5} + 48$$

$$4. f'(x) = \frac{d}{dx}(-4 + x - 3x^3) = 1 - 9x^2 = 1 - 9x^2$$

$$f''(x) = \frac{d}{dx}(1 - 9x^2) = -18x = -18x$$

$$f'''(x) = \frac{d}{dx}(-18x) = -18 = -18$$

$$f^{(4)}(x) = \frac{d}{dx}(-18) = 0 = 0$$

$$5. f'(x) = \frac{d}{dx}(x^{-3} + -4x^{-2} + -4x^{-1} + 5) = -3x^{-4} + 8x^{-3} + 4x^{-2} = \frac{-3}{x^4} + \frac{8}{x^3} + \frac{4}{x^2}$$

$$f''(x) = \frac{d}{dx}(-3x^{-4} + 8x^{-3} + 4x^{-2}) = 12x^{-5} + -24x^{-4} + -8x^{-3} = \frac{12}{x^5} + \frac{-24}{x^4} + \frac{-8}{x^3}$$

$$f'''(x) = \frac{d}{dx}(12x^{-5} + -24x^{-4} + -8x^{-3}) = -60x^{-6} + 96x^{-5} + 24x^{-4} = \frac{-60}{x^6} + \frac{96}{x^5} + \frac{24}{x^4}$$

$$f^{(4)}(x) = \frac{d}{dx}(-60x^{-6} + 96x^{-5} + 24x^{-4}) = 360x^{-7} + -480x^{-6} + -96x^{-5} = \frac{360}{x^7} + \frac{-480}{x^6} + \frac{-96}{x^5}$$

$$6. f'(x) = \frac{d}{dx}(-3x^{-2} + 4x^{-1} + 1 - 5x - 2x^2 + 2x^3 - 3x^4) = 6x^{-3} + -4x^{-2} - 5 - 4x + 6x^2 - 12x^3 = \frac{6}{x^3} + \frac{-4}{x^2} - 5 - 4x + 6x^2 - 12x^3$$

$$f''(x) = \frac{d}{dx}(6x^{-3} + -4x^{-2} - 5 - 4x + 6x^2 - 12x^3) = -18x^{-4} + 8x^{-3} - 4 + 12x - 36x^2 = \frac{-18}{x^4} + \frac{8}{x^3} - 4 + 12x - 36x^2$$

$$f'''(x) = \frac{d}{dx}(-18x^{-4} + 8x^{-3} - 4 + 12x - 36x^2) = 72x^{-5} + -24x^{-4} + 12 - 72x = \frac{72}{x^5} + \frac{-24}{x^4} + 12 - 72x$$

$$f^{(4)}(x) = \frac{d}{dx}(72x^{-5} + -24x^{-4} + 12 - 72x) = -360x^{-6} + 96x^{-5} - 72 = \frac{-360}{x^6} + \frac{96}{x^5} - 72$$

$$7. f'(x) = \frac{d}{dx}(5x^{-1} + 5 - x - 2x^2 + x^3 - 2x^4 + 5x^5) = -5x^{-2} - 1 - 4x + 3x^2 - 8x^3 + 25x^4 = \frac{-5}{x^2} - 1 - 4x + 3x^2 - 8x^3 + 25x^4$$

$$f''(x) = \frac{d}{dx}(-5x^{-2} - 1 - 4x + 3x^2 - 8x^3 + 25x^4) = 10x^{-3} - 4 + 6x - 24x^2 + 100x^3 = \frac{10}{x^3} - 4 + 6x - 24x^2 + 100x^3$$

$$f'''(x) = \frac{d}{dx}(10x^{-3} - 4 + 6x - 24x^2 + 100x^3) = -30x^{-4} + 6 - 48x + 300x^2 = \frac{-30}{x^4} + 6 - 48x + 300x^2$$

$$f^{(4)}(x) = \frac{d}{dx}(-30x^{-4} + 6 - 48x + 300x^2) = 120x^{-5} - 48 + 600x = \frac{120}{x^5} - 48 + 600x$$

$$8. f'(x) = \frac{d}{dx}(5x^{-2} + 4x^{-1} - 2 - 5x) = -10x^{-3} + -4x^{-2} - 5 = \frac{-10}{x^3} + \frac{-4}{x^2} - 5$$

$$f''(x) = \frac{d}{dx}(-10x^{-3} + -4x^{-2} - 5) = 30x^{-4} + 8x^{-3} = \frac{30}{x^4} + \frac{8}{x^3}$$

$$f'''(x) = \frac{d}{dx}(30x^{-4} + 8x^{-3}) = -120x^{-5} + -24x^{-4} = \frac{-120}{x^5} + \frac{-24}{x^4}$$

$$f^{(4)}(x) = \frac{d}{dx}(-120x^{-5} + -24x^{-4}) = 600x^{-6} + 96x^{-5} = \frac{600}{x^6} + \frac{96}{x^5}$$

$$9. f'(x) = \frac{d}{dx}(4x^{-3} + 5x^{-2} + -x^{-1} + 3 + x + 5x^2 + 3x^3) = -12x^{-4} + -10x^{-3} + x^{-2} + 1 + 10x + 9x^2 = \frac{-12}{x^4} + \frac{-10}{x^3} + \frac{1}{x^2} + 1 + 10x + 9x^2$$

$$f''(x) = \frac{d}{dx}(-12x^{-4} + -10x^{-3} + x^{-2} + 1 + 10x + 9x^2) = 48x^{-5} + 30x^{-4} + -2x^{-3} + 10 + 18x = \frac{48}{x^5} + \frac{30}{x^4} + \frac{-2}{x^3} + 10 + 18x$$

$$f'''(x) = \frac{d}{dx}(48x^{-5} + 30x^{-4} + -2x^{-3} + 10 + 18x) = -240x^{-6} + -120x^{-5} + 6x^{-4} + 18 = \frac{-240}{x^6} + \frac{-120}{x^5} + \frac{6}{x^4} + 18$$

$$f^{(4)}(x) = \frac{d}{dx}(-240x^{-6} + -120x^{-5} + 6x^{-4} + 18) = 1440x^{-7} + 600x^{-6} + -24x^{-5} = \frac{1440}{x^7} + \frac{600}{x^6} + \frac{-24}{x^5}$$

$$10. f'(x) = \frac{d}{dx}(-5x^{-3} + 3x^{-1} - 2 + 4x) = 15x^{-4} + -3x^{-2} + 4 = \frac{15}{x^4} + \frac{-3}{x^2} + 4$$

$$f''(x) = \frac{d}{dx}(15x^{-4} + -3x^{-2} + 4) = -60x^{-5} + 6x^{-3} = \frac{-60}{x^5} + \frac{6}{x^3}$$

$$f'''(x) = \frac{d}{dx}(-60x^{-5} + 6x^{-3}) = 300x^{-6} + -18x^{-4} = \frac{300}{x^6} + \frac{-18}{x^4}$$

$$f^{(4)}(x) = \frac{d}{dx}(300x^{-6} + -18x^{-4}) = -1800x^{-7} + 72x^{-5} = \frac{-1800}{x^7} + \frac{72}{x^5}$$