1. Consider the following polynomial function: $f(x) = \dfrac{(x+2)^2(x-1)^4}{4}$

Sketch the grapf of the function $f(x)$.

2. Consider the following polynomial function: $f(x) = \dfrac{(x+1)^3(x-3)}{-3}$

Sketch the grapf of the function $f(x)$.

3. Consider the following polynomial function: $f(x) = \dfrac{(x+3)^3(x-2)^2}{-108}$

Sketch the grapf of the function $f(x)$.

4. Consider the following polynomial function: $f(x) = \dfrac{(x+1)^3(x-2)^2}{-4}$

Sketch the grapf of the function $f(x)$.

5. Consider the following polynomial function: $f(x) = \dfrac{(x+1)(x-2)^3}{-8}$

Sketch the grapf of the function $f(x)$.

6. Consider the following polynomial function: $f(x) = \dfrac{(x+2)^3(x-1)}{-8}$

Sketch the grapf of the function $f(x)$.

7. Consider the following polynomial function: $f(x) = \dfrac{(x+3)^2(x-2)^4}{144}$

Sketch the grapf of the function $f(x)$.

8. Consider the following polynomial function: $f(x) = \dfrac{(x+1)^2(x-2)^2}{4}$

Sketch the grapf of the function $f(x)$.

9. Consider the following polynomial function: $f(x) = \dfrac{(x+3)(x-1)}{-3}$

Sketch the grapf of the function $f(x)$.

10. Consider the following polynomial function: $f(x) = \dfrac{(x+2)^2(x-2)^4}{64}$

Sketch the grapf of the function $f(x)$.

Solutions:

1. $f(x) = \dfrac{(x+2)^2(x-1)^4}{4}$

Domain

The function $f(x)$ is a polynomial function. Therefore the domain is $D_f = \mathbb{R}$.

Symmetry

$f(-x) = \dfrac{(-x+2)^2(-x-1)^4}{4}$

$f(-x) \neq -f(x)$ $f(-x) \neq f(x)$ Therefore the function $f(x)$ is neither even nor odd function.

Zeros

The zeros of the function $f(x)$ are: $x_1 = -2$ and $x_2 = 1$

y-intercept

$y - int = f(0) = \dfrac{(0+2)^2(0-1)^4}{4} = 1.000$

Asymptotes

The function $f(x)$ is a polynomial function of degree 3. Therefore the function does not have any kind of asymptotes.

Critical Numbers

$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{(x+2)^2(x-1)^4}{4} = \dfrac{(x+2)(x-1)^3}{4}[(2)(x-1)+(4)(x+2)]$

Critical numbers are the solutions of the equation $f'(x) = 0$ or

$\dfrac{(x+2)(x-1)^3}{4}[(2)(x-1)+(4)(x+2)] = 0$

So, the critical numbers are: $x_1 = -2$ $x_2 = 1$ $x_3 = \dfrac{(2)(1)+(4)(-2)}{2+4} = -1.000$

Sign Chart for the First Derivative $f'(x)$

x		-2		-1.000		1	
$f(x)$	$\searrow$	0	$\nearrow$	4.000	$\searrow$	0	$\nearrow$
$f'(x)$	$-$	0	$+$	0	$-$	0	$+$

Increasing and Decreasing Intervals

The function $f(x)$ is decreasing over $(-\infty, -2)$ and over $(-1.000, 1)$ and is increasing over $(-2, -1.000)$ and over $(1, \infty)$.

Maximum and Minimum Points

The function $f(x)$ has a maximum point at $P_3(-1.000, 4.000)$ and two minimum points at $P_1(-2, 0)$ and at $P_2(1, 0)$.

Concavity Intervals

The second derivative of the function $f(x)$ is given by:

$f''(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{(x+2)(x-1)^3}{4}[(2)(x-1)+(4)(x--2)]$

$f''(x) = \dfrac{(x-1)^2}{4}[30x^2+60x+18]$

The second derivative $f''(x)$ is zero when $f''(x) = 0$ or:

$\frac{(x-1)^2}{4}[30x^2 + 60x + 18] = 0$

So, the second derivative $f''(x)$ is zero at:

$x_2 = 1$ $x_4 = -1.632$ $x_5 = -0.368$

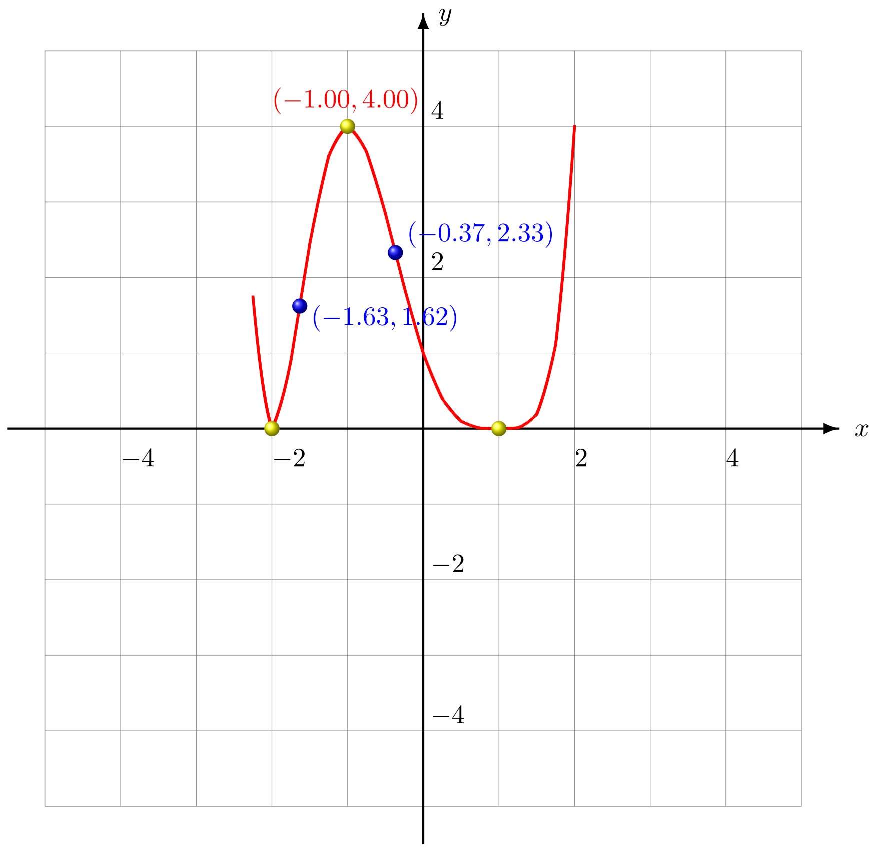
Sign Chart for the Second Derivative $f''(x)$

x		-1.632		-0.368	
$f(x)$	$\smile$	1.622	$\frown$	2.330	$\smile$
$f''(x)$	$+$	0	$-$	0	$+$

Inflection Points
The inflection point(s) is(are):

$P_4 = (-1.632, 1.622)$ $P_5 = (-0.368, 2.330)$

Graph



2. $f(x) = \dfrac{(x+1)^3(x-3)}{-3}$

Domain

The function $f(x)$ is a polynomial function. Therefore the domain is $D_f = \mathbb{R}$.

Symmetry

$f(-x) = \dfrac{(-x+1)^3(-x-3)}{-3}$

$f(-x) \neq -f(x)$ $\quad f(-x) \neq f(x)$ $\quad$ Therefore the function $f(x)$ is neither even nor odd function.

Zeros

The zeros of the function $f(x)$ are: $x_1 = -1$ and $x_2 = 3$

y-intercept

$y - int = f(0) = \dfrac{(0+1)^3(0-3)}{-3} = 1.000$

Asymptotes

The function $f(x)$ is a polynomial function of degree 3. Therefore the function does not have any kind of asymptotes.

Critical Numbers

$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{(x+1)^3(x-3)}{-3} = \dfrac{(x+1)^2}{-3}[(3)(x-3)+(1)(x+1)]$

Critical numbers are the solutions of the equation $f'(x) = 0$ or

$\dfrac{(x+1)^2}{-3}[(3)(x-3)+(1)(x+1)] = 0$

So, the critical numbers are: $x_1 = -1$ $\quad x_3 = \dfrac{(3)(3)+(1)(-1)}{3+1} = 2.000$

Sign Chart for the First Derivative $f'(x)$

x		-1		2.000	
$f(x)$	$\nearrow$	0	$\nearrow$	9.000	$\searrow$
$f'(x)$	$+$	0	$+$	0	$-$

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(-\infty, 2.000$ and is decreasing over $(2.000, \infty)$.

Maximum and Minimum Points

The function $f(x)$ has a maximum point at $P_3(2.000, 9.000)$.

Concavity Intervals

The second derivative of the function $f(x)$ is given by:

$f''(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{(x+1)^2}{-3}[(3)(x-3)+(1)(x--1)]$

$f''(x) = \dfrac{(x--1)^1}{-3}[12x+-12]$

The second derivative $f''(x)$ is zero when $f''(x) = 0$ or:

$\dfrac{(x--1)^1}{-3}[12x+-12] = 0$

So, the second derivative $f''(x)$ is zero at:

$x_1 = -1 \qquad x_4 = 1.000$

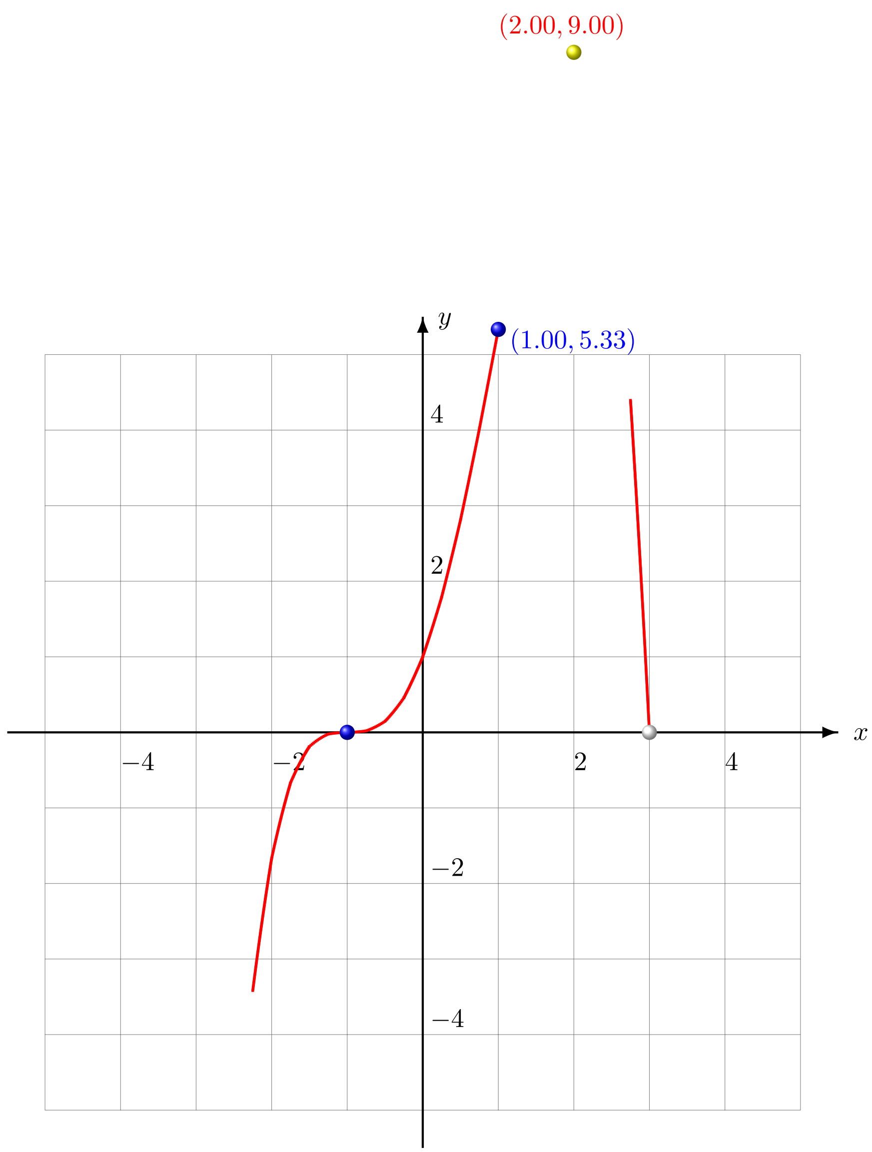
Sign Chart for the Second Derivative $f''(x)$

x		-1		1.000	
$f(x)$	$\frown$	0	$\smile$	5.333	$\frown$
$f''(x)$	$-$	0	$+$	0	$-$

Inflection Points
The inflection point(s) is(are):

$P_1 = (-1, 0) \qquad P_4 = (1.000, 5.333)$

Graph



3. $f(x) = \dfrac{(x+3)^3(x-2)^2}{-108}$

Domain
The function $f(x)$ is a polynomial function. Therefore the domain is $D_f = \mathbb{R}$.

Symmetry
$f(-x) = \dfrac{(-x+3)^3(-x-2)^2}{-108}$

$f(-x) \neq -f(x)$ $\quad f(-x) \neq f(x)$ $\quad$ Therefore the function $f(x)$ is neither even nor odd function.

Zeros
The zeros of the function $f(x)$ are: $x_1 = -3$ and $x_2 = 2$

y-intercept
$y - int = f(0) = \dfrac{(0+3)^3(0-2)^2}{-108} = -1.000$

Asymptotes
The function $f(x)$ is a polynomial function of degree 3. Therefore the function does not have any kind of asymptotes.

Critical Numbers
$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{(x+3)^3(x-2)^2}{-108} = \dfrac{(x+3)^2(x-2)}{-108}[(3)(x-2)+(2)(x+3)]$

Critical numbers are the solutions of the equation $f'(x) = 0$ or

$\dfrac{(x+3)^2(x-2)}{-108}[(3)(x-2)+(2)(x+3)] = 0$

So, the critical numbers are: $x_1 = -3$ $\quad x_2 = 2$ $\quad x_3 = \dfrac{(3)(2)+(2)(-3)}{3+2} = 0.000$

Sign Chart for the First Derivative $f'(x)$

x		-3		0.000		2	
$f(x)$	↘	0	↘	-1.000	↗	0	↘
$f'(x)$	$-$	0	$-$	0	$+$	0	$-$

Increasing and Decreasing Intervals

The function $f(x)$ is decreasing over $(-\infty, 0.000)$ and over $(2, \infty)$ and is increasing over $(0.000, 2)$.

Maximum and Minimum Points

The function $f(x)$ has a minimum point at $P_3(0.000, -1.000)$ and a maximum point at $P_2(2, 0)$.

Concavity Intervals
The second derivative of the function $f(x)$ is given by:

$f''(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{(x+3)^2(x-2)}{-108}[(3)(x-2)+(2)(x--3)]$

$f''(x) = \dfrac{(x+3)}{-108}[20x^2 + 0x + -30]$

The second derivative $f''(x)$ is zero when $f''(x) = 0$ or:

$\dfrac{(x+3)}{-108}[20x^2 + 0x + -30] = 0$

So, the second derivative $f''(x)$ is zero at:

$x_1 = -3$ $x_4 = -1.225$ $x_5 = 1.225$

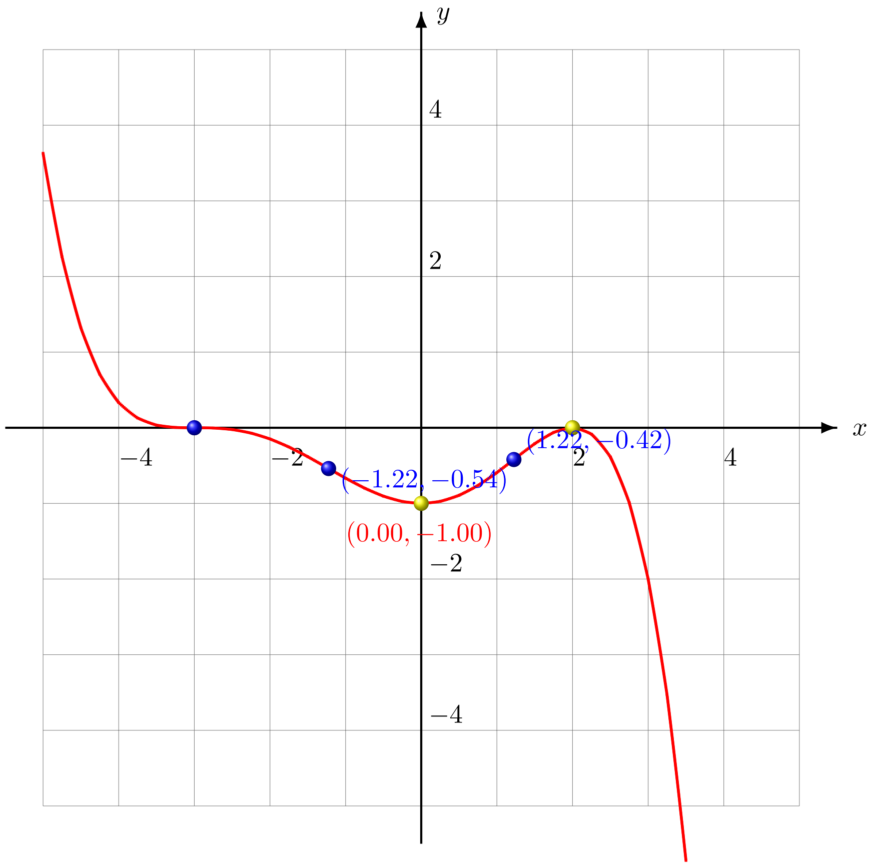
Sign Chart for the Second Derivative $f''(x)$

x		-3		-1.225		1.225	
$f(x)$	$\smile$	0	$\frown$	-0.539	$\smile$	-0.420	$\frown$
$f''(x)$	$+$	0	$-$	0	$+$	0	$-$

Inflection Points
The inflection point(s) is(are):

$P_1 = (-3, 0)$ $P_4 = (-1.225, -0.539)$ $P_5 = (1.225, -0.420)$

Graph



4. $f(x) = \dfrac{(x+1)^3(x-2)^2}{-4}$

Domain
The function $f(x)$ is a polynomial function. Therefore the domain is $D_f = \mathbb{R}$.

Symmetry
$f(-x) = \dfrac{(-x+1)^3(-x-2)^2}{-4}$

$f(-x) \neq -f(x)$ $f(-x) \neq f(x)$ Therefore the function $f(x)$ is neither even nor odd function.

Zeros
The zeros of the function $f(x)$ are: $x_1 = -1$ and $x_2 = 2$

y-intercept
$y - int = f(0) = \dfrac{(0+1)^3(0-2)^2}{-4} = -1.000$

Asymptotes
The function $f(x)$ is a polynomial function of degree 3. Therefore the function does not have any kind of asymptotes.

Critical Numbers
$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{(x+1)^3(x-2)^2}{-4} = \dfrac{(x+1)^2(x-2)}{-4}[(3)(x-2)+(2)(x+1)]$

Critical numbers are the solutions of the equation $f'(x) = 0$ or

$\dfrac{(x+1)^2(x-2)}{-4}[(3)(x-2)+(2)(x+1)] = 0$

So, the critical numbers are: $x_1 = -1$ $x_2 = 2$ $x_3 = \dfrac{(3)(2)+(2)(-1)}{3+2} = 0.800$

Sign Chart for the First Derivative $f'(x)$

x		-1		0.800		2	
$f(x)$	$\searrow$	0	$\searrow$	-2.100	$\nearrow$	0	$\searrow$
$f'(x)$	$-$	0	$-$	0	$+$	0	$-$

Increasing and Decreasing Intervals

The function $f(x)$ is decreasing over $(-\infty, 0.800)$ and over $(2, \infty)$ and is increasing over $(0.800, 2)$.

Maximum and Minimum Points

The function $f(x)$ has a minimum point at $P_3(0.800, -2.100)$ and a maximum point at $P_2(2, 0)$.

Concavity Intervals
The second derivative of the function $f(x)$ is given by:

$f''(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{(x+1)^2(x-2)}{-4}[(3)(x-2)+(2)(x--1)]$

$f''(x) = \dfrac{(x+1)}{-4}[20x^2 + -32x + 2]$

The second derivative $f''(x)$ is zero when $f''(x) = 0$ or:

$\dfrac{(x+1)}{-4}[20x^2 + -32x + 2] = 0$

So, the second derivative $f''(x)$ is zero at:

$x_1 = -1$ $x_4 = 0.065$ $x_5 = 1.535$

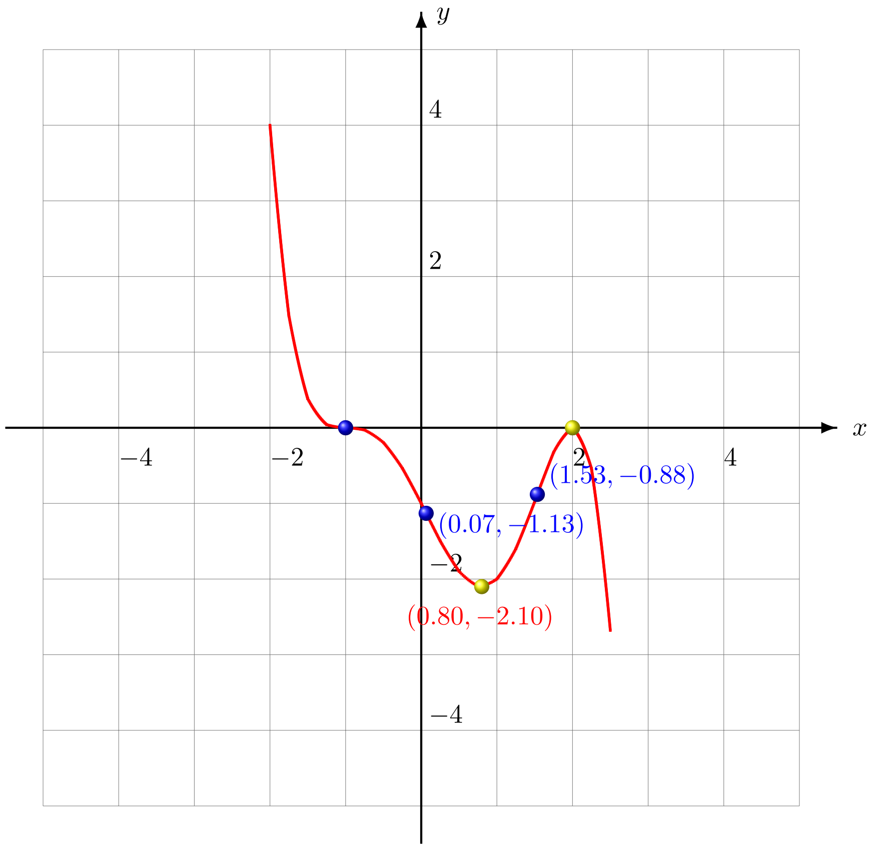
Sign Chart for the Second Derivative $f''(x)$

x		-1		0.065		1.535	
$f(x)$	$\smile$	0	$\frown$	-1.131	$\smile$	-0.881	$\frown$
$f''(x)$	$+$	0	$-$	0	$+$	0	$-$

Inflection Points
The inflection point(s) is(are):

$P_1 = (-1, 0)$ $P_4 = (0.065, -1.131)$ $P_5 = (1.535, -0.881)$

Graph



5. $f(x) = \dfrac{(x+1)(x-2)^3}{-8}$

Domain
The function $f(x)$ is a polynomial function. Therefore the domain is $D_f = \mathbb{R}$.

Symmetry
$f(-x) = \dfrac{(-x+1)(-x-2)^3}{-8}$

$f(-x) \neq -f(x) \qquad f(-x) \neq f(x)$ Therefore the function $f(x)$ is neither even nor odd function.

Zeros
The zeros of the function $f(x)$ are: $x_1 = -1$ and $x_2 = 2$

y-intercept
$y - int = f(0) = \dfrac{(0+1)(0-2)^3}{-8} = 1.000$

Asymptotes
The function $f(x)$ is a polynomial function of degree 3. Therefore the function does not have any kind of asymptotes.

Critical Numbers
$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{(x+1)(x-2)^3}{-8} = \dfrac{(x-2)^2}{-8}[(1)(x-2)+(3)(x+1)]$

Critical numbers are the solutions of the equation $f'(x) = 0$ or

$\dfrac{(x-2)^2}{-8}[(1)(x-2)+(3)(x+1)] = 0$

So, the critical numbers are: $x_2 = 2 \qquad x_3 = \dfrac{(1)(2)+(3)(-1)}{1+3} = -0.250$

Sign Chart for the First Derivative $f'(x)$

x		-0.250		2	
$f(x)$	$\nearrow$	1.068	$\searrow$	0	$\searrow$
$f'(x)$	+	0	−	0	−

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(-\infty, -0.250)$ and is decreasing over (-0.250∞).

Maximum and Minimum Points

The function $f(x)$ has a maximum point at $P_3(-0.250, 1.068)$.

Concavity Intervals
The second derivative of the function $f(x)$ is given by:

$f''(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{(x-2)^2}{-8}[(1)(x-2)+(3)(x--1)]$

$f''(x) = \dfrac{(x-2)^1}{-8}[12x+-6]$

The second derivative $f''(x)$ is zero when $f''(x) = 0$ or:

$\dfrac{(x-2)^1}{-8}[12x+-6] = 0$

So, the second derivative $f''(x)$ is zero at:

$x_2 = 2$ $\qquad x_4 = 0.500$

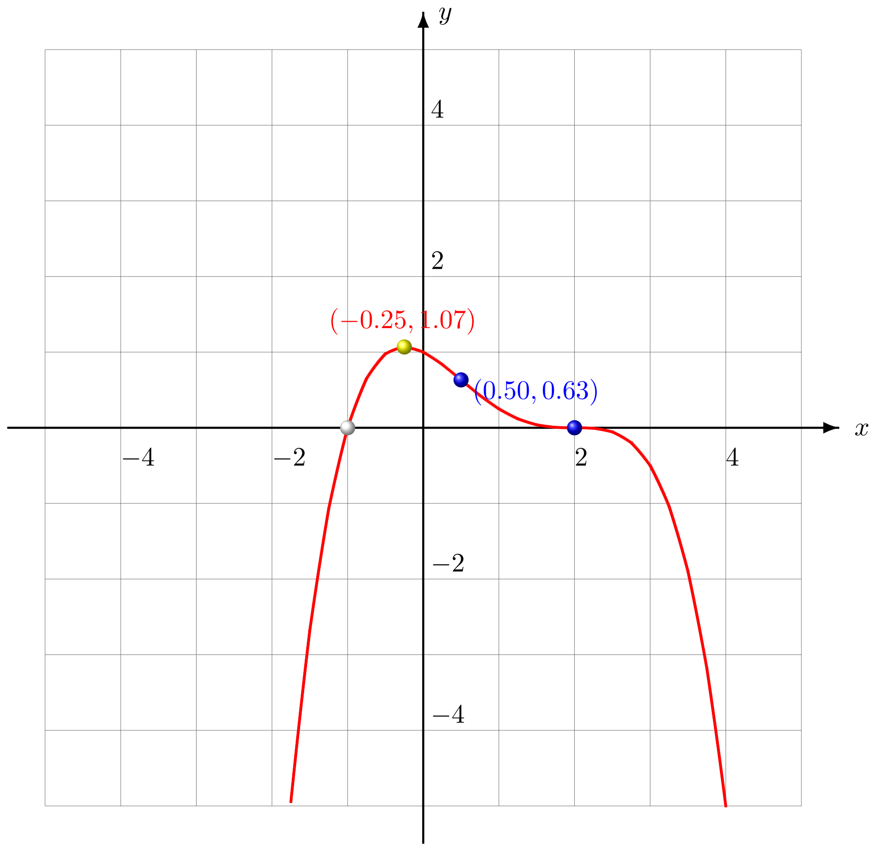
Sign Chart for the Second Derivative $f''(x)$

x		0.500		2	
$f(x)$	$\frown$	0.633	$\smile$	0	$\frown$
$f''(x)$	$-$	0	$+$	0	$-$

Inflection Points
The inflection point(s) is(are):

$P_2 = (2, 0)$ $\qquad P_4 = (0.500, 0.633)$

Graph



6. $f(x) = \dfrac{(x+2)^3(x-1)}{-8}$

Domain
The function $f(x)$ is a polynomial function. Therefore the domain is $D_f = \mathbb{R}$.

Symmetry
$f(-x) = \dfrac{(-x+2)^3(-x-1)}{-8}$

$f(-x) \neq -f(x)$ $\quad f(-x) \neq f(x)$ $\quad$ Therefore the function $f(x)$ is neither even nor odd function.

Zeros
The zeros of the function $f(x)$ are: $x_1 = -2$ and $x_2 = 1$

y-intercept
$y - int = f(0) = \dfrac{(0+2)^3(0-1)}{-8} = 1.000$

Asymptotes
The function $f(x)$ is a polynomial function of degree 3. Therefore the function does not have any kind of asymptotes.

Critical Numbers
$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{(x+2)^3(x-1)}{-8} = \dfrac{(x+2)^2}{-8}[(3)(x-1)+(1)(x+2)]$

Critical numbers are the solutions of the equation $f'(x) = 0$ or

$\dfrac{(x+2)^2}{-8}[(3)(x-1)+(1)(x+2)] = 0$

So, the critical numbers are: $x_1 = -2$ $\quad x_3 = \dfrac{(3)(1)+(1)(-2)}{3+1} = 0.250$

Sign Chart for the First Derivative $f'(x)$

x		-2		0.250	
$f(x)$	$\nearrow$	0	$\nearrow$	1.068	$\searrow$
$f'(x)$	+	0	+	0	−

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(-\infty, 0.250$ and is decreasing over $(0.250, \infty)$.

Maximum and Minimum Points

The function $f(x)$ has a maximum point at $P_3(0.250, 1.068)$.

Concavity Intervals
The second derivative of the function $f(x)$ is given by:

$f''(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{(x+2)^2}{-8}[(3)(x-1)+(1)(x--2)]$

$f''(x) = \dfrac{(x--2)^1}{-8}[12x+6]$

The second derivative $f''(x)$ is zero when $f''(x) = 0$ or:

$\dfrac{(x--2)^1}{-8}[12x+6] = 0$

So, the second derivative $f''(x)$ is zero at:

$x_1 = -2 \qquad x_4 = -0.500$

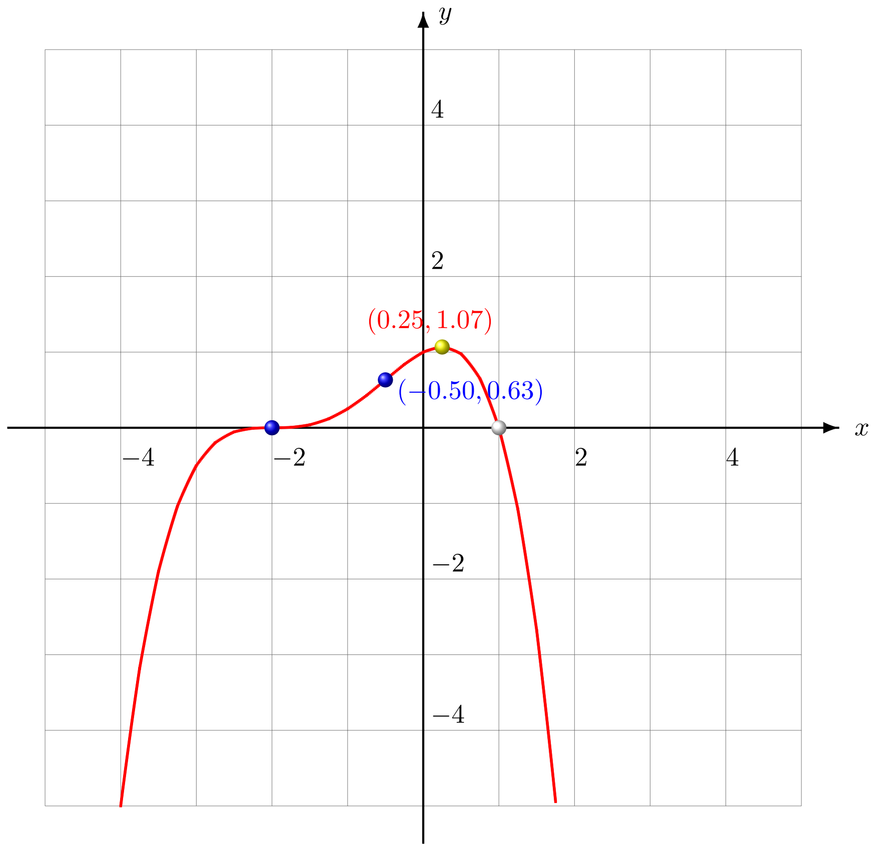
Sign Chart for the Second Derivative $f''(x)$

x		-2		-0.500	
$f(x)$	$\frown$	0	$\smile$	0.633	$\frown$
$f''(x)$	$-$	0	$+$	0	$-$

Inflection Points
The inflection point(s) is(are):

$P_1 = (-2, 0) \qquad P_4 = (-0.500, 0.633)$

Graph



7. $f(x) = \dfrac{(x+3)^2(x-2)^4}{144}$

Domain

The function $f(x)$ is a polynomial function. Therefore the domain is $D_f = \mathbb{R}$.

Symmetry

$f(-x) = \dfrac{(-x+3)^2(-x-2)^4}{144}$

$f(-x) \neq -f(x) \qquad f(-x) \neq f(x) \qquad$ Therefore the function $f(x)$ is neither even nor odd function.

Zeros

The zeros of the function $f(x)$ are: $x_1 = -3 \qquad$ and $\qquad x_2 = 2$

y-intercept

$y - int = f(0) = \dfrac{(0+3)^2(0-2)^4}{144} = 1.000$

Asymptotes

The function $f(x)$ is a polynomial function of degree 3. Therefore the function does not have any kind of asymptotes.

Critical Numbers

$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{(x+3)^2(x-2)^4}{144} = \dfrac{(x+3)(x-2)^3}{144}[(2)(x-2)+(4)(x+3)]$

Critical numbers are the solutions of the equation $f'(x) = 0 \qquad$ or

$\dfrac{(x+3)(x-2)^3}{144}[(2)(x-2)+(4)(x+3)] = 0$

So, the critical numbers are: $x_1 = -3 \qquad x_2 = 2 \qquad x_3 = \dfrac{(2)(2)+(4)(-3)}{2+4} = -1.333$

Sign Chart for the First Derivative $f'(x)$

x		-3		-1.333		2	
$f(x)$	$\searrow$	0	$\nearrow$	2.381	$\searrow$	0	$\nearrow$
$f'(x)$	$-$	0	$+$	0	$-$	0	$+$

Increasing and Decreasing Intervals

The function $f(x)$ is decreasing over $(-\infty, -3)$ and over $(-1.333, 2)$ and is increasing over $(-3, -1.333)$ and over $(2, \infty)$.

Maximum and Minimum Points

The function $f(x)$ has a maximum point at $P_3(-1.333, 2.381)$ and two minimum points at $P_1(-3, 0)$ and at $P_2(2, 0)$.

Concavity Intervals

The second derivative of the function $f(x)$ is given by:

$f''(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{(x+3)(x-2)^3}{144}[(2)(x-2)+(4)(x--3)]$

$f''(x) = \dfrac{(x-2)^2}{144}[30x^2+80x+20]$

The second derivative $f''(x)$ is zero when $f''(x) = 0 \qquad$ or:

$$\frac{(x-2)^2}{144}[30x^2+80x+20]=0$$

So, the second derivative $f''(x)$ is zero at:

$x_2 = 2 \qquad x_4 = -2.387 \qquad x_5 = -0.279$

Sign Chart for the Second Derivative $f''(x)$

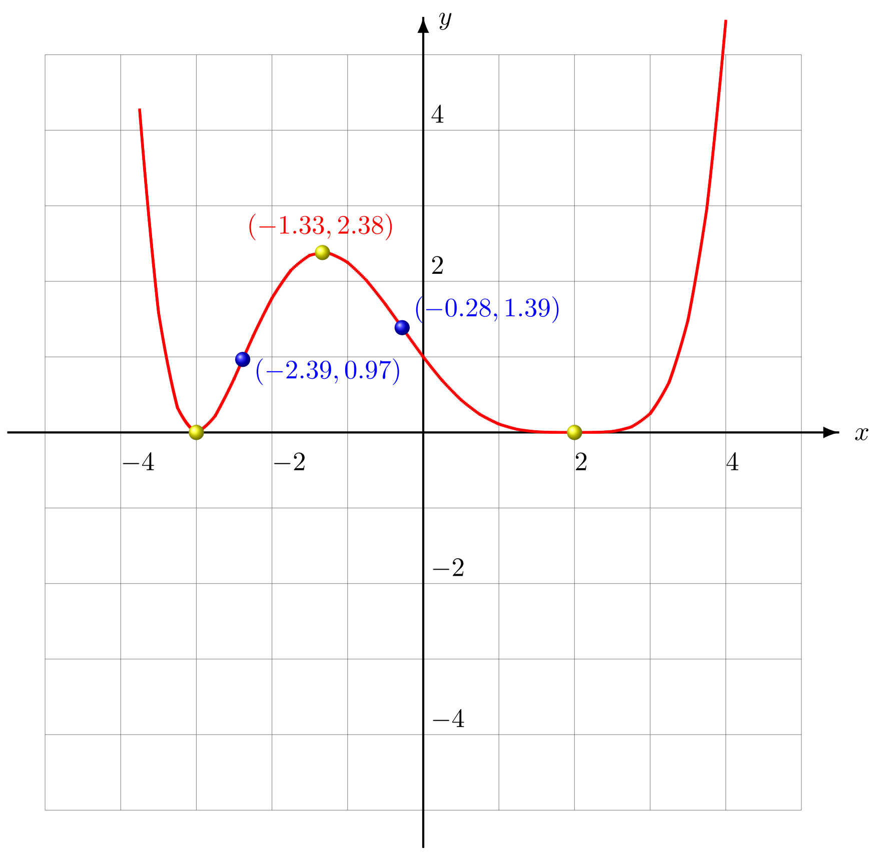
x		-2.387		-0.279	
$f(x)$	$\smile$	0.966	$\frown$	1.387	$\smile$
$f''(x)$	$+$	0	$-$	0	$+$

Inflection Points

The inflection point(s) is(are):

$P_4 = (-2.387, 0.966) \qquad P_5 = (-0.279, 1.387)$

Graph



8. $f(x) = \dfrac{(x+1)^2(x-2)^2}{4}$

Domain
The function $f(x)$ is a polynomial function. Therefore the domain is $D_f = \mathbb{R}$.

Symmetry
$f(-x) = \dfrac{(-x+1)^2(-x-2)^2}{4}$

$f(-x) \neq -f(x)$ $f(-x) \neq f(x)$ Therefore the function $f(x)$ is neither even nor odd function.

Zeros
The zeros of the function $f(x)$ are: $x_1 = -1$ and $x_2 = 2$

y-intercept
$y - int = f(0) = \dfrac{(0+1)^2(0-2)^2}{4} = 1.000$

Asymptotes
The function $f(x)$ is a polynomial function of degree 3. Therefore the function does not have any kind of asymptotes.

Critical Numbers
$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{(x+1)^2(x-2)^2}{4} = \dfrac{(x+1)(x-2)}{4}[(2)(x-2)+(2)(x+1)]$

Critical numbers are the solutions of the equation $f'(x) = 0$ or

$\dfrac{(x+1)(x-2)}{4}[(2)(x-2)+(2)(x+1)] = 0$

So, the critical numbers are: $x_1 = -1$ $x_2 = 2$ $x_3 = \dfrac{(2)(2)+(2)(-1)}{2+2} = 0.500$

Sign Chart for the First Derivative $f'(x)$

x		-1		0.500		2	
$f(x)$	$\searrow$	0	$\nearrow$	1.266	$\searrow$	0	$\nearrow$
$f'(x)$	$-$	0	$+$	0	$-$	0	$+$

Increasing and Decreasing Intervals

The function $f(x)$ is decreasing over $(-\infty, -1)$ and over $(0.500, 2)$ and is increasing over $(-1, 0.500)$ and over $(2, \infty)$.

Maximum and Minimum Points

The function $f(x)$ has a maximum point at $P_3(0.500, 1.266)$ and two minimum points at $P_1(-1, 0)$ and at $P_2(2, 0)$.

Concavity Intervals
The second derivative of the function $f(x)$ is given by:

$f''(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{(x+1)(x-2)}{4}[(2)(x-2)+(2)(x--1)]$

$f''(x) = \dfrac{1}{4}[12x^2 + -12x + -6]$

The second derivative $f''(x)$ is zero when $f''(x) = 0$ or:

$\frac{1}{4}[12x^2 + -12x + -6] = 0$

So, the second derivative $f''(x)$ is zero at:

$x_4 = -0.366 \qquad x_5 = 1.366$

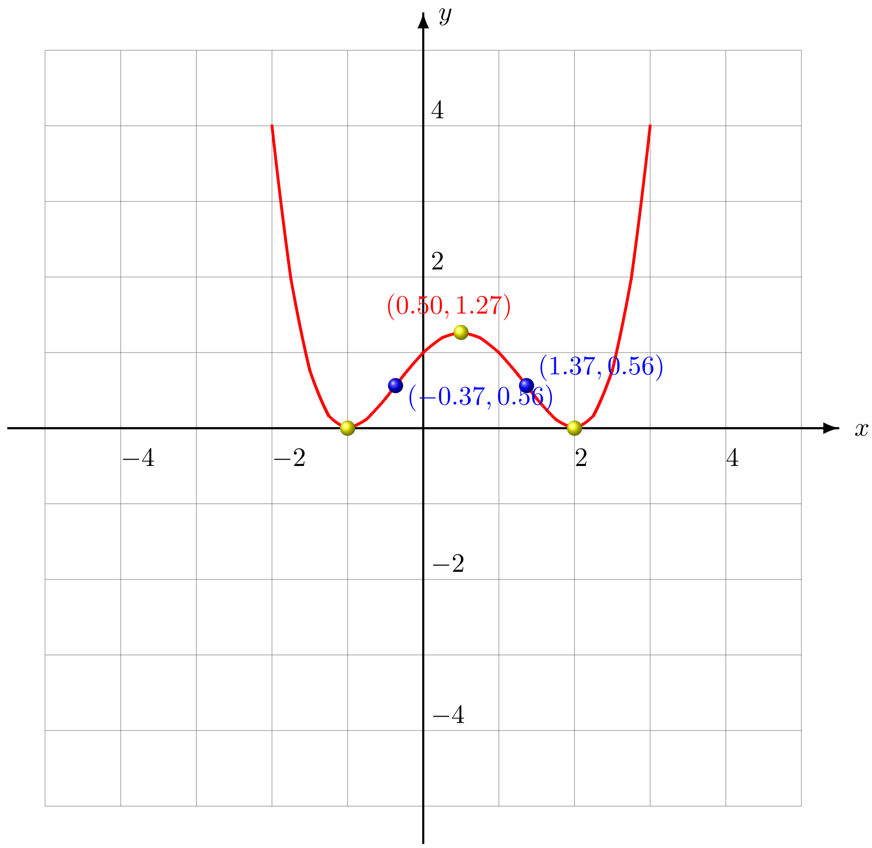
Sign Chart for the Second Derivative $f''(x)$

x		-0.366		1.366	
$f(x)$	$\smile$	0.563	$\frown$	0.563	$\smile$
$f''(x)$	$+$	0	$-$	0	$+$

Inflection Points
The inflection point(s) is(are):

$P_4 = (-0.366, 0.563) \qquad P_5 = (1.366, 0.563)$

Graph



9. $f(x) = \frac{(x+3)(x-1)}{-3}$

Domain
The function $f(x)$ is a polynomial function. Therefore the domain is $D_f = \mathbb{R}$.

Symmetry
$f(-x) = \frac{(-x+3)(-x-1)}{-3}$

$f(-x) \neq -f(x) \qquad f(-x) \neq f(x)$ Therefore the function $f(x)$ is neither even nor odd function.

Zeros
The zeros of the function $f(x)$ are: $x_1 = -3$ and $x_2 = 1$

y-intercept
$y - int = f(0) = \frac{(0+3)(0-1)}{-3} = 1.000$

Asymptotes
The function $f(x)$ is a polynomial function of degree 3. Therefore the function does not have any kind of asymptotes.

Critical Numbers
$f'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\frac{(x+3)(x-1)}{-3} = \frac{1}{-3}[(1)(x-1)+(1)(x+3)]$

Critical numbers are the solutions of the equation $f'(x) = 0$ or

$\frac{1}{-3}[(1)(x-1)+(1)(x+3)] = 0$

So, the critical numbers are: $x_3 = \frac{(1)(1)+(1)(-3)}{1+1} = -1.000$

Sign Chart for the First Derivative $f'(x)$

x		-1.000	
$f(x)$	$\nearrow$	1.333	$\searrow$
$f'(x)$	$+$	0	$-$

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(-\infty, -1.000$ and is decreasing over $(-1.000, \infty)$.

Maximum and Minimum Points

The function $f(x)$ has a maximum point at $P_3(-1.000, 1.333)$.

Concavity Intervals
The second derivative of the function $f(x)$ is given by:

$f''(x) = \frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{-3}[(1)(x-1)+(1)(x--3)]$

$f''(x) = 2$

The second derivative $f''(x)$ is zero when $f''(x) = 0$ or:

$2 = 0$

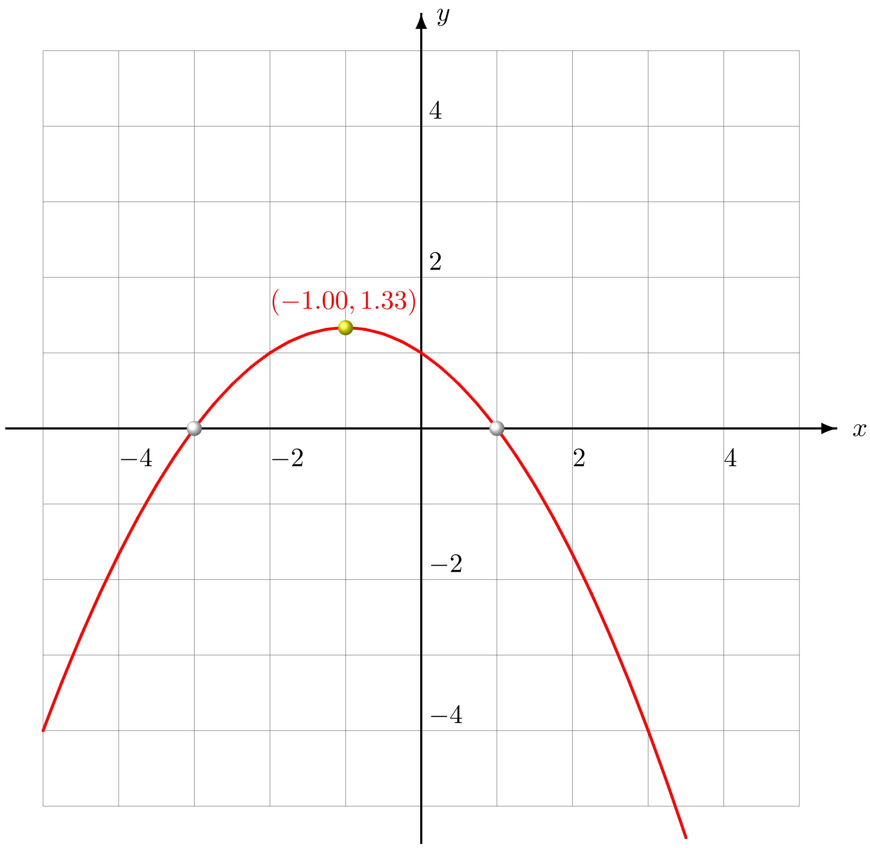
The second derivative $f''(x)$ cannot be zero.

Sign Chart for the Second Derivative $f''(x)$

x	
$f(x)$	$\frown$
$f''(x)$	$-$

Inflection Points
There is no inflection point.

Graph



10. $f(x) = \dfrac{(x+2)^2(x-2)^4}{64}$

Domain
The function $f(x)$ is a polynomial function. Therefore the domain is $D_f = \mathbb{R}$.

Symmetry
$f(-x) = \dfrac{(-x+2)^2(-x-2)^4}{64}$
$f(-x) \neq -f(x)$ $f(-x) \neq f(x)$ Therefore the function $f(x)$ is neither even nor odd function.

Zeros
The zeros of the function $f(x)$ are: $x_1 = -2$ and $x_2 = 2$

y-intercept
$y - int = f(0) = \dfrac{(0+2)^2(0-2)^4}{64} = 1.000$

Asymptotes
The function $f(x)$ is a polynomial function of degree 3. Therefore the function does not have any kind of asymptotes.

Critical Numbers
$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{(x+2)^2(x-2)^4}{64} = \dfrac{(x+2)(x-2)^3}{64}[(2)(x-2)+(4)(x+2)]$
Critical numbers are the solutions of the equation $f'(x) = 0$ or
$\dfrac{(x+2)(x-2)^3}{64}[(2)(x-2)+(4)(x+2)] = 0$
So, the critical numbers are: $x_1 = -2$ $x_2 = 2$ $x_3 = \dfrac{(2)(2)+(4)(-2)}{2+4} = -0.667$

Sign Chart for the First Derivative $f'(x)$

x		-2		-0.667		2	
$f(x)$	$\searrow$	0	$\nearrow$	1.405	$\searrow$	0	$\nearrow$
$f'(x)$	$-$	0	$+$	0	$-$	0	$+$

Increasing and Decreasing Intervals

The function $f(x)$ is decreasing over $(-\infty, -2)$ and over $(-0.667, 2)$ and is increasing over $(-2, -0.667)$ and over $(2, \infty)$.

Maximum and Minimum Points

The function $f(x)$ has a maximum point at $P_3(-0.667, 1.405)$ and two minimum points at $P_1(-2, 0)$ and at $P_2(2, 0)$.

Concavity Intervals
The second derivative of the function $f(x)$ is given by:

$f''(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{(x+2)(x-2)^3}{64}[(2)(x-2)+(4)(x--2)]$

$f''(x) = \dfrac{(x-2)^2}{64}[30x^2 + 40x + -8]$

The second derivative $f''(x)$ is zero when $f''(x) = 0$ or:

$\frac{(x-2)^2}{64}[30x^2 + 40x + -8] = 0$

So, the second derivative $f''(x)$ is zero at:

$x_2 = 2 \qquad x_4 = -1.510 \qquad x_5 = 0.177$

Sign Chart for the Second Derivative $f''(x)$

x		-1.510		0.177	
$f(x)$	$\smile$	0.570	$\frown$	0.818	$\smile$
$f''(x)$	$+$	0	$-$	0	$+$

Inflection Points

The inflection point(s) is(are):

$P_4 = (-1.510, 0.570) \qquad P_5 = (0.177, 0.818)$

Graph