1. Consider the following polynomial function: $f(x) = \dfrac{(x+1)(x-4)}{x}$

Sketch the grapf of the function $f(x)$.

2. Consider the following polynomial function: $f(x) = \dfrac{(x+2)(x-2)}{x}$

Sketch the grapf of the function $f(x)$.

3. Consider the following polynomial function: $f(x) = \dfrac{(x+2)(x+1)}{x-4}$

Sketch the grapf of the function $f(x)$.

4. Consider the following polynomial function: $f(x) = \dfrac{(x+4)(x+1)}{x-4}$

Sketch the grapf of the function $f(x)$.

5. Consider the following polynomial function: $f(x) = \dfrac{(x+3)(x-1)}{x+1}$

Sketch the grapf of the function $f(x)$.

6. Consider the following polynomial function: $f(x) = \dfrac{(x+2)(x+4)}{x-4}$

Sketch the grapf of the function $f(x)$.

7. Consider the following polynomial function: $f(x) = \dfrac{(x+3)(x-2)}{x-1}$

Sketch the grapf of the function $f(x)$.

8. Consider the following polynomial function: $f(x) = \dfrac{(x-3)(x-4)}{x+4}$

Sketch the grapf of the function $f(x)$.

9. Consider the following polynomial function: $f(x) = \dfrac{(x+2)(x-3)}{x+3}$

Sketch the grapf of the function $f(x)$.

10. Consider the following polynomial function: $f(x) = \dfrac{(x-2)(x+4)}{x+2}$

Sketch the grapf of the function $f(x)$.

Solutions:

1. $f(x) = \dfrac{(x+1)(x-4)}{x}$

Domain

The function $f(x)$ is a rational function. The domain of $f(x)$ is $D_f = \mathbb{R}\backslash\{0\}$.

Symmetry

$f(-x) = \dfrac{(-x+1)(-x-4)}{-x}$

$f(-x) \neq -f(x) \qquad f(-x) \neq f(x)$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros

The zero(s) of the function $f(x)$ are given by: $\dfrac{(x+1)(x-4)}{x} = 0 \qquad \text{or} \qquad (x+1)(x-4) = 0$

Therefore teh zeros of the function $f(x)$ are: $x_1 = -1 \qquad x_2 = 4$

Sign Chart for $f(x)$

x	$(-\infty,-1)$	-1	$(-1,0)$	0	$(0,4)$	4	$(4,\infty)$
$f(x)$	$-$	0	$+$	DNE	$-$	0	$+$

y-intercept

$y - int = f(0) = DNE$ (Does Not Exist)

Asymptotes

The function $f(x)$ has a vertical asymptote at $x = 0$.

The function $f(x)$ can be written as: $f(x) = \dfrac{(x+1)(x-4)}{x} = x - 3 + \dfrac{-4}{x-0}$

The function $f(x)$ has an oblique asymptote given by the equation: $y = x - 3$

Critical Numbers

$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{(x+1)(x-4)}{x} = \dfrac{x^2+0x+4}{x^2}$

Critical numbers are the solutions of the equation $f'(x) = 0 \qquad \text{or} \qquad \dfrac{x^2+0x+4}{x^2} = 0$

The critical numbers are: $\qquad x = 0$

Sign Chart for First Derivative $f'(x)$

x	$(-\infty,0)$	0	$(0,\infty)$
$f(x)$	$\nearrow$	DNE	$\nearrow$
$f'(x)$	$+$	DNE	$+$

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(-\infty,0) \qquad (0,\infty)$

Maximum and Minimum Points

Concavity Intervals

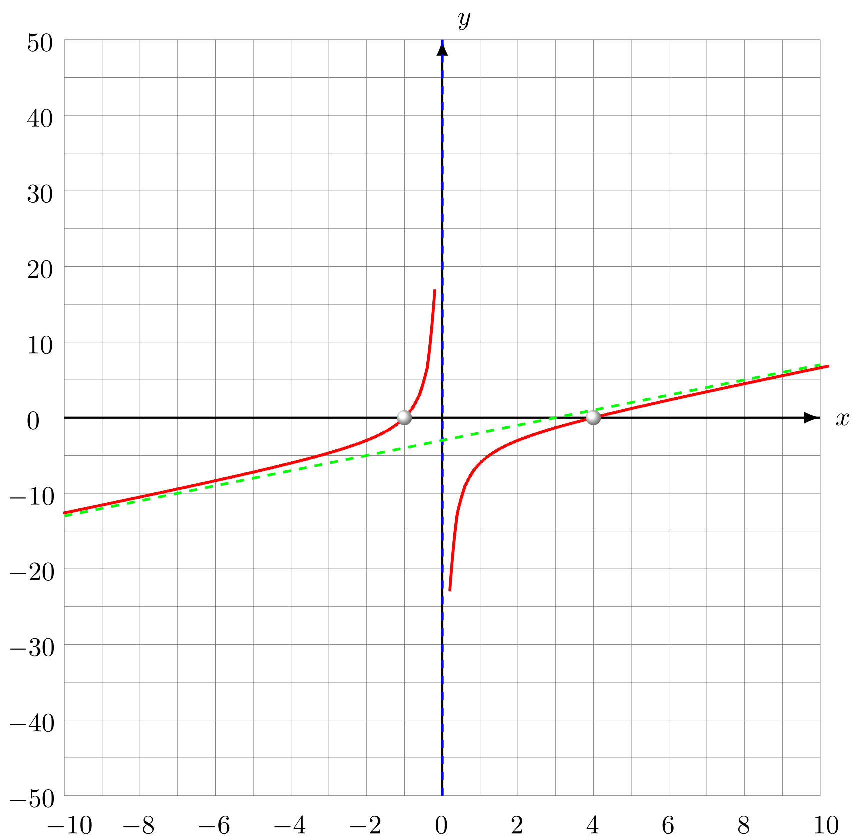
The second derivative of the function $f(x)$ is given by: $f''(x) = \frac{\mathrm{d}}{\mathrm{d}x} \frac{x^2 + 0x + 4}{x^2} = \frac{-8}{(x-0)^3}$

The second derivative of the function $f(x)$ is not zero at any x. Therefore the function $f(x)$ does not have any inflection points.

Sign Chart for Second Derivative $f''(x)$

x	$(-\infty, 0)$	0	$(0, \infty)$
$f(x)$	$\smile$	DNE	$\frown$
$f''(x)$	$+$	DNE	$-$

Graph



2. $f(x) = \dfrac{(x+2)(x-2)}{x}$

Domain

The function $f(x)$ is a rational function. The domain of $f(x)$ is $D_f = \mathbb{R}\backslash\{0\}$.

Symmetry

$f(-x) = \dfrac{(-x+2)(-x-2)}{-x}$

$f(-x) = -f(x) \qquad f(-x) \neq f(x)$

Therefore the function $f(x)$ is an odd function.

Zeros

The zero(s) of the function $f(x)$ are given by: $\dfrac{(x+2)(x-2)}{x} = 0 \qquad$ or $\qquad (x+2)(x-2) = 0$

Therefore teh zeros of the function $f(x)$ are: $x_1 = -2 \qquad x_2 = 2$

Sign Chart for $f(x)$

x	$(-\infty,-2)$	-2	$(-2,0)$	0	$(0,2)$	2	$(2,\infty)$
$f(x)$	$-$	0	$+$	DNE	$-$	0	$+$

y-intercept

$y - int = f(0) = DNE$ (Does Not Exist)

Asymptotes

The function $f(x)$ has a vertical asymptote at $x = 0$.

The function $f(x)$ can be written as: $f(x) = \dfrac{(x+2)(x-2)}{x} = x + 0 + \dfrac{-4}{x-0}$

The function $f(x)$ has an oblique asymptote given by the equation: $y = x + 0$

Critical Numbers

$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{(x+2)(x-2)}{x} = \dfrac{x^2+0x+4}{x^2}$

Critical numbers are the solutions of the equation $f'(x) = 0 \qquad$ or $\qquad \dfrac{x^2+0x+4}{x^2} = 0$

The critical numbers are: $\qquad x = 0$

Sign Chart for First Derivative $f'(x)$

x	$(-\infty,0)$	0	$(0,\infty)$
$f(x)$	$\nearrow$	DNE	$\nearrow$
$f'(x)$	$+$	DNE	$+$

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(-\infty,0) \qquad (0,\infty)$

Maximum and Minimum Points

Concavity Intervals

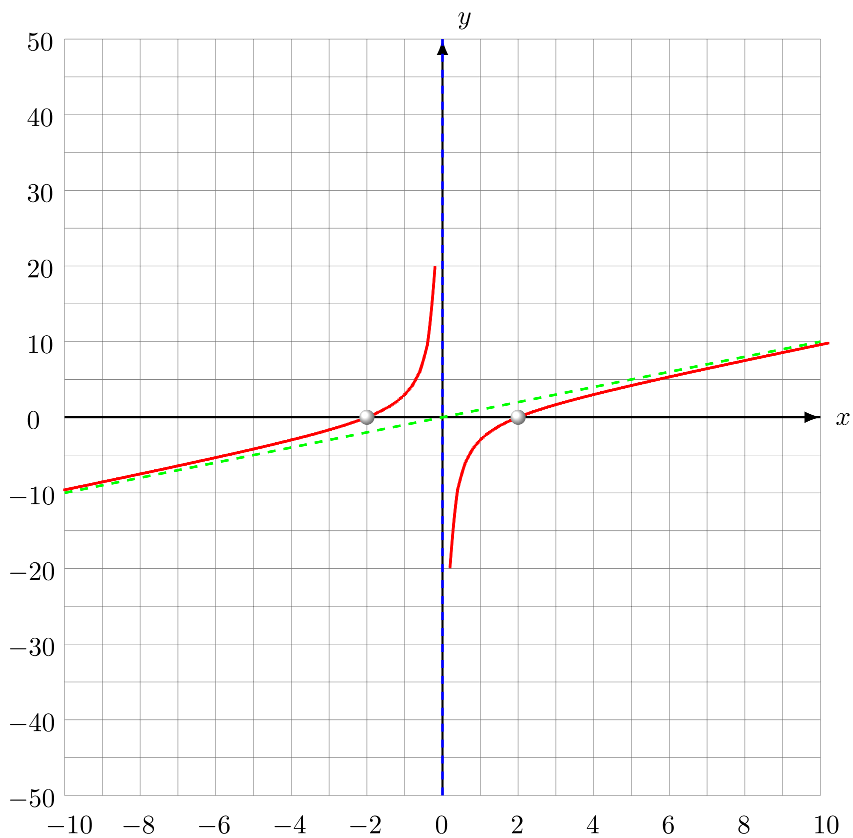
The second derivative of the function $f(x)$ is given by: $f''(x) = \frac{\mathrm{d}}{\mathrm{d}x}\frac{x^2+0x+4}{x^2} = \frac{-8}{(x-0)^3}$

The second derivative of the function $f(x)$ is not zero at any x. Therefore the function $f(x)$ does not have any inflection points.

Sign Chart for Second Derivative $f''(x)$

x	$(-\infty, 0)$	0	$(0, \infty)$
$f(x)$	$\smile$	DNE	$\frown$
$f''(x)$	$+$	DNE	$-$

Graph



3. $f(x) = \dfrac{(x+2)(x+1)}{x-4}$

Domain

The function $f(x)$ is a rational function. The domain of $f(x)$ is $D_f = \mathbb{R}\backslash\{4\}$.

Symmetry

$f(-x) = \dfrac{(-x+2)(-x+1)}{-x-4}$

$f(-x) \neq -f(x) \qquad f(-x) \neq f(x)$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros

The zero(s) of the function $f(x)$ are given by: $\dfrac{(x+2)(x+1)}{x-4} = 0$ or $(x+2)(x+1) = 0$

Therefore teh zeros of the function $f(x)$ are: $x_1 = -2 \qquad x_2 = -1$

Sign Chart for $f(x)$

x	$(-\infty,-2)$	-2	$(-2,-1)$	-1	$(-1,4)$	4	$(4,\infty)$
$f(x)$	$-$	0	$+$	0	$-$	DNE	$+$

y-intercept

$y - int = f(0) = \dfrac{(0+2)(0+1)}{0-4} = -0.500$

Asymptotes

The function $f(x)$ has a vertical asymptote at $x = 4$.

The function $f(x)$ can be written as: $f(x) = \dfrac{(x+2)(x+1)}{x-4} = x + 7 + \dfrac{30}{x-4}$

The function $f(x)$ has an oblique asymptote given by the equation: $y = x + 7$

Critical Numbers

$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{(x+2)(x+1)}{x-4} = \dfrac{x^2 - 8x - 14}{(x-4)^2}$

Critical numbers are the solutions of the equation $f'(x) = 0$ or $\dfrac{x^2 - 8x - 14}{(x-4)^2} = 0$

The critical numbers are: $x = -1.477 \qquad x = 4 \qquad x = 9.477$

Sign Chart for First Derivative $f'(x)$

x	$(-\infty,-1.477)$	-1.477	$(-1.477,4)$	4	$(4,9.477)$	9.477	$(9.477,\infty)$
$f(x)$	$\nearrow$	0.046	$\searrow$	DNE	$\searrow$	21.954	$\nearrow$
$f'(x)$	$+$	0	$-$	DNE	$-$	0	$+$

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(-\infty,-1.477) \qquad (9.477,\infty)$

The function $f(x)$ is decreasing over $(-1.477,4) \qquad (4,9.477)$

Maximum and Minimum Points

The function $f(x)$ has a maximum point at $(-1.477, 0.046)$
The function $f(x)$ has a minimum point at $(9.477, 21.954)$

Concavity Intervals

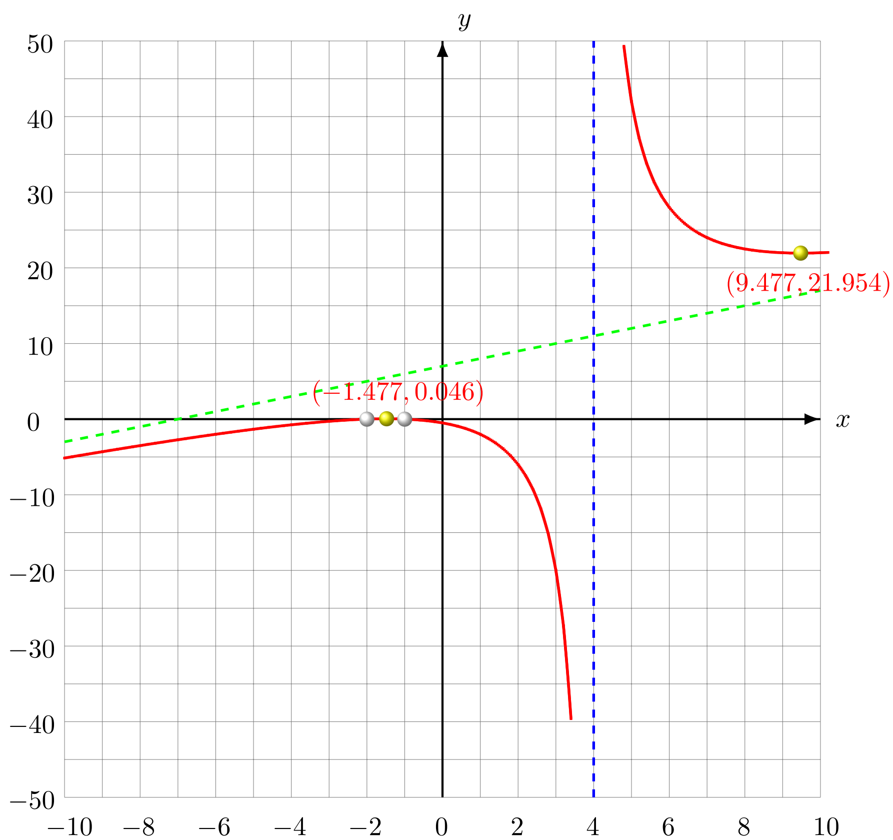
The second derivative of the function $f(x)$ is given by: $f''(x) = \frac{\mathrm{d}}{\mathrm{d}x}\frac{x^2-8x-14}{(x-4)^2} = \frac{60}{(x-4)^3}$

The second derivative of the function $f(x)$ is not zero at any x. Therefore the function $f(x)$ does not have any inflection points.

Sign Chart for Second Derivative $f''(x)$

x	$(-\infty, 4)$	4	$(4, \infty)$
$f(x)$	$\frown$	DNE	$\smile$
$f''(x)$	$-$	DNE	$+$

Graph



4. $f(x) = \dfrac{(x+4)(x+1)}{x-4}$

Domain

The function $f(x)$ is a rational function. The domain of $f(x)$ is $D_f = \mathbb{R}\backslash\{4\}$.

Symmetry

$f(-x) = \dfrac{(-x+4)(-x+1)}{-x-4}$

$f(-x) \neq -f(x) \qquad f(-x) \neq f(x)$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros

The zero(s) of the function $f(x)$ are given by: $\dfrac{(x+4)(x+1)}{x-4} = 0$ or $(x+4)(x+1) = 0$

Therefore teh zeros of the function $f(x)$ are: $x_1 = -4 \qquad x_2 = -1$

Sign Chart for $f(x)$

x	$(-\infty,-4)$	-4	$(-4,-1)$	-1	$(-1,4)$	4	$(4,\infty)$
$f(x)$	$-$	0	$+$	0	$-$	DNE	$+$

y-intercept

$y - int = f(0) = \dfrac{(0+4)(0+1)}{0-4} = -1.000$

Asymptotes

The function $f(x)$ has a vertical asymptote at $x = 4$.

The function $f(x)$ can be written as: $f(x) = \dfrac{(x+4)(x+1)}{x-4} = x + 9 + \dfrac{40}{x-4}$

The function $f(x)$ has an oblique asymptote given by the equation: $y = x + 9$

Critical Numbers

$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{(x+4)(x+1)}{x-4} = \dfrac{x^2-8x-24}{(x-4)^2}$

Critical numbers are the solutions of the equation $f'(x) = 0$ or $\dfrac{x^2-8x-24}{(x-4)^2} = 0$

The critical numbers are: $x = -2.325 \qquad x = 4 \qquad x = 10.325$

Sign Chart for First Derivative $f'(x)$

x	$(-\infty,-2.325)$	-2.325	$(-2.325,4)$	4	$(4,10.325)$	10.325	$(10.325,\infty)$
$f(x)$	$\nearrow$	0.351	$\searrow$	DNE	$\searrow$	25.649	$\nearrow$
$f'(x)$	$+$	0	$-$	DNE	$-$	0	$+$

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(-\infty,-2.325) \qquad (10.325,\infty)$

The function $f(x)$ is decreasing over $(-2.325,4) \qquad (4,10.325)$

Maximum and Minimum Points

The function $f(x)$ has a maximum point at $(-2.325, 0.351)$
The function $f(x)$ has a minimum point at $(10.325, 25.649)$

Concavity Intervals

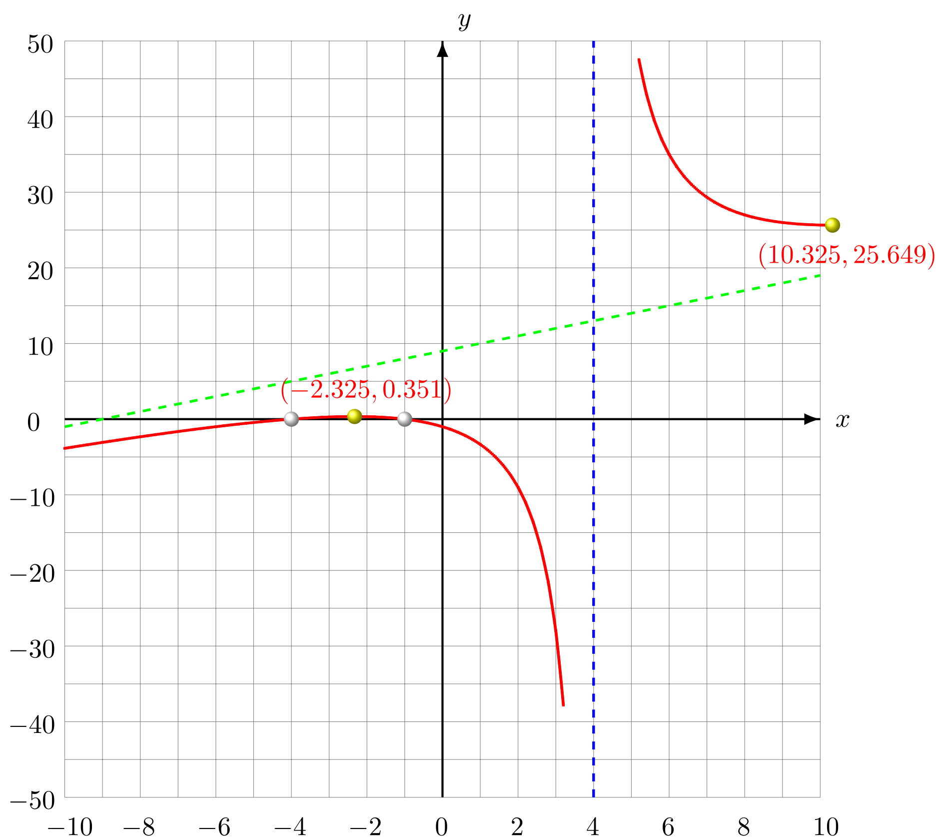
The second derivative of the function $f(x)$ is given by: $f''(x) = \frac{\mathrm{d}}{\mathrm{d}x}\frac{x^2 - 8x - 24}{(x-4)^2} = \frac{80}{(x-4)^3}$

The second derivative of the function $f(x)$ is not zero at any x. Therefore the function $f(x)$ does not have any inflection points.

Sign Chart for Second Derivative $f''(x)$

x	$(-\infty, 4)$	4	$(4, \infty)$
$f(x)$	$\frown$	DNE	$\smile$
$f''(x)$	$-$	DNE	$+$

Graph



5. $f(x) = \dfrac{(x+3)(x-1)}{x+1}$

Domain
The function $f(x)$ is a rational function. The domain of $f(x)$ is $D_f = \mathbb{R}\backslash\{-1\}$.

Symmetry
$f(-x) = \dfrac{(-x+3)(-x-1)}{-x+1}$

$f(-x) \neq -f(x) \qquad f(-x) \neq f(x)$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros
The zero(s) of the function $f(x)$ are given by: $\dfrac{(x+3)(x-1)}{x+1} = 0$ or $(x+3)(x-1) = 0$

Therefore teh zeros of the function $f(x)$ are: $x_1 = -3 \qquad x_2 = 1$

Sign Chart for $f(x)$

x	$(-\infty,-3)$	-3	$(-3,-1)$	-1	$(-1,1)$	1	$(1,\infty)$
$f(x)$	$-$	0	$+$	DNE	$-$	0	$+$

y-intercept
$y - int = f(0) = \dfrac{(0+3)(0-1)}{0+1} = -3.000$

Asymptotes
The function $f(x)$ has a vertical asymptote at $x = -1$.

The function $f(x)$ can be written as: $f(x) = \dfrac{(x+3)(x-1)}{x+1} = x + 1 + \dfrac{-4}{x+1}$

The function $f(x)$ has an oblique asymptote given by the equation: $y = x + 1$

Critical Numbers
$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{(x+3)(x-1)}{x+1} = \dfrac{x^2+2x+5}{(x+1)^2}$

Critical numbers are the solutions of the equation $f'(x) = 0$ or $\dfrac{x^2+2x+5}{(x+1)^2} = 0$

The critical numbers are: $x = -1$

Sign Chart for First Derivative $f'(x)$

x	$(-\infty,-1)$	-1	$(-1,\infty)$
$f(x)$	$\nearrow$	DNE	$\nearrow$
$f'(x)$	$+$	DNE	$+$

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(-\infty,-1) \qquad (-1,\infty)$

Maximum and Minimum Points

Concavity Intervals

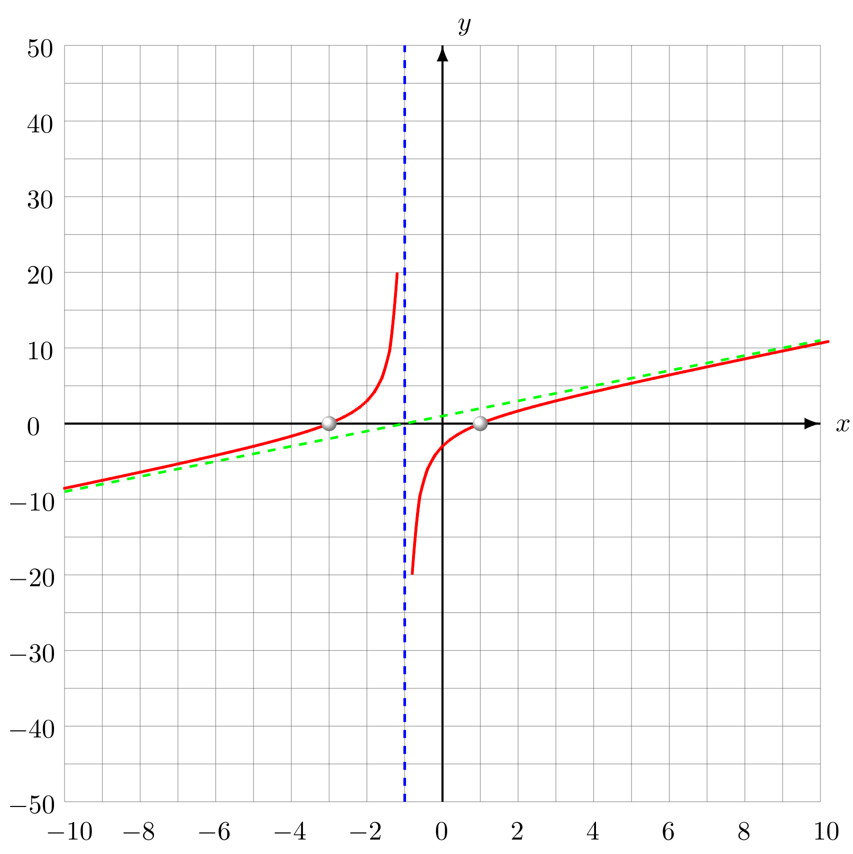
The second derivative of the function $f(x)$ is given by: $f''(x) = \frac{\mathrm{d}}{\mathrm{d}x}\frac{x^2+2x+5}{(x+1)^2} = \frac{-8}{(x+1)^3}$

The second derivative of the function $f(x)$ is not zero at any x. Therefore the function $f(x)$ does not have any inflection points.

Sign Chart for Second Derivative $f''(x)$

x	$(-\infty, -1)$	-1	$(-1, \infty)$
$f(x)$	$\smile$	DNE	$\frown$
$f''(x)$	$+$	DNE	$-$

Graph



6. $f(x) = \dfrac{(x+2)(x+4)}{x-4}$

Domain
The function $f(x)$ is a rational function. The domain of $f(x)$ is $D_f = \mathbb{R}\backslash\{4\}$.

Symmetry
$f(-x) = \dfrac{(-x+2)(-x+4)}{-x-4}$

$f(-x) \neq -f(x) \qquad f(-x) \neq f(x)$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros
The zero(s) of the function $f(x)$ are given by: $\dfrac{(x+2)(x+4)}{x-4} = 0$ or $(x+2)(x+4) = 0$

Therefore teh zeros of the function $f(x)$ are: $x_1 = -2 \qquad x_2 = -4$

Sign Chart for $f(x)$

x	$(-\infty,-4)$	-4	$(-4,-2)$	-2	$(-2,4)$	4	$(4,\infty)$
$f(x)$	$-$	0	$+$	0	$-$	DNE	$+$

y-intercept
$y - int = f(0) = \dfrac{(0+2)(0+4)}{0-4} = -2.000$

Asymptotes
The function $f(x)$ has a vertical asymptote at $x = 4$.

The function $f(x)$ can be written as: $f(x) = \dfrac{(x+2)(x+4)}{x-4} = x + 10 + \dfrac{48}{x-4}$

The function $f(x)$ has an oblique asymptote given by the equation: $y = x + 10$

Critical Numbers
$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{(x+2)(x+4)}{x-4} = \dfrac{x^2-8x-32}{(x-4)^2}$

Critical numbers are the solutions of the equation $f'(x) = 0$ or $\dfrac{x^2-8x-32}{(x-4)^2} = 0$

The critical numbers are: $x = -2.928 \qquad x = 4 \qquad x = 10.928$

Sign Chart for First Derivative $f'(x)$

x	$(-\infty,-2.928)$	-2.928	$(-2.928,4)$	4	$(4,10.928)$	10.928	$(10.928,\infty)$
$f(x)$	$\nearrow$	0.144	$\searrow$	DNE	$\searrow$	27.856	$\nearrow$
$f'(x)$	$+$	0	$-$	DNE	$-$	0	$+$

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(-\infty,-2.928) \qquad (10.928,\infty)$

The function $f(x)$ is decreasing over $(-2.928,4) \qquad (4,10.928)$

Maximum and Minimum Points

The function $f(x)$ has a maximum point at $(-2.928, 0.144)$
The function $f(x)$ has a minimum point at $(10.928, 27.856)$

Concavity Intervals

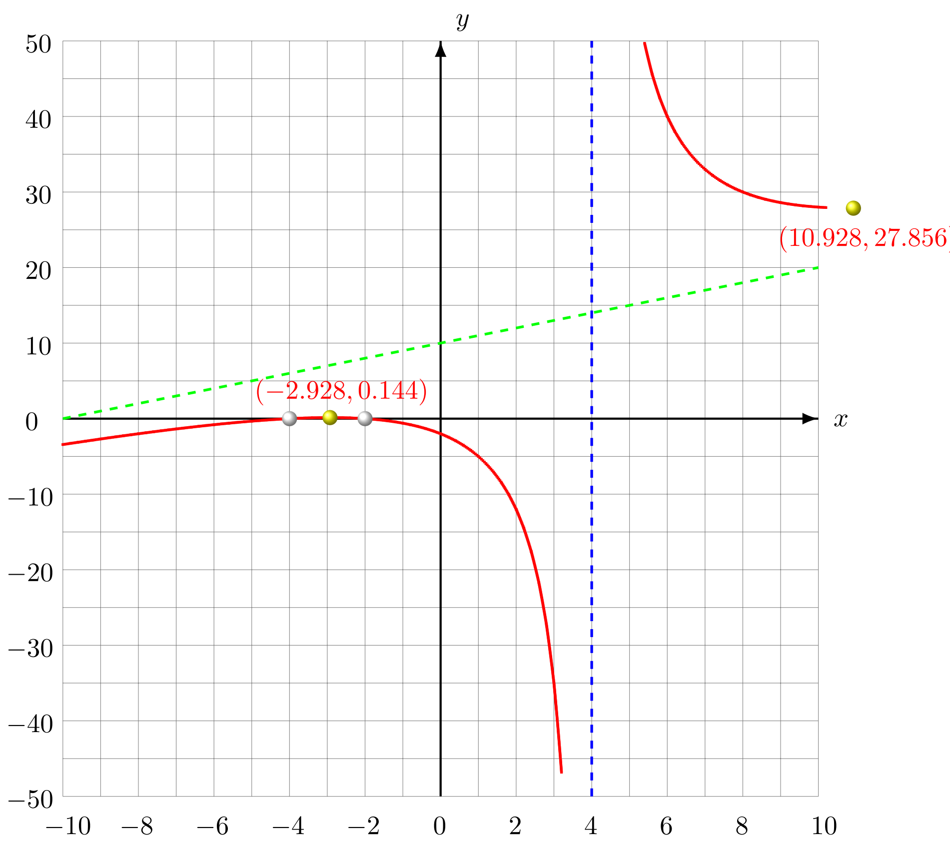
The second derivative of the function $f(x)$ is given by: $f''(x) = \frac{\mathrm{d}}{\mathrm{d}x}\frac{x^2 - 8x - 32}{(x-4)^2} = \frac{96}{(x-4)^3}$

The second derivative of the function $f(x)$ is not zero at any x. Therefore the function $f(x)$ does not have any inflection points.

Sign Chart for Second Derivative $f''(x)$

x	$(-\infty, 4)$	4	$(4, \infty)$
$f(x)$	$\frown$	DNE	$\smile$
$f''(x)$	$-$	DNE	$+$

Graph



7. $f(x) = \dfrac{(x+3)(x-2)}{x-1}$

Domain

The function $f(x)$ is a rational function. The domain of $f(x)$ is $D_f = \mathbb{R}\backslash\{1\}$.

Symmetry

$f(-x) = \dfrac{(-x+3)(-x-2)}{-x-1}$

$f(-x) \neq -f(x) \qquad f(-x) \neq f(x)$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros

The zero(s) of the function $f(x)$ are given by: $\dfrac{(x+3)(x-2)}{x-1} = 0$ or $(x+3)(x-2) = 0$

Therefore teh zeros of the function $f(x)$ are: $x_1 = -3 \qquad x_2 = 2$

Sign Chart for $f(x)$

x	$(-\infty,-3)$	-3	$(-3,1)$	1	$(1,2)$	2	$(2,\infty)$
$f(x)$	$-$	0	$+$	DNE	$-$	0	$+$

y-intercept

$y - int = f(0) = \dfrac{(0+3)(0-2)}{0-1} = 6.000$

Asymptotes

The function $f(x)$ has a vertical asymptote at $x = 1$.

The function $f(x)$ can be written as: $f(x) = \dfrac{(x+3)(x-2)}{x-1} = x + 2 + \dfrac{-4}{x-1}$

The function $f(x)$ has an oblique asymptote given by the equation: $y = x + 2$

Critical Numbers

$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{(x+3)(x-2)}{x-1} = \dfrac{x^2-2x+5}{(x-1)^2}$

Critical numbers are the solutions of the equation $f'(x) = 0$ or $\dfrac{x^2-2x+5}{(x-1)^2} = 0$

The critical numbers are: $x = 1$

Sign Chart for First Derivative $f'(x)$

x	$(-\infty,1)$	1	$(1,\infty)$
$f(x)$	$\nearrow$	DNE	$\nearrow$
$f'(x)$	$+$	DNE	$+$

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(-\infty,1) \qquad (1,\infty)$

Maximum and Minimum Points

Concavity Intervals

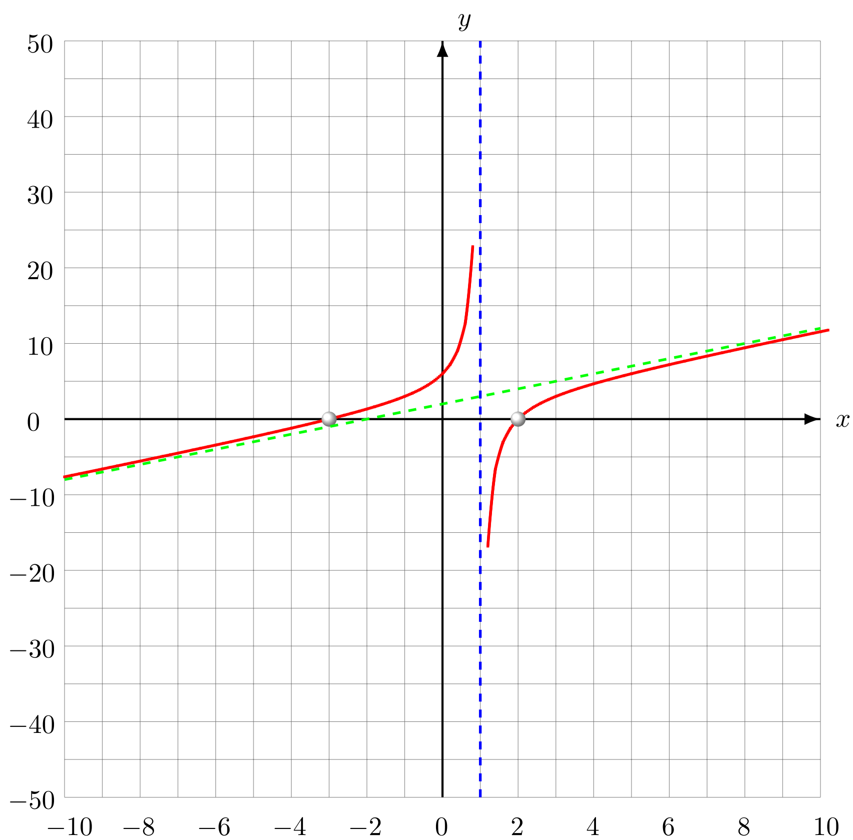
The second derivative of the function $f(x)$ is given by: $f''(x) = \frac{\mathrm{d}}{\mathrm{d}x}\frac{x^2 - 2x + 5}{(x-1)^2} = \frac{-8}{(x-1)^3}$

The second derivative of the function $f(x)$ is not zero at any x. Therefore the function $f(x)$ does not have any inflection points.

Sign Chart for Second Derivative $f''(x)$

x	$(-\infty, 1)$	1	$(1, \infty)$
$f(x)$	$\smile$	DNE	$\frown$
$f''(x)$	$+$	DNE	$-$

Graph



8. $f(x) = \dfrac{(x-3)(x-4)}{x+4}$

Domain
The function $f(x)$ is a rational function. The domain of $f(x)$ is $D_f = \mathbb{R}\backslash\{-4\}$.

Symmetry
$f(-x) = \dfrac{(-x-3)(-x-4)}{-x+4}$

$f(-x) \neq -f(x) \qquad f(-x) \neq f(x)$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros
The zero(s) of the function $f(x)$ are given by: $\dfrac{(x-3)(x-4)}{x+4} = 0 \qquad \text{or} \qquad (x-3)(x-4) = 0$

Therefore teh zeros of the function $f(x)$ are: $x_1 = 3 \qquad x_2 = 4$

Sign Chart for $f(x)$

x	$(-\infty,-4)$	-4	$(-4,3)$	3	$(3,4)$	4	$(4,\infty)$
$f(x)$	$-$	DNE	$+$	0	$-$	0	$+$

y-intercept
$y - int = f(0) = \dfrac{(0-3)(0-4)}{0+4} = 3.000$

Asymptotes
The function $f(x)$ has a vertical asymptote at $x = -4$.

The function $f(x)$ can be written as: $f(x) = \dfrac{(x-3)(x-4)}{x+4} = x - 11 + \dfrac{56}{x+4}$

The function $f(x)$ has an oblique asymptote given by the equation: $y = x - 11$

Critical Numbers
$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{(x-3)(x-4)}{x+4} = \dfrac{x^2+8x-40}{(x+4)^2}$

Critical numbers are the solutions of the equation $f'(x) = 0 \qquad \text{or} \qquad \dfrac{x^2+8x-40}{(x+4)^2} = 0$

The critical numbers are: $x = -11.483 \qquad x = -4 \qquad x = 3.483$

Sign Chart for First Derivative $f'(x)$

x	$(-\infty,-11.483)$	-11.483	$(-11.483,-4)$	-4	$(-4,3.483)$	3.483	$(3.483,\infty)$
$f(x)$	$\nearrow$	-29.967	$\searrow$	DNE	$\searrow$	-0.033	$\nearrow$
$f'(x)$	$+$	0	$-$	DNE	$-$	0	$+$

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(-\infty,-11.483) \qquad (3.483,\infty)$

The function $f(x)$ is decreasing over $(-11.483,-4) \qquad (-4,3.483)$

Maximum and Minimum Points

The function $f(x)$ has a maximum point at $(-11.483, -29.967)$
The function $f(x)$ has a minimum point at $(3.483, -0.033)$

Concavity Intervals

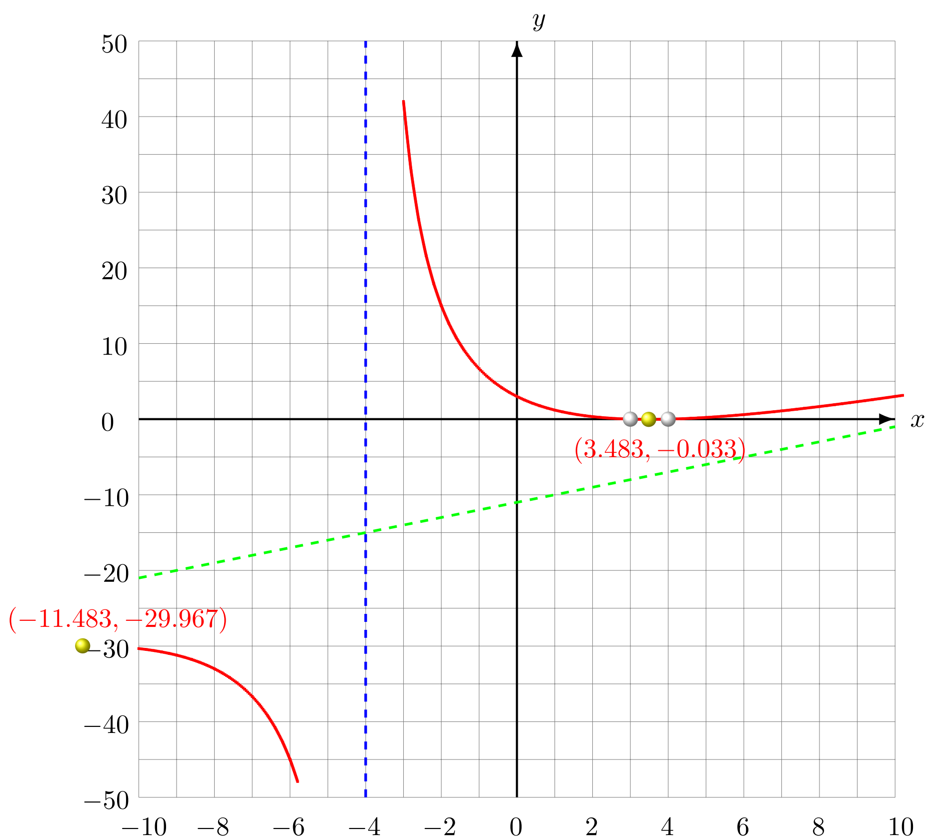
The second derivative of the function $f(x)$ is given by: $f''(x) = \frac{\mathrm{d}}{\mathrm{d}x} \frac{x^2 + 8x - 40}{(x+4)^2} = \frac{112}{(x+4)^3}$

The second derivative of the function $f(x)$ is not zero at any x. Therefore the function $f(x)$ does not have any inflection points.

Sign Chart for Second Derivative $f''(x)$

x	$(-\infty, -4)$	-4	$(-4, \infty)$
$f(x)$	$\frown$	DNE	$\smile$
$f''(x)$	$-$	DNE	$+$

Graph



9. $f(x) = \dfrac{(x+2)(x-3)}{x+3}$

Domain

The function $f(x)$ is a rational function. The domain of $f(x)$ is $D_f = \mathbb{R}\backslash\{-3\}$.

Symmetry

$f(-x) = \dfrac{(-x+2)(-x-3)}{-x+3}$

$f(-x) \neq -f(x) \qquad f(-x) \neq f(x)$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros

The zero(s) of the function $f(x)$ are given by: $\dfrac{(x+2)(x-3)}{x+3} = 0 \qquad \text{or} \qquad (x+2)(x-3) = 0$

Therefore teh zeros of the function $f(x)$ are: $x_1 = -2 \qquad x_2 = 3$

Sign Chart for $f(x)$

x	$(-\infty,-3)$	-3	$(-3,-2)$	-2	$(-2,3)$	3	$(3,\infty)$
$f(x)$	$-$	DNE	$+$	0	$-$	0	$+$

y-intercept

$y-int = f(0) = \dfrac{(0+2)(0-3)}{0+3} = -2.000$

Asymptotes

The function $f(x)$ has a vertical asymptote at $x = -3$.

The function $f(x)$ can be written as: $f(x) = \dfrac{(x+2)(x-3)}{x+3} = x - 4 + \dfrac{6}{x+3}$

The function $f(x)$ has an oblique asymptote given by the equation: $y = x - 4$

Critical Numbers

$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{(x+2)(x-3)}{x+3} = \dfrac{x^2+6x+3}{(x+3)^2}$

Critical numbers are the solutions of the equation $f'(x) = 0 \qquad \text{or} \qquad \dfrac{x^2+6x+3}{(x+3)^2} = 0$

The critical numbers are: $x = -5.449 \qquad x = -3 \qquad x = -0.551$

Sign Chart for First Derivative $f'(x)$

x	$(-\infty,-5.449)$	-5.449	$(-5.449,-3)$	-3	$(-3,-0.551)$	-0.551	$(-0.551,\infty)$
$f(x)$	$\nearrow$	-11.899	$\searrow$	DNE	$\searrow$	-2.101	$\nearrow$
$f'(x)$	$+$	0	$-$	DNE	$-$	0	$+$

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(-\infty,-5.449) \qquad (-0.551,\infty)$

The function $f(x)$ is decreasing over $(-5.449,-3) \qquad (-3,-0.551)$

Maximum and Minimum Points

The function $f(x)$ has a maximum point at $(-5.449, -11.899)$
The function $f(x)$ has a minimum point at $(-0.551, -2.101)$

Concavity Intervals

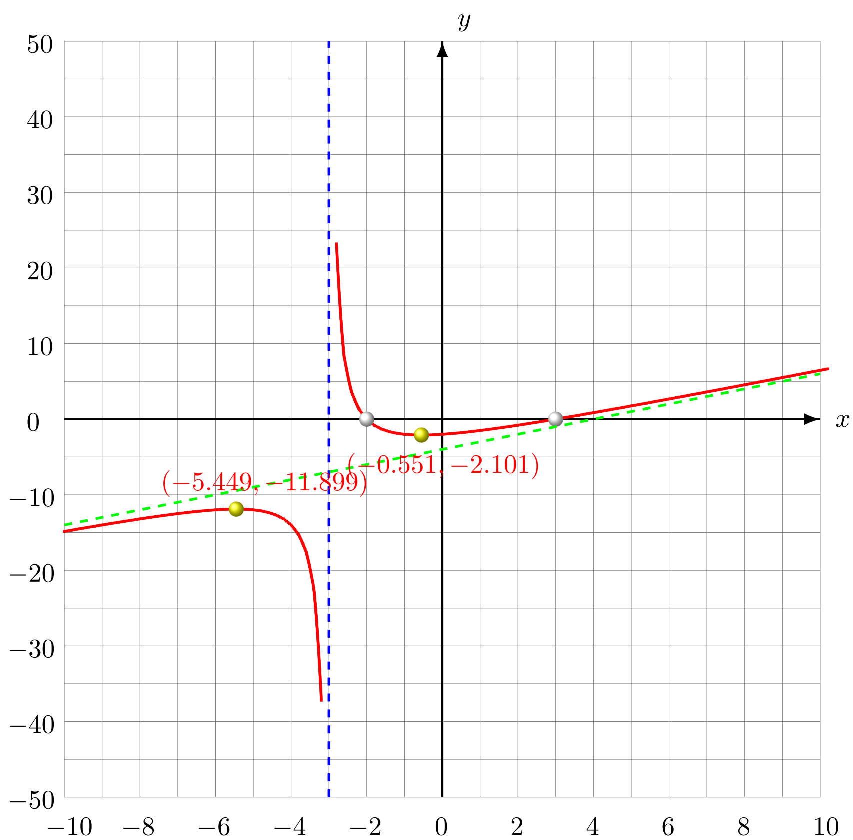
The second derivative of the function $f(x)$ is given by: $f''(x) = \frac{\mathrm{d}}{\mathrm{d}x}\frac{x^2+6x+3}{(x+3)^2} = \frac{12}{(x+3)^3}$

The second derivative of the function $f(x)$ is not zero at any x. Therefore the function $f(x)$ does not have any inflection points.

Sign Chart for Second Derivative $f''(x)$

x	$(-\infty, -3)$	-3	$(-3, \infty)$
$f(x)$	$\frown$	DNE	$\smile$
$f''(x)$	$-$	DNE	$+$

Graph



10. $f(x) = \dfrac{(x-2)(x+4)}{x+2}$

Domain
The function $f(x)$ is a rational function. The domain of $f(x)$ is $D_f = \mathbb{R}\backslash\{-2\}$.

Symmetry
$f(-x) = \dfrac{(-x-2)(-x+4)}{-x+2}$

$f(-x) \neq -f(x) \qquad f(-x) \neq f(x)$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros
The zero(s) of the function $f(x)$ are given by: $\dfrac{(x-2)(x+4)}{x+2} = 0$ or $(x-2)(x+4) = 0$

Therefore teh zeros of the function $f(x)$ are: $x_1 = 2 \qquad x_2 = -4$

Sign Chart for $f(x)$

x	$(-\infty,-4)$	-4	$(-4,-2)$	-2	$(-2,2)$	2	$(2,\infty)$
$f(x)$	$-$	0	$+$	DNE	$-$	0	$+$

y-intercept
$y - int = f(0) = \dfrac{(0-2)(0+4)}{0+2} = -4.000$

Asymptotes
The function $f(x)$ has a vertical asymptote at $x = -2$.

The function $f(x)$ can be written as: $f(x) = \dfrac{(x-2)(x+4)}{x+2} = x + 0 + \dfrac{-8}{x+2}$

The function $f(x)$ has an oblique asymptote given by the equation: $y = x + 0$

Critical Numbers
$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{(x-2)(x+4)}{x+2} = \dfrac{x^2+4x+12}{(x+2)^2}$

Critical numbers are the solutions of the equation $f'(x) = 0$ or $\dfrac{x^2+4x+12}{(x+2)^2} = 0$

The critical numbers are: $x = -2$

Sign Chart for First Derivative $f'(x)$

x	$(-\infty,-2)$	-2	$(-2,\infty)$
$f(x)$	$\nearrow$	DNE	$\nearrow$
$f'(x)$	$+$	DNE	$+$

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(-\infty,-2) \qquad (-2,\infty)$

Maximum and Minimum Points

Concavity Intervals

The second derivative of the function $f(x)$ is given by: $f''(x) = \frac{\mathrm{d}}{\mathrm{d}x}\frac{x^2+4x+12}{(x+2)^2} = \frac{-16}{(x+2)^3}$

The second derivative of the function $f(x)$ is not zero at any x. Therefore the function $f(x)$ does not have any inflection points.

Sign Chart for Second Derivative $f''(x)$

x	$(-\infty, -2)$	-2	$(-2, \infty)$
$f(x)$	$\smile$	DNE	$\frown$
$f''(x)$	$+$	DNE	$-$

Graph