1. Consider the following polynomial function: $f(x) = \frac{1}{9}(x-1)^2\sqrt[5]{(x+2)^3}$

Sketch the grapf of the function $f(x)$.

2. Consider the following polynomial function: $f(x) = \frac{1}{16}\frac{(x-3)^2}{\sqrt{x+1}}$

Sketch the grapf of the function $f(x)$.

3. Consider the following polynomial function: $f(x) = \frac{1}{5}(x-2)\sqrt[3]{(x+3)^4}$

Sketch the grapf of the function $f(x)$.

4. Consider the following polynomial function: $f(x) = \frac{1}{16}(x-2)^2\sqrt[5]{(x+2)^3}$

Sketch the grapf of the function $f(x)$.

5. Consider the following polynomial function: $f(x) = \frac{1}{1296}(x-3)^4\sqrt[5]{(x+3)^3}$

Sketch the grapf of the function $f(x)$.

6. Consider the following polynomial function: $f(x) = \frac{1}{16}\frac{(x-1)^2}{\sqrt[5]{(x+3)^2}}$

Sketch the grapf of the function $f(x)$.

7. Consider the following polynomial function: $f(x) = \frac{1}{16}\frac{(x-1)^2}{\sqrt[4]{(x+3)^3}}$

Sketch the grapf of the function $f(x)$.

8. Consider the following polynomial function: $f(x) = \frac{1}{9}(x-3)^2\sqrt[4]{x}$

Sketch the grapf of the function $f(x)$.

9. Consider the following polynomial function: $f(x) = \frac{1}{8}(x-1)^3\sqrt[5]{(x+1)^3}$

Sketch the grapf of the function $f(x)$.

10. Consider the following polynomial function: $f(x) = \frac{1}{1}x^3\sqrt[4]{(x+1)^3}$

Sketch the grapf of the function $f(x)$.

Solutions:

1. $f(x) = \frac{1}{9}(x-1)^2\sqrt[5]{(x+2)^3}$

Domain

The domain is $D_f = \mathbb{R}$.

Symmetry

$f(-x) = \frac{1}{9}(-x-1)^2\sqrt[5]{(-x+2)^3}$

$f(-x) \neq -f(x) \qquad f(-x) \neq f(x)$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros

The zero(s) of the function $f(x)$ is(are):

$x_1 = 1 \qquad x_2 = -2$

Sign Chart for $f(x)$

x	$(-\infty,-2)$	-2	$(-2,1)$	1	$(1,\infty)$
$f(x)$	$-$	0	$+$	0	$+$

y-intercept

$y-int = f(0) = \frac{1}{9}(0-1)^2\sqrt[5]{(0+2)^3} = 0.168$

Asymptotes

The function $f(x)$ does not have any kind of asymptotes.

Critical Numbers

$f'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{9}(x-1)^2\sqrt[5]{(x+2)^3} = \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{9}(x-1)^2(x+2)^{\frac{3}{5}} = \frac{1}{45}\frac{x-1}{\sqrt[5]{(x+2)^2}}[13x+17]$

Critical numbers are the solutions of the equation $f'(x) = 0$ or

$\frac{1}{45}\frac{x-1}{\sqrt[5]{(x+2)^2}}[13x+17] = 0$

$x_3 = \frac{-17}{13} = -1.308$

The critical number(s) is(are): $x = -2$ $x = -1.308$ $x = 1$ px3= 1

Sign Chart for First Derivative $f'(x)$

x	$(-\infty,-2)$	-2	$(-2,-1.308)$	-1.308	$(-1.308,1)$	1	$(1,\infty)$
$f(x)$	$\nearrow$	0	$\nearrow$	0.475	$\searrow$	0	$\nearrow$
$f'(x)$	$+$	DNE	$+$	0	$-$	0	$+$

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(-\infty,-2)$ $\quad (-2,-1.308) \quad (1,\infty)$

The function $f(x)$ is decreasing over $(-1.308,1)$

Maximum and Minimum Points

The function $f(x)$ has a maximum point at $(-1.308, 0.475)$
The function $f(x)$ has a minimum point at $(1, 0.000)$

Concavity Intervals
The second derivative of the function $f(x)$ is given by:

$$f''(x) = \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{45}\frac{x-1}{\sqrt[5]{(x+2)^2}}[13x+17] = \frac{1}{225}\frac{1}{\sqrt[5]{(x+2)^7}}(104x^2+272x+74)$$

The second derivative $f''(x)$ is zero when $f''(x) = 0$ or: $\frac{1}{225}\frac{1}{\sqrt[5]{(x+2)^7}}(104x^2+272x+74) = 0$

The second derivative $f''(x)$ is zero at: $x_4 = -2.307$ $x_5 = -0.308$

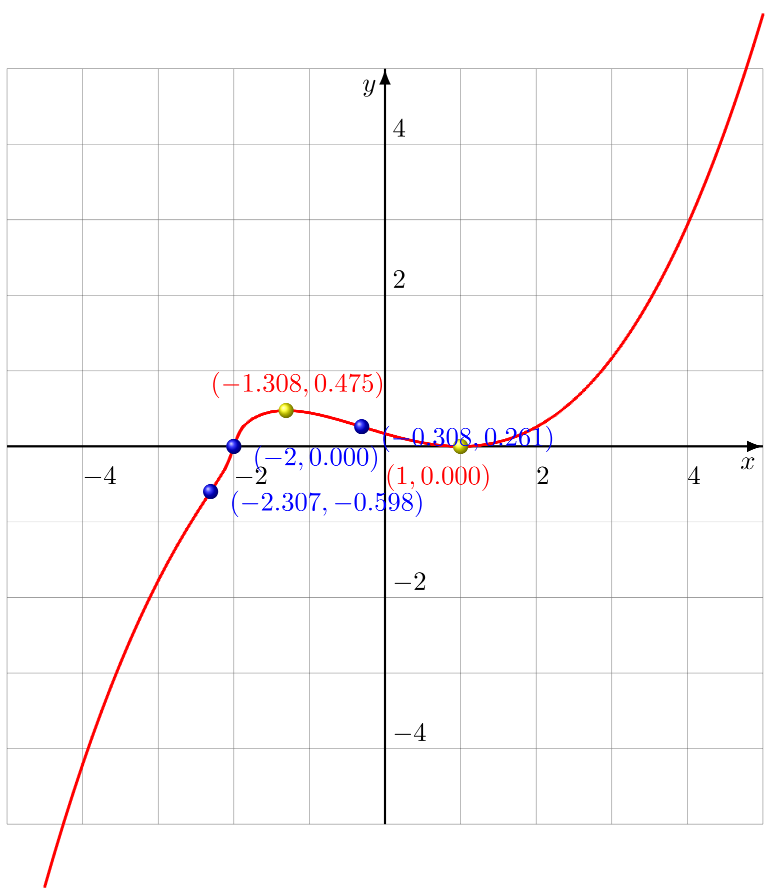
The second derivative $f''(x)$ does not exist at: $x_2 = -2$

Sign Chart for the Second Derivative $f''(x)$

x	$(-\infty, -2.307)$	-2.307	$(-2.307, -2)$	-2	$(-2, -0.308)$	-0.308	$(-0.308, \infty)$
$f(x)$	$\frown$	-0.598	$\smile$	0	$\frown$	0.261	$\smile$
$f''(x)$	$-$	0	$+$	DNE	$-$	0	$+$

Inflection Points
The inflection point(s) is(are): (-2.307,-0.598) (-2,0.000) (-0.308,0.261)

Graph



2. $f(x) = \frac{1}{16}\frac{(x-3)^2}{\sqrt{x+1}}$

Domain

The domain is $D_f = (-1, \infty)$.

Symmetry

$f(-x) = \frac{1}{16}\frac{(-x-3)^2}{\sqrt{-x+1}}$

$f(-x) \neq -f(x) \qquad f(-x) \neq f(x)$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros

The zero(s) of the function $f(x)$ is(are):

$x_1 = 3$

Sign Chart for $f(x)$

x	$(-\infty,-1)$	-1	$(-1,3)$	3	$(3,\infty)$
$f(x)$	DNE	DNE	$+$	0	$+$

y-intercept

$y - int = f(0) = \frac{1}{16}\frac{(0-3)^2}{\sqrt{0+1}} = 0.563$

Asymptotes

The function $f(x)$ has a vertical asymptote at $x = -1$.

Critical Numbers

$f'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{16}\frac{(x-3)^2}{\sqrt{x+1}} = \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{16}(x-3)^2(x+1)^{\frac{-1}{2}} = \frac{1}{32}\frac{x-3}{\sqrt{(x+1)^3}}[3x+7]$

Critical numbers are the solutions of the equation $f'(x) = 0$ or

$\frac{1}{32}\frac{x-3}{\sqrt{(x+1)^3}}[3x+7] = 0$

$x_3 = \frac{-7}{3} = -2.333$

The critical number(s) is(are): $x = -1 \qquad x = 3$

Sign Chart for First Derivative $f'(x)$

x	$(-\infty,-1)$	-1	$(-1,3)$	3	$(3,\infty)$
$f(x)$	DNE	DNE	$\searrow$	0	$\nearrow$
$f'(x)$	DNE	DNE	$-$	0	$+$

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(3, \infty)$

The function $f(x)$ is decreasing over $(-1, 3)$

Maximum and Minimum Points

The function $f(x)$ has a minimum point at $(3, 0.000)$

Concavity Intervals
The second derivative of the function $f(x)$ is given by:

$$f''(x) = \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{32}\frac{x-3}{\sqrt{(x+1)^3}}[3x+7] = \frac{1}{64}\frac{1}{\sqrt{(x+1)^5}}(3x^2+14x+59)$$

The second derivative $f''(x)$ is zero when $f''(x) = 0$ or: $\frac{1}{64}\frac{1}{\sqrt{(x+1)^5}}(3x^2+14x+59) = 0$

The second derivative $f''(x)$ is zero at:

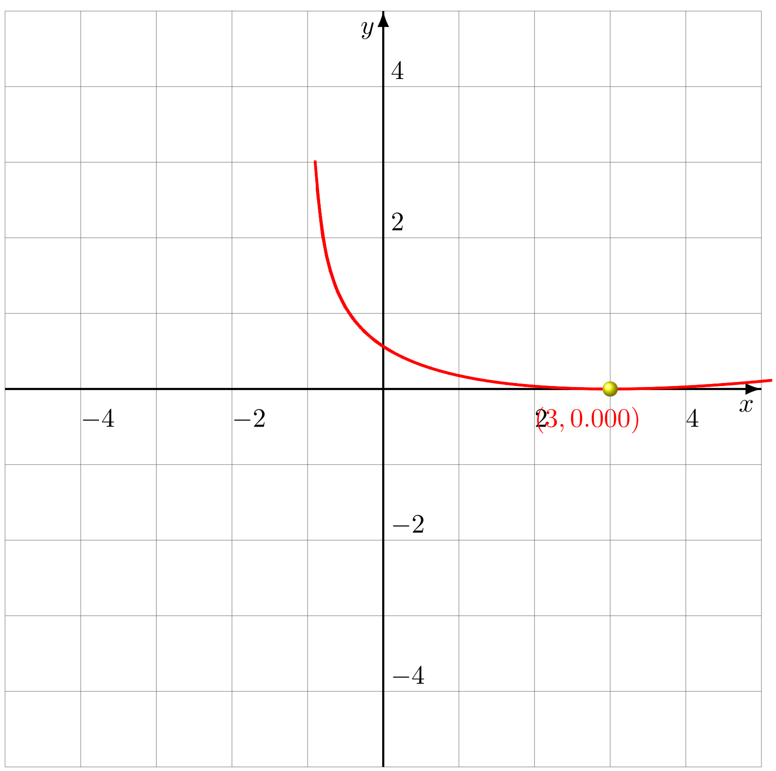
The second derivative $f''(x)$ does not exist at: $x_2 = -1$

Sign Chart for the Second Derivative $f''(x)$

x	$(-\infty, -1)$	-1	$(-1, \infty)$
$f(x)$	DNE	DNE	$\smile$
$f''(x)$	DNE	DNE	$+$

Inflection Points
The inflection point(s) is(are):

Graph



3. $f(x) = \frac{1}{5}(x-2)\sqrt[3]{(x+3)^4}$

Domain
The domain is $D_f = \mathbb{R}$.

Symmetry
$f(-x) = \frac{1}{5}(-x-2)\sqrt[3]{(-x+3)^4}$

$f(-x) \neq -f(x) \qquad f(-x) \neq f(x)$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros
The zero(s) of the function $f(x)$ is(are):

$x_1 = 2 \qquad x_2 = -3$

Sign Chart for $f(x)$

x	$(-\infty,-3)$	-3	$(-3,2)$	2	$(2,\infty)$
$f(x)$	$-$	0	$-$	0	$+$

y-intercept
$y - int = f(0) = \frac{1}{5}(0-2)\sqrt[3]{(0+3)^4} = -1.731$

Asymptotes
The function $f(x)$ does not have any kind of asymptotes.

Critical Numbers
$f'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{5}(x-2)\sqrt[3]{(x+3)^4} = \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{5}(x-2)(x+3)^{\frac{4}{3}} = \frac{1}{15}\sqrt[3]{x+3}[7x+1]$

Critical numbers are the solutions of the equation $f'(x) = 0$ or

$\frac{1}{15}\sqrt[3]{x+3}[7x+1] = 0$

$x_3 = \frac{-1}{7} = -0.143$

The critical number(s) is(are): $x = -3 \qquad x = -0.143 \qquad$ px3= 1

Sign Chart for First Derivative $f'(x)$

x	$(-\infty,-3)$	-3	$(-3,-0.143)$	-0.143	$(-0.143,\infty)$
$f(x)$	$\nearrow$	0	$\searrow$	-1.738	$\nearrow$
$f'(x)$	$+$	0	$-$	0	$+$

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(-\infty,-3) \qquad (-0.143,\infty)$

The function $f(x)$ is decreasing over $(-3,-0.143)$

Maximum and Minimum Points

The function $f(x)$ has a maximum point at $(-3, 0.000)$

The function $f(x)$ has a minimum point at $(-0.143, -1.738)$

Concavity Intervals
The second derivative of the function $f(x)$ is given by:

$$f''(x) = \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{15}\sqrt[3]{x+3}[7x+1] = \frac{1}{45}\frac{1}{\sqrt[3]{(x+3)^2}}(28x+64)$$

The second derivative $f''(x)$ is zero when $f''(x) = 0$ or: $\frac{1}{45}\frac{1}{\sqrt[3]{(x+3)^2}}(28x+64) = 0$

The second derivative $f''(x)$ is zero at: $x_4 = -2.286$

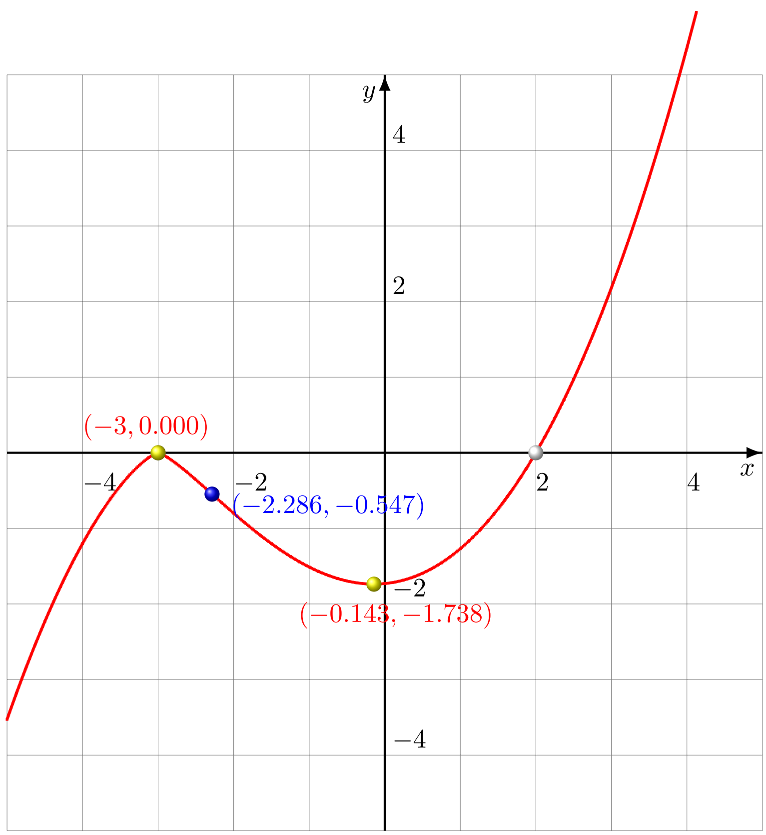
The second derivative $f''(x)$ does not exist at: $x_2 = -3$

Sign Chart for the Second Derivative $f''(x)$

x	$(-\infty, -3)$	-3	$(-3, -2.286)$	-2.286	$(-2.286, \infty)$
$f(x)$	$\frown$	0	$\frown$	-0.547	$\smile$
$f''(x)$	$-$	DNE	$-$	0	$+$

Inflection Points
The inflection point(s) is(are): (-2.286,-0.547)

Graph



4. $f(x) = \dfrac{1}{16}(x-2)^2 \sqrt[5]{(x+2)^3}$

Domain

The domain is $D_f = \mathbb{R}$.

Symmetry

$f(-x) = \dfrac{1}{16}(-x-2)^2 \sqrt[5]{(-x+2)^3}$

$f(-x) \neq -f(x) \qquad f(-x) \neq f(x)$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros

The zero(s) of the function $f(x)$ is(are):

$x_1 = 2 \qquad x_2 = -2$

Sign Chart for $f(x)$

x	$(-\infty,-2)$	-2	$(-2,2)$	2	$(2,\infty)$
$f(x)$	$-$	0	$+$	0	$+$

y-intercept

$y - int = f(0) = \dfrac{1}{16}(0-2)^2 \sqrt[5]{(0+2)^3} = 0.379$

Asymptotes

The function $f(x)$ does not have any kind of asymptotes.

Critical Numbers

$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{1}{16}(x-2)^2 \sqrt[5]{(x+2)^3} = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{1}{16}(x-2)^2(x+2)^{\frac{3}{5}} = \dfrac{1}{80}\dfrac{x-2}{\sqrt[5]{(x+2)^2}}[13x+14]$

Critical numbers are the solutions of the equation $f'(x) = 0$ or

$\dfrac{1}{80}\dfrac{x-2}{\sqrt[5]{(x+2)^2}}[13x+14] = 0$

$x_3 = \dfrac{-14}{13} = -1.077$

The critical number(s) is(are): $x = -2 \qquad x = -1.077 \qquad x = 2$ px3= 1

Sign Chart for First Derivative $f'(x)$

x	$(-\infty,-2)$	-2	$(-2,-1.077)$	-1.077	$(-1.077,2)$	2	$(2,\infty)$
$f(x)$	$\nearrow$	0	$\nearrow$	0.564	$\searrow$	0	$\nearrow$
$f'(x)$	$+$	DNE	$+$	0	$-$	0	$+$

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(-\infty,-2) \qquad (-2,-1.077) \qquad (2,\infty)$

The function $f(x)$ is decreasing over $(-1.077,2)$

Maximum and Minimum Points

The function $f(x)$ has a maximum point at $(-1.077, 0.564)$
The function $f(x)$ has a minimum point at $(2, 0.000)$

Concavity Intervals
The second derivative of the function $f(x)$ is given by:

$$f''(x) = \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{80}\frac{x-2}{\sqrt[5]{(x+2)^2}}[13x+14] = \frac{1}{400}\frac{1}{\sqrt[5]{(x+2)^7}}(104x^2+224x+-64)$$

The second derivative $f''(x)$ is zero when $f''(x) = 0$ or: $\frac{1}{400}\frac{1}{\sqrt[5]{(x+2)^7}}(104x^2+224x+-64) = 0$

The second derivative $f''(x)$ is zero at: $x_4 = -2.409$ $x_5 = 0.255$

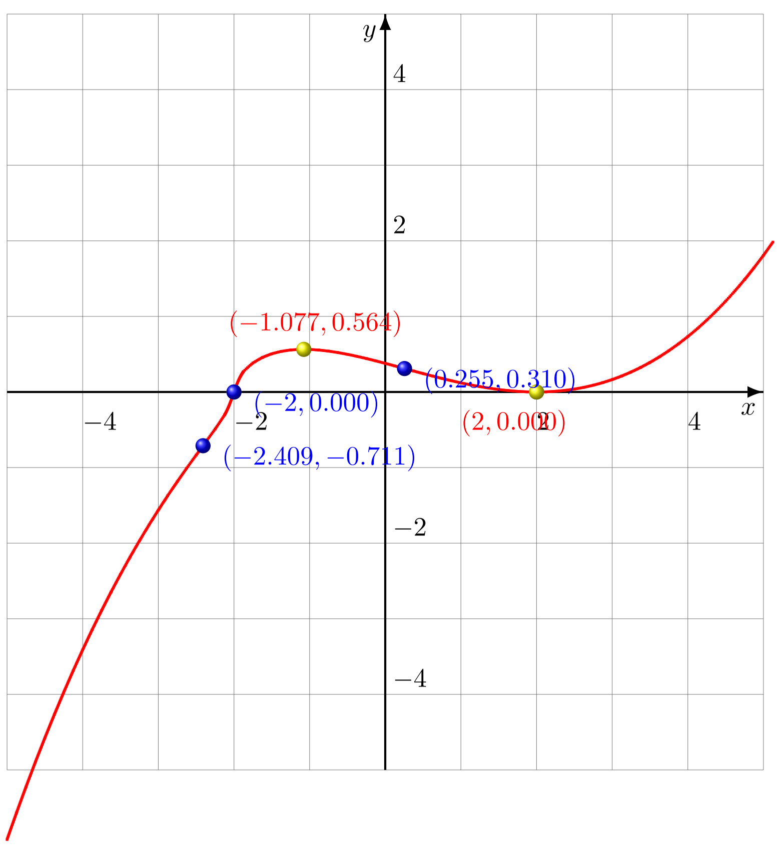
The second derivative $f''(x)$ does not exist at: $x_2 = -2$

Sign Chart for the Second Derivative $f''(x)$

x	$(-\infty, -2.409)$	-2.409	$(-2.409, -2)$	-2	$(-2, 0.255)$	0.255	$(0.255, \infty)$
$f(x)$	$\frown$	-0.711	$\smile$	0	$\frown$	0.310	$\smile$
$f''(x)$	$-$	0	$+$	DNE	$-$	0	$+$

Inflection Points
The inflection point(s) is(are): (-2.409,-0.711) (-2,0.000) (0.255,0.310)

Graph



5. $f(x) = \frac{1}{1296}(x-3)^4\sqrt[5]{(x+3)^3}$

Domain
The domain is $D_f = \mathbb{R}$.

Symmetry
$f(-x) = \frac{1}{1296}(-x-3)^4\sqrt[5]{(-x+3)^3}$

$f(-x) \neq -f(x) \qquad f(-x) \neq f(x)$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros
The zero(s) of the function $f(x)$ is(are):

$x_1 = 3 \qquad x_2 = -3$

Sign Chart for $f(x)$

x	$(-\infty,-3)$	-3	$(-3,3)$	3	$(3,\infty)$
$f(x)$	$-$	0	$+$	0	$+$

y-intercept
$y - int = f(0) = \frac{1}{1296}(0-3)^4\sqrt[5]{(0+3)^3} = 0.121$

Asymptotes
The function $f(x)$ does not have any kind of asymptotes.

Critical Numbers
$f'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{1296}(x-3)^4\sqrt[5]{(x+3)^3} = \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{1296}(x-3)^4(x+3)^{\frac{3}{5}} = \frac{1}{6480}\frac{(x-3)^3}{\sqrt[5]{(x+3)^2}}[23x+51]$

Critical numbers are the solutions of the equation $f'(x) = 0$ or

$\frac{1}{6480}\frac{(x-3)^3}{\sqrt[5]{(x+3)^2}}[23x+51] = 0$

$x_3 = \frac{-51}{23} = -2.217$

The critical number(s) is(are): $x = -3$ $x = -2.217$ $x = 3$ px3= 1

Sign Chart for First Derivative $f'(x)$

x	$(-\infty,-3)$	-3	$(-3,-2.217)$	-2.217	$(-2.217,3)$	3	$(3,\infty)$
$f(x)$	$\nearrow$	0	$\nearrow$	0.494	$\searrow$	0	$\nearrow$
$f'(x)$	$+$	DNE	$+$	0	$-$	0	$+$

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(-\infty,-3)$ $(-3,-2.217)$ $(3,\infty)$

The function $f(x)$ is decreasing over $(-2.217,3)$

Maximum and Minimum Points

The function $f(x)$ has a maximum point at $(-2.217, 0.494)$
The function $f(x)$ has a minimum point at $(3, 0.000)$

Concavity Intervals
The second derivative of the function $f(x)$ is given by:

$$f''(x) = \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{6480}\frac{(x-3)^3}{\sqrt[5]{(x+3)^2}}[23x+51] = \frac{1}{32400}\frac{(x-3)^2}{\sqrt[5]{(x+3)^7}}(414x^2+1836x+1566)$$

The second derivative $f''(x)$ is zero when $f''(x) = 0$ or: $\frac{1}{32400}\frac{(x-3)^2}{\sqrt[5]{(x+3)^7}}(414x^2+1836x+1566) = 0$

The second derivative $f''(x)$ is zero at: $x_4 = -3.282$ $x_5 = -1.152$ $x_1 = 3.000$

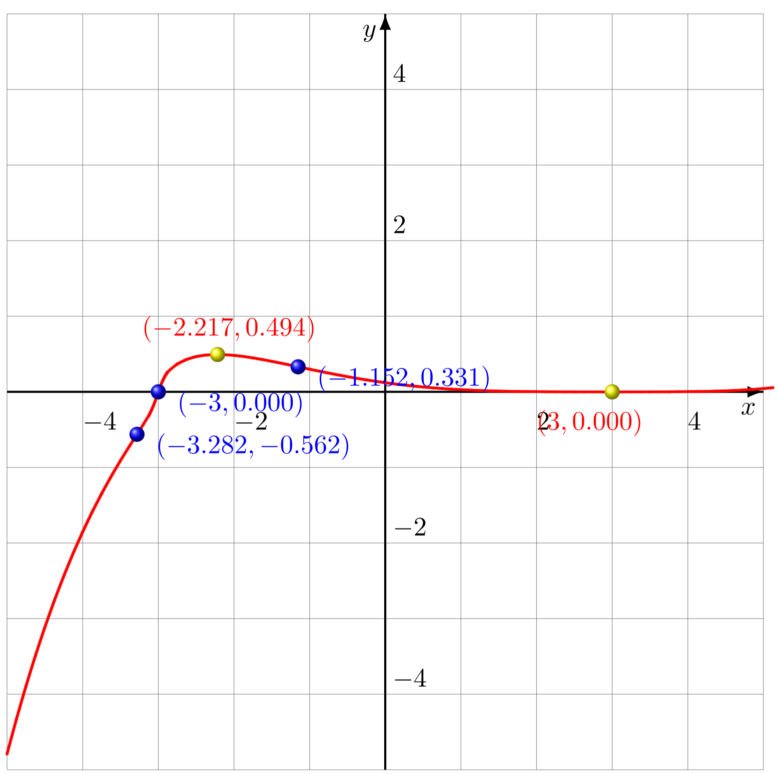
The second derivative $f''(x)$ does not exist at: $x_2 = -3$

Sign Chart for the Second Derivative $f''(x)$

x	$(-\infty, -3.282)$	-3.282	$(-3.282, -3)$	-3	$(-3, -1.152)$	-1.152	$(-1.152, 3)$	3	$(3, \infty)$
$f(x)$	$\frown$	-0.563	$\smile$	0	$\frown$	0.332	$\smile$	0	$\smile$
$f''(x)$	$-$	0	$+$	DNE	$-$	0	$+$	0	$+$

Inflection Points
The inflection point(s) is(are): (-3.282,-0.562) (-3,0.000) (-1.152,0.331)

Graph



6. $f(x) = \dfrac{1}{16}\dfrac{(x-1)^2}{\sqrt[5]{(x+3)^2}}$

Domain

The domain is $D_f = \mathbb{R}\backslash\{-3\}$.

Symmetry

$f(-x) = \dfrac{1}{16}\dfrac{(-x-1)^2}{\sqrt[5]{(-x+3)^2}}$

$f(-x) \neq -f(x) \qquad f(-x) \neq f(x)$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros

The zero(s) of the function $f(x)$ is(are):

$x_1 = 1$

Sign Chart for $f(x)$

x	$(-\infty, -3)$	-3	$(-3, 1)$	1	$(1, \infty)$
$f(x)$	$+$	DNE	$+$	0	$+$

y-intercept

$y - int = f(0) = \dfrac{1}{16}\dfrac{(0-1)^2}{\sqrt[5]{(0+3)^2}} = 0.040$

Asymptotes

The function $f(x)$ has a vertical asymptote at $x = -3$.

Critical Numbers

$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{1}{16}\dfrac{(x-1)^2}{\sqrt[5]{(x+3)^2}} = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{1}{16}(x-1)^2(x+3)^{\frac{-2}{5}} = \dfrac{1}{80}\dfrac{x-1}{\sqrt[5]{(x+3)^7}}[8x+32]$

Critical numbers are the solutions of the equation $f'(x) = 0$ or

$\dfrac{1}{80}\dfrac{x-1}{\sqrt[5]{(x+3)^7}}[8x+32] = 0$

$x_3 = \dfrac{-32}{8} = -4.000$

The critical number(s) is(are): $x = -3 \qquad x = -4 \qquad x = 1$ px3=

Sign Chart for First Derivative $f'(x)$

x	$(-\infty, -4)$	-4	$(-4, -3)$	-3	$(-3, 1)$	1	$(1, \infty)$
$f(x)$	$\searrow$	1.563	$\nearrow$	DNE	$\searrow$	0	$\nearrow$
$f'(x)$	$-$	0	$+$	DNE	$-$	0	$+$

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(-4, -3) \qquad (1, \infty)$

The function $f(x)$ is decreasing over $(-\infty, -4) \qquad (-3, 1)$

Maximum and Minimum Points

The function $f(x)$ has a minimum point at $(-4, 1.563)$ $(1, 0.000)$

Concavity Intervals

The second derivative of the function $f(x)$ is given by:

$$f''(x) = \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{80}\frac{x-1}{\sqrt[5]{(x+3)^7}}[8x+32] = \frac{1}{400}\frac{1}{\sqrt[5]{(x+3)^{12}}}(24x^2+192x+584)$$

The second derivative $f''(x)$ is zero when $f''(x) = 0$ or: $\frac{1}{400}\frac{1}{\sqrt[5]{(x+3)^{12}}}(24x^2+192x+584) = 0$

The second derivative $f''(x)$ is zero at:

The second derivative $f''(x)$ does not exist at: $x_2 = -3$

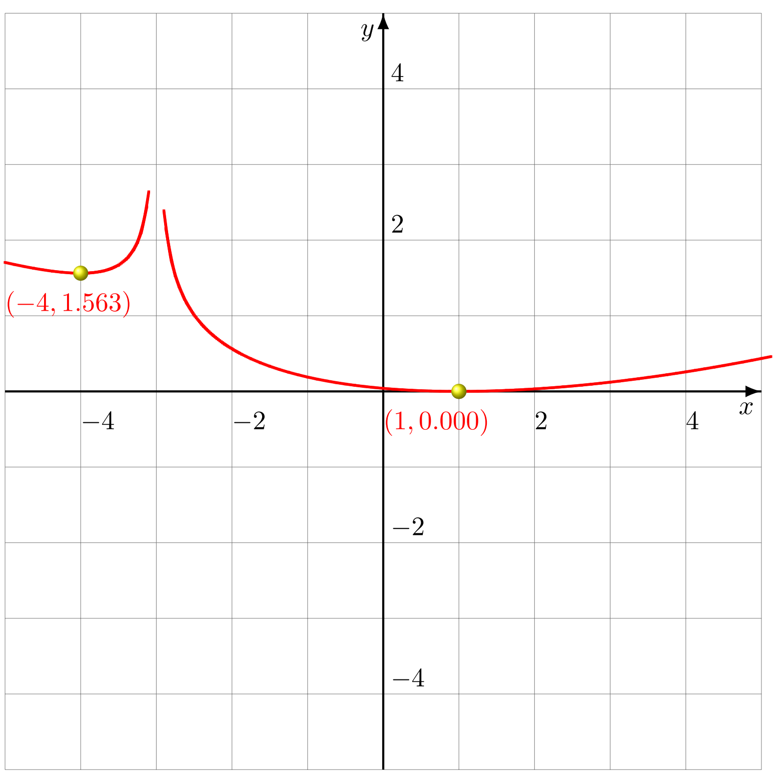
Sign Chart for the Second Derivative $f''(x)$

x	$(-\infty, -3)$	-3	$(-3, \infty)$
$f(x)$	$\smile$	DNE	$\smile$
$f''(x)$	$+$	DNE	$+$

Inflection Points

The inflection point(s) is(are):

Graph



7. $f(x) = \frac{1}{16}\frac{(x-1)^2}{\sqrt[4]{(x+3)^3}}$

Domain
The domain is $D_f = (-3, \infty)$.

Symmetry
$f(-x) = \frac{1}{16}\frac{(-x-1)^2}{\sqrt[4]{(-x+3)^3}}$

$f(-x) \neq -f(x) \qquad f(-x) \neq f(x)$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros
The zero(s) of the function $f(x)$ is(are):

$x_1 = 1$

Sign Chart for $f(x)$

x	$(-\infty,-3)$	-3	$(-3,1)$	1	$(1,\infty)$
$f(x)$	DNE	DNE	$+$	0	$+$

y-intercept
$y - int = f(0) = \frac{1}{16}\frac{(0-1)^2}{\sqrt[4]{(0+3)^3}} = 0.027$

Asymptotes
The function $f(x)$ has a vertical asymptote at $x = -3$.

Critical Numbers
$f'(x) = \frac{d}{dx}\frac{1}{16}\frac{(x-1)^2}{\sqrt[4]{(x+3)^3}} = \frac{d}{dx}\frac{1}{16}(x-1)^2(x+3)^{\frac{-3}{4}} = \frac{1}{64}\frac{x-1}{\sqrt[4]{(x+3)^7}}[5x+27]$

Critical numbers are the solutions of the equation $f'(x) = 0$ or

$\frac{1}{64}\frac{x-1}{\sqrt[4]{(x+3)^7}}[5x+27] = 0$

$x_3 = \frac{-27}{5} = -5.400$

The critical number(s) is(are): $x = -3 \qquad x = 1$

Sign Chart for First Derivative $f'(x)$

x	$(-\infty,-3)$	-3	$(-3,1)$	1	$(1,\infty)$
$f(x)$	DNE	DNE	$\searrow$	0	$\nearrow$
$f'(x)$	DNE	DNE	$-$	0	$+$

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(1,\infty)$

The function $f(x)$ is decreasing over $(-3,1)$

Maximum and Minimum Points

The function $f(x)$ has a minimum point at $(1, 0.000)$

Concavity Intervals
The second derivative of the function $f(x)$ is given by:

$$f''(x) = \frac{\mathrm{d}}{\mathrm{d}x} \frac{1}{64} \frac{x-1}{\sqrt[4]{(x+3)^7}}[5x+27] = \frac{1}{256} \frac{1}{\sqrt[4]{(x+3)^{11}}}(5x^2+54x+453)$$

The second derivative $f''(x)$ is zero when $f''(x) = 0$ or: $\frac{1}{256} \frac{1}{\sqrt[4]{(x+3)^{11}}}(5x^2+54x+453) = 0$

The second derivative $f''(x)$ is zero at:

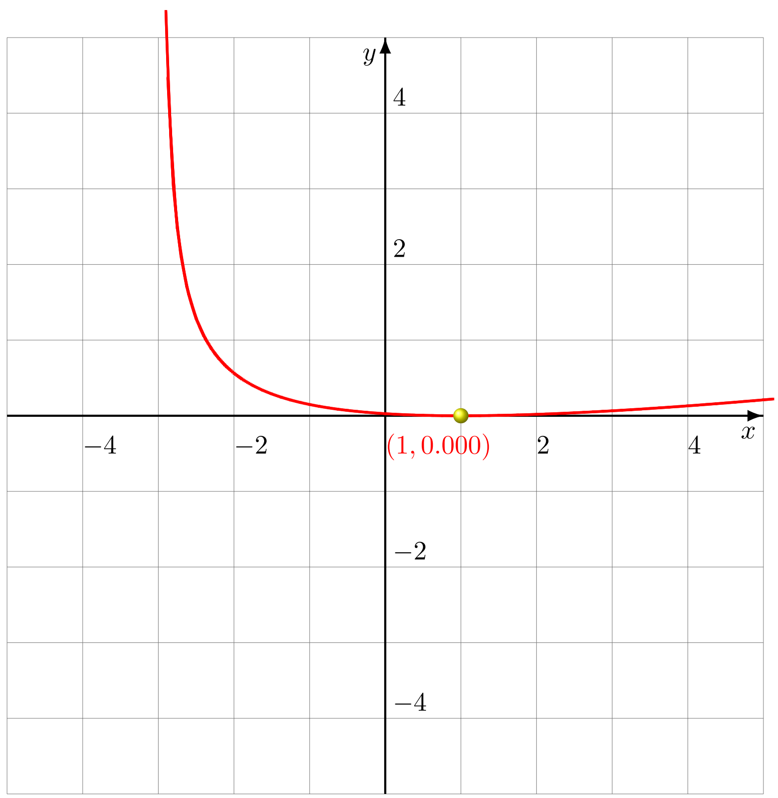
The second derivative $f''(x)$ does not exist at: $x_2 = -3$

Sign Chart for the Second Derivative $f''(x)$

x	$(-\infty, -3)$	-3	$(-3, \infty)$
$f(x)$	DNE	DNE	$\smile$
$f''(x)$	DNE	DNE	$+$

Inflection Points
The inflection point(s) is(are):

Graph



8. $f(x) = \frac{1}{9}(x-3)^2\sqrt[4]{x}$

Domain
The domain is $D_f = [0, \infty)$.

Symmetry
$f(-x) = \frac{1}{9}(-x-3)^2\sqrt[4]{-x}$

$f(-x) \neq -f(x) \qquad f(-x) \neq f(x)$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros
The zero(s) of the function $f(x)$ is(are):

$x_1 = 3 \qquad x_2 = 0$

Sign Chart for $f(x)$

x	$(-\infty, 0)$	0	$(0, 3)$	3	$(3, \infty)$
$f(x)$	DNE	0	$+$	0	$+$

y-intercept
$y - int = f(0) = \frac{1}{9}(0-3)^2\sqrt[4]{0} = 0.000$

Asymptotes
The function $f(x)$ does not have any kind of asymptotes.

Critical Numbers
$f'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{9}(x-3)^2\sqrt[4]{x} = \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{9}(x-3)^2x^{\frac{1}{4}} = \frac{1}{36}\frac{x-3}{\sqrt[4]{x^3}}[9x + -3]$

Critical numbers are the solutions of the equation $f'(x) = 0$ or

$\frac{1}{36}\frac{x-3}{\sqrt[4]{x^3}}[9x + -3] = 0$

$x_3 = \frac{3}{9} = 0.333$

The critical number(s) is(are): $x = 0 \qquad x = 0.333 \qquad x = 3$

Sign Chart for First Derivative $f'(x)$

x	$(-\infty, 0)$	0	$(0, 0.333)$	0.333	$(0.333, 3)$	3	$(3, \infty)$
$f(x)$	DNE	0	$\nearrow$	0.600	$\searrow$	0	$\nearrow$
$f'(x)$	DNE	DNE	$+$	0	$-$	0	$+$

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(0, 0.333) \qquad (3, \infty)$

The function $f(x)$ is decreasing over $(0.333, 3)$

Maximum and Minimum Points

The function $f(x)$ has a maximum point at $(0.333, 0.600)$

The function $f(x)$ has a minimum point at $(3, 0.000)$

Concavity Intervals

The second derivative of the function $f(x)$ is given by:

$$f''(x) = \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{36}\frac{x-3}{\sqrt[4]{x^3}}[9x+-3] = \frac{1}{144}\frac{1}{\sqrt[4]{x^7}}(45x^2+-30x+-27)$$

The second derivative $f''(x)$ is zero when $f''(x) = 0$ or: $\frac{1}{144}\frac{1}{\sqrt[4]{x^7}}(45x^2+-30x+-27) = 0$

The second derivative $f''(x)$ is zero at: $x_5 = 1.177$

The second derivative $f''(x)$ does not exist at: $x_2 = 0$

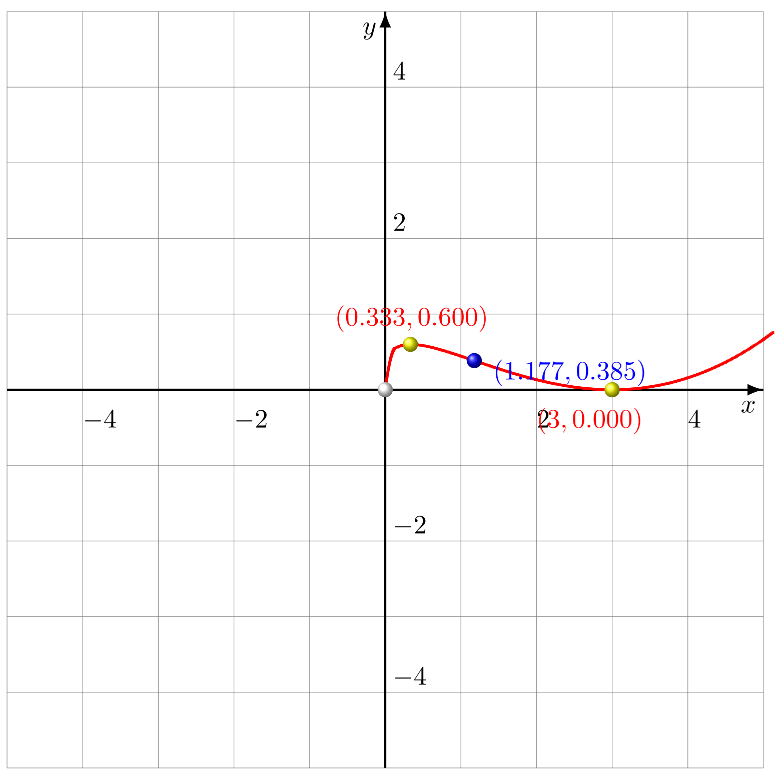
Sign Chart for the Second Derivative $f''(x)$

x	$(-\infty, 0)$	0	$(0, 1.177)$	1.177	$(1.177, \infty)$
$f(x)$	DNE	0	$\frown$	0.385	$\smile$
$f''(x)$	DNE	DNE	$-$	0	$+$

Inflection Points

The inflection point(s) is(are): (1.177,0.385)

Graph



9. $f(x) = \frac{1}{8}(x-1)^3 \sqrt[5]{(x+1)^3}$

Domain
The domain is $D_f = \mathbb{R}$.

Symmetry
$f(-x) = \frac{1}{8}(-x-1)^3 \sqrt[5]{(-x+1)^3}$

$f(-x) \neq -f(x) \qquad f(-x) \neq f(x)$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros
The zero(s) of the function $f(x)$ is(are):

$x_1 = 1 \qquad x_2 = -1$

Sign Chart for $f(x)$

x	$(-\infty,-1)$	-1	$(-1,1)$	1	$(1,\infty)$
$f(x)$	$+$	0	$-$	0	$+$

y-intercept
$y - int = f(0) = \frac{1}{8}(0-1)^3 \sqrt[5]{(0+1)^3} = -0.125$

Asymptotes
The function $f(x)$ does not have any kind of asymptotes.

Critical Numbers
$f'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{8}(x-1)^3 \sqrt[5]{(x+1)^3} = \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{8}(x-1)^3(x+1)^{\frac{3}{5}} = \frac{1}{40}\frac{(x-1)^2}{\sqrt[5]{(x+1)^2}}[18x+12]$

Critical numbers are the solutions of the equation $f'(x) = 0$ or

$\frac{1}{40}\frac{(x-1)^2}{\sqrt[5]{(x+1)^2}}[18x+12] = 0$

$x_3 = \frac{-12}{18} = -0.667$

The critical number(s) is(are): $x = -1$ $x = -0.667$ $x = 1$ px3= 1

Sign Chart for First Derivative $f'(x)$

x	$(-\infty,-1)$	-1	$(-1,-0.667)$	-0.667	$(-0.667,1)$	1	$(1,\infty)$
$f(x)$	$\searrow$	0	$\searrow$	-0.299	$\nearrow$	0	$\nearrow$
$f'(x)$	$-$	DNE	$-$	0	$+$	0	$+$

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(-0.667, 1)$ $(1,\infty)$

The function $f(x)$ is decreasing over $(-\infty,-1)$ $(-1,-0.667)$

Maximum and Minimum Points

The function $f(x)$ has a minimum point at $(-0.667, -0.299)$

Concavity Intervals
The second derivative of the function $f(x)$ is given by:

$$f''(x) = \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{40}\frac{(x-1)^2}{\sqrt[5]{(x+1)^2}}[18x+12] = \frac{1}{200}\frac{x-1}{\sqrt[5]{(x+1)^7}}(234x^2+312x+54)$$

The second derivative $f''(x)$ is zero when $f''(x) = 0$ or: $\frac{1}{200}\frac{x-1}{\sqrt[5]{(x+1)^7}}(234x^2+312x+54) = 0$

The second derivative $f''(x)$ is zero at: $x_4 = -1.129$ $x_5 = -0.204$ $x_1 = 1.000$

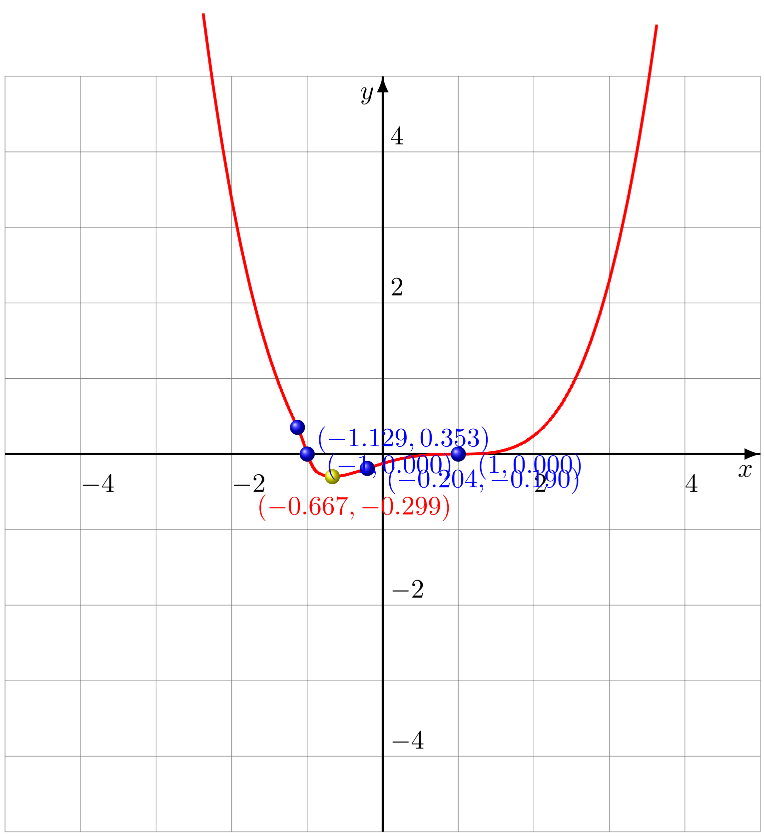
The second derivative $f''(x)$ does not exist at: $x_2 = -1$

Sign Chart for the Second Derivative $f''(x)$

x	$(-\infty, -1.129)$	-1.129	$(-1.129, -1)$	-1	$(-1, -0.204)$	-0.204	$(-0.204, 1)$	1	$(1, \infty)$
$f(x)$	$\smile$	0.353	$\frown$	0	$\smile$	-0.190	$\frown$	0	$\smile$
$f''(x)$	$+$	0	$-$	DNE	$+$	0	$-$	0	$+$

Inflection Points
The inflection point(s) is(are): (-1.129,0.353) (-1,0.000) (-0.204,-0.190) (1,0.000)

Graph



10. $f(x) = \frac{1}{1}x^3\sqrt[4]{(x+1)^3}$

Domain

The domain is $D_f = [-1, \infty)$.

Symmetry

$f(-x) = \frac{1}{1} - x^3\sqrt[4]{(-x+1)^3}$

$f(-x) \neq -f(x) \qquad f(-x) \neq f(x)$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros

The zero(s) of the function $f(x)$ is(are):

$x_1 = 0 \qquad x_2 = -1$

Sign Chart for $f(x)$

x	$(-\infty, -1)$	-1	$(-1, 0)$	0	$(0, \infty)$
$f(x)$	DNE	0	$-$	0	$+$

y-intercept

$y - int = f(0) = \frac{1}{1}0^3\sqrt[4]{(0+1)^3} = 0.000$

Asymptotes

The function $f(x)$ does not have any kind of asymptotes.

Critical Numbers

$f'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{1}x^3\sqrt[4]{(x+1)^3} = \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{1}x^3(x+1)^{\frac{3}{4}} = \frac{1}{4}\frac{x^2}{\sqrt[4]{x+1}}[15x+12]$

Critical numbers are the solutions of the equation $f'(x) = 0$ or

$\frac{1}{4}\frac{x^2}{\sqrt[4]{x+1}}[15x+12] = 0$

$x_3 = \frac{-12}{15} = -0.800$

The critical number(s) is(are): $x = -1 \qquad x = -0.800 \qquad x = 0$

Sign Chart for First Derivative $f'(x)$

x	$(-\infty, -1)$	-1	$(-1, -0.800)$	-0.800	$(-0.800, 0)$	0	$(0, \infty)$
$f(x)$	DNE	0	$\searrow$	-0.153	$\nearrow$	0	$\nearrow$
$f'(x)$	DNE	DNE	$-$	0	$+$	0	$+$

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(-0.800, 0) \qquad (0, \infty)$

The function $f(x)$ is decreasing over $(-1, -0.800)$

Maximum and Minimum Points

The function $f(x)$ has a minimum point at $(-0.800, -0.153)$

Concavity Intervals
The second derivative of the function $f(x)$ is given by:

$$f''(x) = \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{4}\frac{x^2}{\sqrt[4]{x+1}}[15x+12] = \frac{1}{16}\frac{x}{\sqrt[4]{(x+1)^5}}(165x^2+264x+96)$$

The second derivative $f''(x)$ is zero when $f''(x) = 0$ or: $\frac{1}{16}\frac{x}{\sqrt[4]{(x+1)^5}}(165x^2+264x+96) = 0$

The second derivative $f''(x)$ is zero at: $x_5 = -0.559$ $x_1 = 0.000$

The second derivative $f''(x)$ does not exist at: $x_2 = -1$

Sign Chart for the Second Derivative $f''(x)$

x	$(-\infty, -1)$	-1	$(-1, -0.559)$	-0.559	$(-0.559, 0)$	0	$(0, \infty)$
$f(x)$	DNE	0	$\smile$	-0.094	$\frown$	0	$\smile$
$f''(x)$	DNE	DNE	$+$	0	$-$	0	$+$

Inflection Points
The inflection point(s) is(are): (-0.559,-0.095) (0,0.000)

Graph