

1. Sketch the graph of the following polynomial function: $f(x) = \frac{-50 - 36x + 12x^2 - 2x^3}{18}$

2. Sketch the graph of the following polynomial function: $f(x) = \frac{-8 + 18x - 12x^2 + 2x^3}{30}$

3. Sketch the graph of the following polynomial function: $f(x) = \frac{6x - 2x^3}{30}$

4. Sketch the graph of the following polynomial function: $f(x) = \frac{-8 + 6x^2 + 2x^3}{-30}$

5. Sketch the graph of the following polynomial function: $f(x) = \frac{-108x + 36x^2 - 4x^3}{-30}$

6. Sketch the graph of the following polynomial function: $f(x) = \frac{12x^2 - 4x^3}{-24}$

7. Sketch the graph of the following polynomial function: $f(x) = \frac{6x - 4x^3}{24}$

8. Sketch the graph of the following polynomial function: $f(x) = \frac{14 + 6x - 6x^2 + 2x^3}{30}$

9. Sketch the graph of the following polynomial function: $f(x) = \frac{4 - 18x + 12x^2 - 2x^3}{-18}$

10. Sketch the graph of the following polynomial function: $f(x) = \frac{12x + 12x^2 + 2x^3}{-24}$

Solutions:

$$1. f(x) = \frac{-50 - 36x + 12x^2 - 2x^3}{18}$$

Domain

The function $f(x)$ is a polynomial function. Therefore the domain is $D_f = \mathbb{R}$.

Symmetry

$$f(-x) = \frac{-50 - 36(-x) + 12(-x)^2 - 2(-x)^3}{-6} = \frac{-50 + 36x + 12x^2 + 2x^3}{-6}$$

$$f(-x) \neq f(x) \quad f(-x) \neq -f(x)$$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros

$f(-1) = 0$. Therefore $x = -1$ is a zero of $f(x)$ and $(x + 1)$ is a factor of $f(x)$. By factoring $f(x)$:

$$f(x) = \frac{1}{18}(x + 1)(-50 + 14x - 2x^2)$$

The other zeros are given by: $-50 + 14x - 2x^2 = 0$. The discriminant of this quadratic equation is:

$$\Delta = (14)^2 - 4(-2)(-50) = -204$$

There are no more real zeros.

The zero(s) of the function $f(x)$ is(are): $x_1 = -1$

y-intercept

$$y - \text{int} = f(0) = \frac{-50}{18} = -2.778$$

Asymptotes

The function $f(x)$ is a polynomial function of degree 3. Therefore the function does not have any kind of asymptotes.

Critical Numbers

$$f'(x) = \frac{-36 + 24x - 6x^2}{18}$$

Critical numbers are the solutions of the equation $f'(x) = 0$ or $-36 + 24x - 6x^2 = 0$ or $-6 + 4x - x^2 = 0$

There are no critical numbers.

Sign Chart for the First Derivative $f'(x)$

| | |
|---------|---|
| x | |
| $f(x)$ | ↓ |
| $f'(x)$ | − |

Increasing and Decreasing Intervals

The function $f(x)$ is decreasing over $(-\infty, \infty)$.

Maximum and Minimum Points

The function $f(x)$ has no minimum or maximum points

Concavity Intervals

The second derivative of the function $f(x)$ is given by: $f''(x) = \frac{24 - 12x}{18}$

The second derivative $f''(x)$ is zero when: $f''(x) = 0 \quad 24 - 12x = 0 \quad x_6 = 2 \quad y_6 = f(x_6) = -5.000$

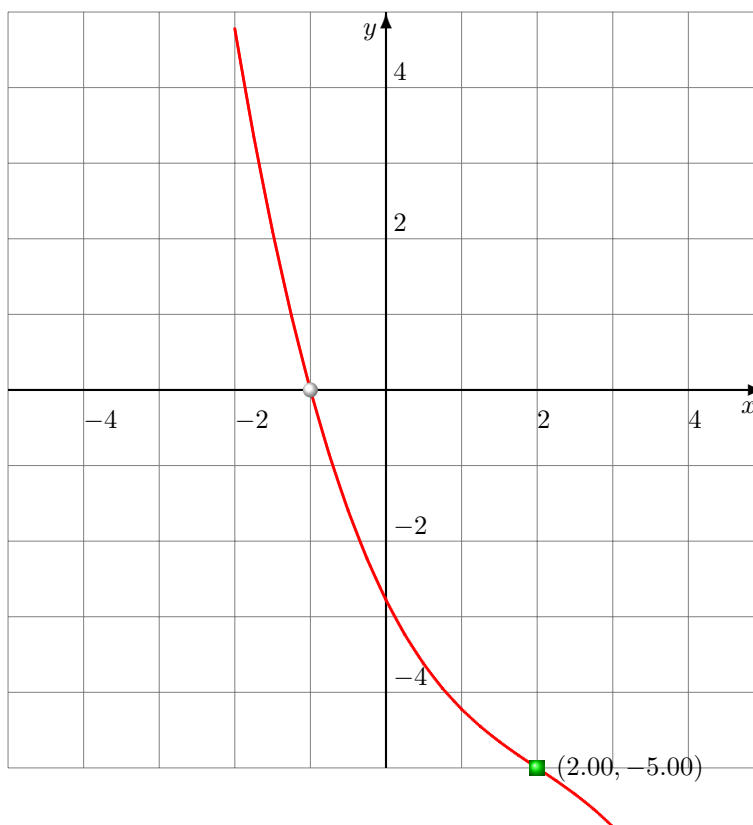
Sign Chart for the Second Derivative $f''(x)$

| | | | |
|----------|---|--------|---|
| x | | 2.000 | |
| $f(x)$ | ∪ | -5.000 | ∩ |
| $f''(x)$ | + | 0 | - |

Inflection Points

There is an inflection point at $P_6 = (2.000, -5.000)$

Graph



$$2. f(x) = \frac{-8 + 18x - 12x^2 + 2x^3}{30}$$

Domain

The function $f(x)$ is a polynomial function. Therefore the domain is $D_f = \mathbb{R}$.

Symmetry

$$f(-x) = \frac{-8 + 18(-x) - 12(-x)^2 + 2(-x)^3}{6} = \frac{-8 - 18x - 12x^2 - 2x^3}{6}$$

$$f(-x) \neq f(x) \quad f(-x) \neq -f(x)$$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros

$f(1) = 0$. Therefore $x = 1$ is a zero of $f(x)$ and $(x - 1)$ is a factor of $f(x)$. By factoring $f(x)$:

$$f(x) = \frac{1}{30}(x - 1)(8 - 10x + 2x^2)$$

The other zeros are given by: $8 - 10x + 2x^2 = 0$. The discriminant of this quadratic equation is:

$$\Delta = (-10)^2 - 4(2)(8) = 36$$

There are two more real zeros given by:

$$x = \frac{10 \pm \sqrt{(36)}}{2(2)}$$

$$x_2 = \frac{10 - \sqrt{(36)}}{4} = 1.000$$

$$x_3 = \frac{10 + \sqrt{(36)}}{4} = 4.000$$

The zero(s) of the function $f(x)$ is(are): $x_1 = 1$ $x_2 = 1.000$ $x_3 = 4.000$

y-intercept

$$y - \text{int} = f(0) = \frac{-8}{30} = -0.267$$

Asymptotes

The function $f(x)$ is a polynomial function of degree 3. Therefore the function does not have any kind of asymptotes.

Critical Numbers

$$f'(x) = \frac{18 - 24x + 6x^2}{30}$$

Critical numbers are the solutions of the equation $f'(x) = 0$ or $18 - 24x + 6x^2 = 0$ or $3 - 4x + x^2 = 0$

There are two critical numbers given by:

$$x = \frac{4 \pm \sqrt{(4)}}{2(1)}$$

$$x_4 = \frac{4 - \sqrt{(4)}}{2} = 1.000 \quad y_4 = f(x_4) = 0.000$$

$$x_5 = \frac{4 + \sqrt{(4)}}{2} = 3.000 \quad y_5 = f(x_5) = -0.267$$

Sign Chart for the First Derivative $f'(x)$

| | | | | | |
|---------|---|-------|---|--------|---|
| x | | 1.000 | | 3.000 | |
| $f(x)$ | ↑ | 0.000 | ↓ | -0.267 | ↑ |
| $f'(x)$ | + | 0 | - | 0 | + |

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(-\infty, 1.000)$ and over $(3.000, \infty)$ and is decreasing over $(1.000, 3.000)$.

Maximum and Minimum Points

The function $f(x)$ has a maximum point at $P_4(1.000, 0.000)$ and a minimum point at $P_5(3.000, -0.267)$.

Concavity Intervals

The second derivative of the function $f(x)$ is given by: $f''(x) = \frac{-24 + 12x}{30}$

The second derivative $f''(x)$ is zero when: $f''(x) = 0 \quad -24 + 12x = 0 \quad x_6 = 2 \quad y_6 = f(x_6) = -0.133$

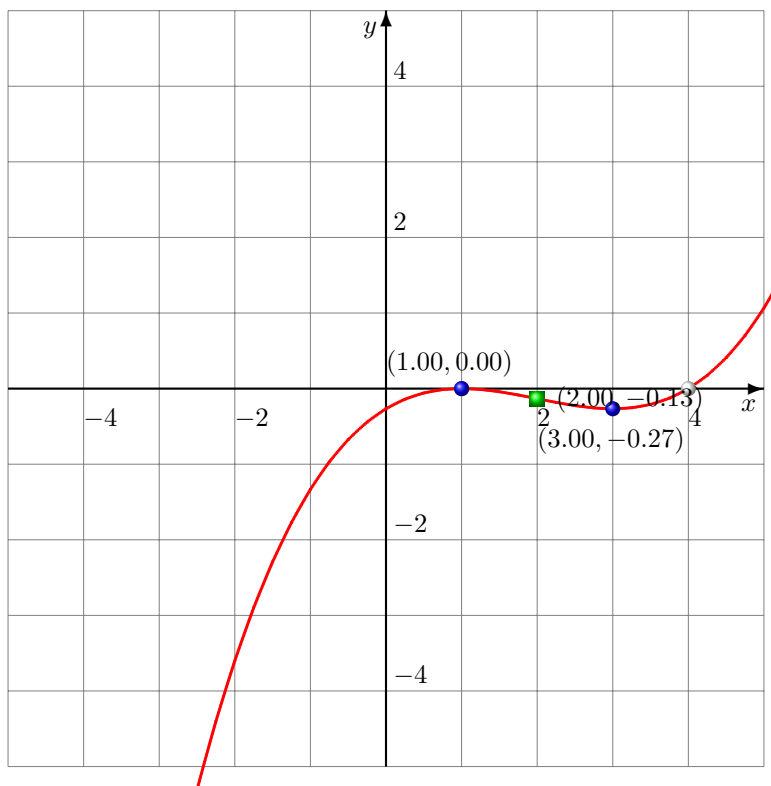
Sign Chart for the Second Derivative $f''(x)$

| | | | |
|----------|---|--------|---|
| x | | 2.000 | |
| $f(x)$ | ∩ | -0.133 | ∪ |
| $f''(x)$ | - | 0 | + |

Inflection Points

There is an inflection point at $P_6 = (2.000, -0.133)$

Graph



$$3. f(x) = \frac{6x - 2x^3}{30}$$

Domain

The function $f(x)$ is a polynomial function. Therefore the domain is $D_f = \mathbb{R}$.

Symmetry

$$f(-x) = \frac{6(-x) - 2(-x)^3}{-6} = \frac{-6x + 2x^3}{-6}$$

$$f(-x) \neq f(x) \quad f(-x) = -f(x)$$

Therefore the function $f(x)$ is an odd function.

Zeros

$f(0) = 0$. Therefore $x = 0$ is a zero of $f(x)$ and $(x - 0)$ is a factor of $f(x)$. By factoring $f(x)$:

$$f(x) = \frac{1}{30}(x - 0)(6 - 2x^2)$$

The other zeros are given by: $6 - 2x^2 = 0$. The discriminant of this quadratic equation is:

$$\Delta = (0)^2 - 4(-2)(6) = 48$$

There are two more real zeros given by:

$$x = \frac{0 \pm \sqrt{(48)}}{2(-2)}$$

$$x_2 = \frac{0 + \sqrt{(48)}}{-4} = -1.732$$

$$x_3 = \frac{0 - \sqrt{(48)}}{-4} = 1.732$$

The zero(s) of the function $f(x)$ is(are): $x_1 = 0$ $x_2 = -1.732$ $x_3 = 1.732$

y-intercept

$$y - \text{int} = f(0) = \frac{0}{30} = 0.000$$

Asymptotes

The function $f(x)$ is a polynomial function of degree 3. Therefore the function does not have any kind of asymptotes.

Critical Numbers

$$f'(x) = \frac{6 - 6x^2}{30}$$

Critical numbers are the solutions of the equation $f'(x) = 0$ or $6 - 6x^2 = 0$ or $1 - x^2 = 0$

There are two critical numbers given by:

$$x = \frac{0 \pm \sqrt{(4)}}{2(-1)}$$

$$x_4 = \frac{0 + \sqrt{(4)}}{-2} = -1.000 \quad y_4 = f(x_4) = -0.133$$

$$x_5 = \frac{0 - \sqrt{(4)}}{-2} = 1.000 \quad y_5 = f(x_5) = 0.133$$

Sign Chart for the First Derivative $f'(x)$

| | | | | | |
|---------|---|--------|---|-------|---|
| x | | -1.000 | | 1.000 | |
| $f(x)$ | ↓ | -0.133 | ↑ | 0.133 | ↓ |
| $f'(x)$ | - | 0 | + | 0 | - |

Increasing and Decreasing Intervals

The function $f(x)$ is decreasing over $(-\infty, -1.000)$ and over $(1.000, \infty)$ and is increasing over $(-1.000, 1.000)$.

Maximum and Minimum Points

The function $f(x)$ has a minimum point at $P_4(-1.000, -0.133)$ and a maximum point at $P_5(1.000, 0.133)$.

Concavity Intervals

The second derivative of the function $f(x)$ is given by: $f''(x) = \frac{-12x}{30}$

The second derivative $f''(x)$ is zero when: $f''(x) = 0 \quad -12x = 0 \quad x_6 = 0 \quad y_6 = f(x_6) = 0.000$

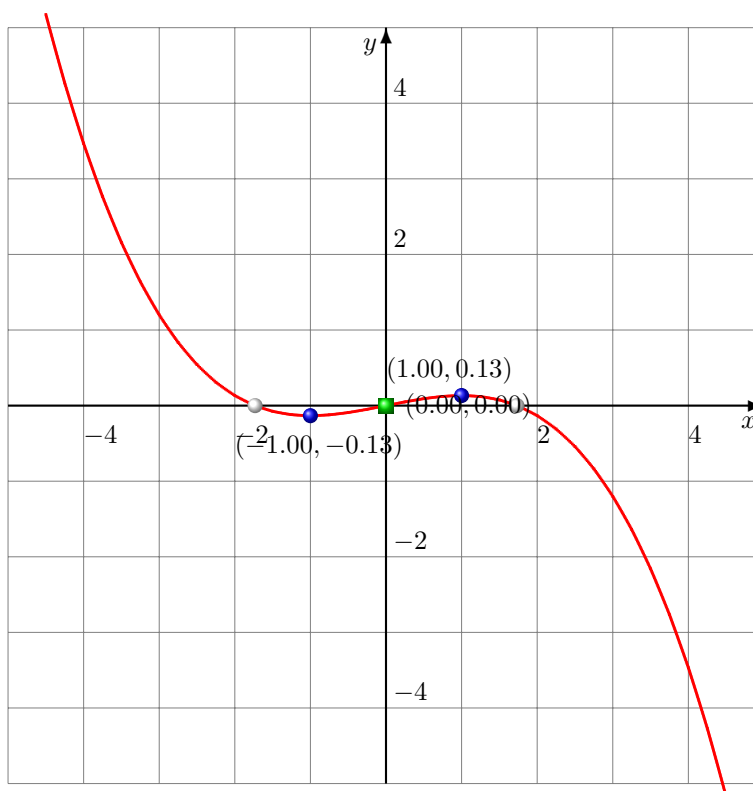
Sign Chart for the Second Derivative $f''(x)$

| | | | |
|----------|---|-------|---|
| x | | 0.000 | |
| $f(x)$ | ∪ | 0.000 | ∩ |
| $f''(x)$ | + | 0 | - |

Inflection Points

There is an inflection point at $P_6 = (0.000, 0.000)$

Graph



$$4. f(x) = \frac{-8 + 6x^2 + 2x^3}{-30}$$

Domain

The function $f(x)$ is a polynomial function. Therefore the domain is $D_f = \mathbb{R}$.

Symmetry

$$f(-x) = \frac{-8 + 6(-x)^2 + 2(-x)^3}{6} = \frac{-8 + 6x^2 - 2x^3}{6}$$

$$f(-x) \neq f(x) \quad f(-x) \neq -f(x)$$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros

$f(-2) = 0$. Therefore $x = -2$ is a zero of $f(x)$ and $(x + 2)$ is a factor of $f(x)$. By factoring $f(x)$:

$$f(x) = \frac{1}{-30}(x + 2)(-4 + 2x + 2x^2)$$

The other zeros are given by: $-4 + 2x + 2x^2 = 0$. The discriminant of this quadratic equation is:

$$\Delta = (2)^2 - 4(2)(-4) = 36$$

There are two more real zeros given by:

$$x = \frac{-2 \pm \sqrt{(36)}}{2(2)}$$

$$x_2 = \frac{-2 - \sqrt{(36)}}{4} = -2.000$$

$$x_3 = \frac{-2 + \sqrt{(36)}}{4} = 1.000$$

The zero(s) of the function $f(x)$ is(are): $x_1 = -2$ $x_2 = -2.000$ $x_3 = 1.000$

y-intercept

$$y - \text{int} = f(0) = \frac{-8}{-30} = 0.267$$

Asymptotes

The function $f(x)$ is a polynomial function of degree 3. Therefore the function does not have any kind of asymptotes.

Critical Numbers

$$f'(x) = \frac{12x + 6x^2}{-30}$$

Critical numbers are the solutions of the equation $f'(x) = 0$ or $12x + 6x^2 = 0$ or $2x + x^2 = 0$

There are two critical numbers given by:

$$x = \frac{-2 \pm \sqrt{(4)}}{2(1)}$$

$$x_4 = \frac{-2 - \sqrt{(4)}}{2} = -2.000 \quad y_4 = f(x_4) = 0.000$$

$$x_5 = \frac{-2 + \sqrt{(4)}}{2} = 0.000 \quad y_5 = f(x_5) = 0.267$$

Sign Chart for the First Derivative $f'(x)$

| | | | | | |
|---------|---|--------|---|-------|---|
| x | | -2.000 | | 0.000 | |
| $f(x)$ | ↓ | 0.000 | ↑ | 0.267 | ↓ |
| $f'(x)$ | - | 0 | + | 0 | - |

Increasing and Decreasing Intervals

The function $f(x)$ is decreasing over $(-\infty, -2.000)$ and over $(0.000, \infty)$ and is increasing over $(-2.000, 0.000)$.

Maximum and Minimum Points

The function $f(x)$ has a minimum point at $P_4(-2.000, 0.000)$ and a maximum point at $P_5(0.000, 0.267)$.

Concavity Intervals

The second derivative of the function $f(x)$ is given by: $f''(x) = \frac{12 + 12x}{-30}$

The second derivative $f''(x)$ is zero when: $f''(x) = 0 \quad 12 + 12x = 0 \quad x_6 = -1 \quad y_6 = f(x_6) = 0.133$

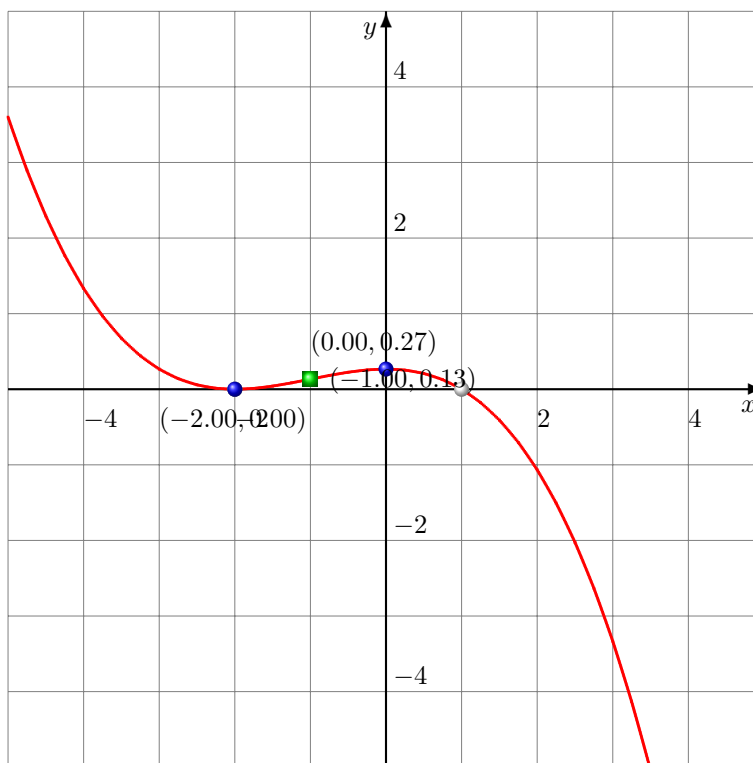
Sign Chart for the Second Derivative $f''(x)$

| | | | |
|----------|---|--------|---|
| x | | -1.000 | |
| $f(x)$ | ∪ | 0.133 | ∩ |
| $f''(x)$ | + | 0 | - |

Inflection Points

There is an inflection point at $P_6 = (-1.000, 0.133)$

Graph



$$5. f(x) = \frac{-108x + 36x^2 - 4x^3}{-30}$$

Domain

The function $f(x)$ is a polynomial function. Therefore the domain is $D_f = \mathbb{R}$.

Symmetry

$$f(-x) = \frac{-108(-x) + 36(-x)^2 - 4(-x)^3}{-12} = \frac{108x + 36x^2 + 4x^3}{-12}$$

$$f(-x) \neq f(x) \quad f(-x) \neq -f(x)$$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros

$f(0) = 0$. Therefore $x = 0$ is a zero of $f(x)$ and $(x - 0)$ is a factor of $f(x)$. By factoring $f(x)$:

$$f(x) = \frac{1}{-30}(x - 0)(-108 + 36x - 4x^2)$$

The other zeros are given by: $-108 + 36x - 4x^2 = 0$. The discriminant of this quadratic equation is:

$$\Delta = (36)^2 - 4(-4)(-108) = -432$$

There are no more real zeros.

The zero(s) of the function $f(x)$ is(are): $x_1 = 0$

y-intercept

$$y - int = f(0) = \frac{0}{-30} = 0.000$$

Asymptotes

The function $f(x)$ is a polynomial function of degree 3. Therefore the function does not have any kind of asymptotes.

Critical Numbers

$$f'(x) = \frac{-108 + 72x - 12x^2}{-30}$$

Critical numbers are the solutions of the equation $f'(x) = 0$ or $-108 + 72x - 12x^2 = 0$ or $-18 + 12x - 2x^2 = 0$

There are two critical numbers given by:

$$x = \frac{-12 \pm \sqrt{(0)}}{2(-2)}$$

$$x_4 = \frac{-12 + \sqrt{(0)}}{-4} = 3.000 \quad y_4 = f(x_4) = 3.600$$

$$x_5 = \frac{-12 - \sqrt{(0)}}{-4} = 3.000 \quad y_5 = f(x_5) = 3.600$$

Sign Chart for the First Derivative $f'(x)$

| | | | |
|---------|---|-------|---|
| x | | 3.000 | |
| $f(x)$ | ↑ | 3.600 | ↑ |
| $f'(x)$ | + | 0 | + |

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(-\infty, \infty)$.

Maximum and Minimum Points

The function $f(x)$ has no minimum or maximum points

Concavity Intervals

The second derivative of the function $f(x)$ is given by: $f''(x) = \frac{72 - 24x}{-30}$

The second derivative $f''(x)$ is zero when: $f''(x) = 0 \quad 72 - 24x = 0 \quad x_6 = 3 \quad y_6 = f(x_6) = 3.600$

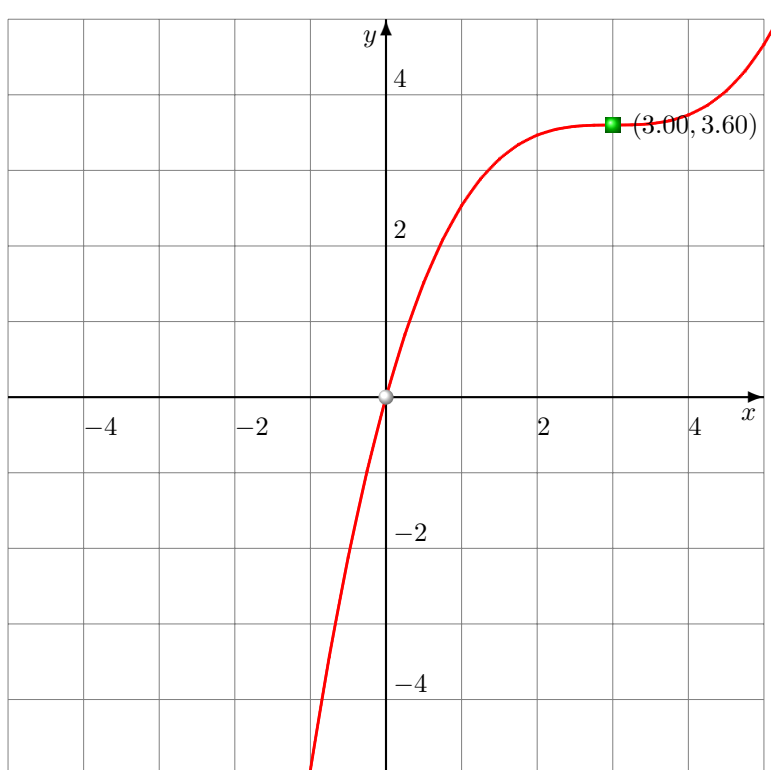
Sign Chart for the Second Derivative $f''(x)$

| | | | |
|----------|---|-------|---|
| x | | 3.000 | |
| $f(x)$ | ∩ | 3.600 | ∪ |
| $f''(x)$ | - | 0 | + |

Inflection Points

There is an inflection point at $P_6 = (3.000, 3.600)$

Graph



$$6. f(x) = \frac{12x^2 - 4x^3}{-24}$$

Domain

The function $f(x)$ is a polynomial function. Therefore the domain is $D_f = \mathbb{R}$.

Symmetry

$$f(-x) = \frac{12(-x)^2 - 4(-x)^3}{-12} = \frac{12x^2 + 4x^3}{-12}$$

$$f(-x) \neq f(x) \quad f(-x) \neq -f(x)$$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros

$f(0) = 0$. Therefore $x = 0$ is a zero of $f(x)$ and $(x - 0)$ is a factor of $f(x)$. By factoring $f(x)$:

$$f(x) = \frac{1}{-24}(x - 0)(12x - 4x^2)$$

The other zeros are given by: $12x - 4x^2 = 0$. The discriminant of this quadratic equation is:

$$\Delta = (12)^2 - 4(-4)(0) = 144$$

There are two more real zeros given by:

$$x = \frac{-12 \pm \sqrt{(144)}}{2(-4)}$$

$$x_2 = \frac{-12 + \sqrt{(144)}}{-8} = 0.000$$

$$x_3 = \frac{-12 - \sqrt{(144)}}{-8} = 3.000$$

The zero(s) of the function $f(x)$ is(are): $x_1 = 0$ $x_2 = 0.000$ $x_3 = 3.000$

y-intercept

$$y - \text{int} = f(0) = \frac{0}{-24} = 0.000$$

Asymptotes

The function $f(x)$ is a polynomial function of degree 3. Therefore the function does not have any kind of asymptotes.

Critical Numbers

$$f'(x) = \frac{24x - 12x^2}{-24}$$

Critical numbers are the solutions of the equation $f'(x) = 0$ or $24x - 12x^2 = 0$ or $4x - 2x^2 = 0$

There are two critical numbers given by:

$$x = \frac{-4 \pm \sqrt{(16)}}{2(-2)}$$

$$x_4 = \frac{-4 + \sqrt{(16)}}{-4} = 0.000 \quad y_4 = f(x_4) = 0.000$$

$$x_5 = \frac{-4 - \sqrt{(16)}}{-4} = 2.000 \quad y_5 = f(x_5) = -0.667$$

Sign Chart for the First Derivative $f'(x)$

| | | | | | |
|---------|---|-------|---|--------|---|
| x | | 0.000 | | 2.000 | |
| $f(x)$ | ↑ | 0.000 | ↓ | -0.667 | ↑ |
| $f'(x)$ | + | 0 | - | 0 | + |

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(-\infty, 0.000)$ and over $(2.000, \infty)$ and is decreasing over $(0.000, 2.000)$.

Maximum and Minimum Points

The function $f(x)$ has a maximum point at $P_4(0.000, 0.000)$ and a minimum point at $P_5(2.000, -0.667)$.

Concavity Intervals

The second derivative of the function $f(x)$ is given by: $f''(x) = \frac{24 - 24x}{-24}$

The second derivative $f''(x)$ is zero when: $f''(x) = 0 \quad 24 - 24x = 0 \quad x_6 = 1 \quad y_6 = f(x_6) = -0.333$

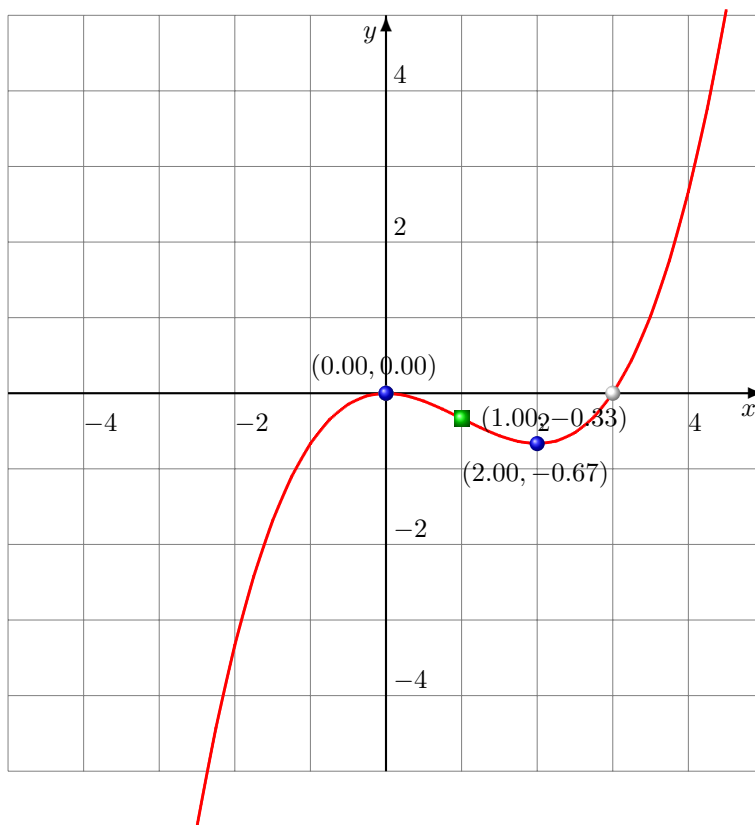
Sign Chart for the Second Derivative $f''(x)$

| | | | |
|----------|---|--------|---|
| x | | 1.000 | |
| $f(x)$ | ∩ | -0.333 | ∪ |
| $f''(x)$ | - | 0 | + |

Inflection Points

There is an inflection point at $P_6 = (1.000, -0.333)$

Graph



$$7. f(x) = \frac{6x - 4x^3}{24}$$

Domain

The function $f(x)$ is a polynomial function. Therefore the domain is $D_f = \mathbb{R}$.

Symmetry

$$f(-x) = \frac{6(-x) - 4(-x)^3}{-12} = \frac{-6x + 4x^3}{-12}$$

$$f(-x) \neq f(x) \quad f(-x) = -f(x)$$

Therefore the function $f(x)$ is an odd function.

Zeros

$f(0) = 0$. Therefore $x = 0$ is a zero of $f(x)$ and $(x - 0)$ is a factor of $f(x)$. By factoring $f(x)$:

$$f(x) = \frac{1}{24}(x - 0)(6 - 4x^2)$$

The other zeros are given by: $6 - 4x^2 = 0$. The discriminant of this quadratic equation is:

$$\Delta = (0)^2 - 4(-4)(6) = 96$$

There are two more real zeros given by:

$$x = \frac{0 \pm \sqrt{(96)}}{2(-4)}$$

$$x_2 = \frac{0 + \sqrt{(96)}}{-8} = -1.225$$

$$x_3 = \frac{0 - \sqrt{(96)}}{-8} = 1.225$$

The zero(s) of the function $f(x)$ is(are): $x_1 = 0$ $x_2 = -1.225$ $x_3 = 1.225$

y-intercept

$$y - int = f(0) = \frac{0}{24} = 0.000$$

Asymptotes

The function $f(x)$ is a polynomial function of degree 3. Therefore the function does not have any kind of asymptotes.

Critical Numbers

$$f'(x) = \frac{6 - 12x^2}{24}$$

Critical numbers are the solutions of the equation $f'(x) = 0$ or $6 - 12x^2 = 0$ or $1 - 2x^2 = 0$

There are two critical numbers given by:

$$x = \frac{0 \pm \sqrt{(8)}}{2(-2)}$$

$$x_4 = \frac{0 + \sqrt{(8)}}{-4} = -0.707 \quad y_4 = f(x_4) = -0.118$$

$$x_5 = \frac{0 - \sqrt{(8)}}{-4} = 0.707 \quad y_5 = f(x_5) = 0.118$$

Sign Chart for the First Derivative $f'(x)$

| | | | | | |
|---------|---|--------|---|-------|---|
| x | | -0.707 | | 0.707 | |
| $f(x)$ | ↓ | -0.118 | ↑ | 0.118 | ↓ |
| $f'(x)$ | - | 0 | + | 0 | - |

Increasing and Decreasing Intervals

The function $f(x)$ is decreasing over $(-\infty, -0.707)$ and over $(0.707, \infty)$ and is increasing over $(-0.707, 0.707)$.

Maximum and Minimum Points

The function $f(x)$ has a minimum point at $P_4(-0.707, -0.118)$ and a maximum point at $P_5(0.707, 0.118)$.

Concavity Intervals

The second derivative of the function $f(x)$ is given by: $f''(x) = \frac{-24x}{24}$

The second derivative $f''(x)$ is zero when: $f''(x) = 0 \quad -24x = 0 \quad x_6 = 0 \quad y_6 = f(x_6) = 0.000$

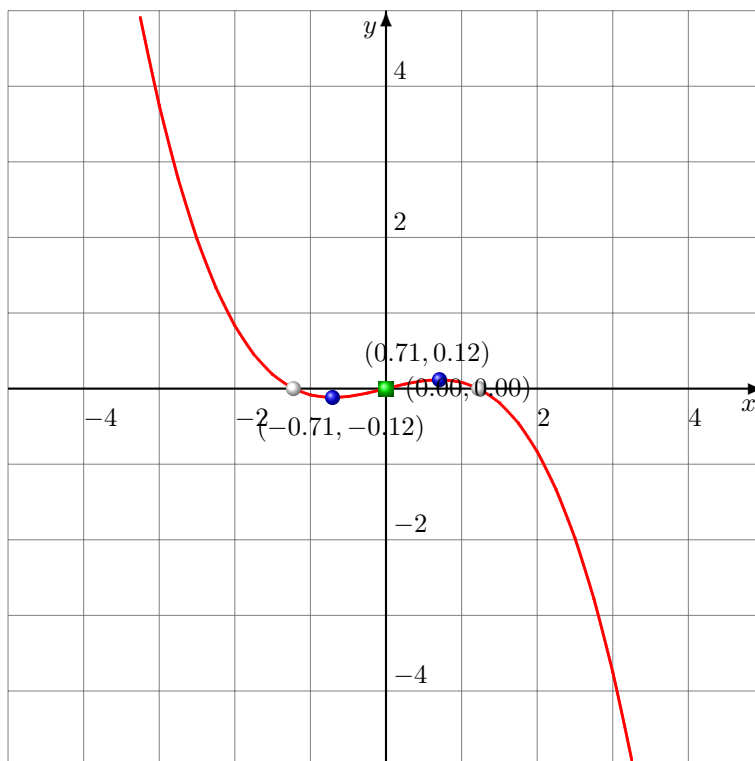
Sign Chart for the Second Derivative $f''(x)$

| | | | |
|----------|---|-------|---|
| x | | 0.000 | |
| $f(x)$ | ∪ | 0.000 | ∩ |
| $f''(x)$ | + | 0 | - |

Inflection Points

There is an inflection point at $P_6 = (0.000, 0.000)$

Graph



$$8. f(x) = \frac{14 + 6x - 6x^2 + 2x^3}{30}$$

Domain

The function $f(x)$ is a polynomial function. Therefore the domain is $D_f = \mathbb{R}$.

Symmetry

$$f(-x) = \frac{14 + 6(-x) - 6(-x)^2 + 2(-x)^3}{6} = \frac{14 - 6x - 6x^2 - 2x^3}{6}$$

$$f(-x) \neq f(x) \quad f(-x) \neq -f(x)$$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros

$f(-1) = 0$. Therefore $x = -1$ is a zero of $f(x)$ and $(x + 1)$ is a factor of $f(x)$. By factoring $f(x)$:

$$f(x) = \frac{1}{30}(x + 1)(14 - 8x + 2x^2)$$

The other zeros are given by: $14 - 8x + 2x^2 = 0$. The discriminant of this quadratic equation is:

$$\Delta = (-8)^2 - 4(2)(14) = -48$$

There are no more real zeros.

The zero(s) of the function $f(x)$ is(are): $x_1 = -1$

y-intercept

$$y - int = f(0) = \frac{14}{30} = 0.467$$

Asymptotes

The function $f(x)$ is a polynomial function of degree 3. Therefore the function does not have any kind of asymptotes.

Critical Numbers

$$f'(x) = \frac{6 - 12x + 6x^2}{30}$$

Critical numbers are the solutions of the equation $f'(x) = 0$ or $6 - 12x + 6x^2 = 0$ or $1 - 2x + x^2 = 0$

There are two critical numbers given by:

$$x = \frac{2 \pm \sqrt{(0)}}{2(1)}$$

$$x_4 = \frac{2 - \sqrt{(0)}}{2} = 1.000 \quad y_4 = f(x_4) = 0.533$$

$$x_5 = \frac{2 + \sqrt{(0)}}{2} = 1.000 \quad y_5 = f(x_5) = 0.533$$

Sign Chart for the First Derivative $f'(x)$

| | | | |
|---------|---|-------|---|
| x | | 1.000 | |
| $f(x)$ | ↑ | 0.533 | ↑ |
| $f'(x)$ | + | 0 | + |

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(-\infty, \infty)$.

Maximum and Minimum Points

The function $f(x)$ has no minimum or maximum points

Concavity Intervals

The second derivative of the function $f(x)$ is given by: $f''(x) = \frac{-12 + 12x}{30}$

The second derivative $f''(x)$ is zero when: $f''(x) = 0 \quad -12 + 12x = 0 \quad x_6 = 1 \quad y_6 = f(x_6) = 0.533$

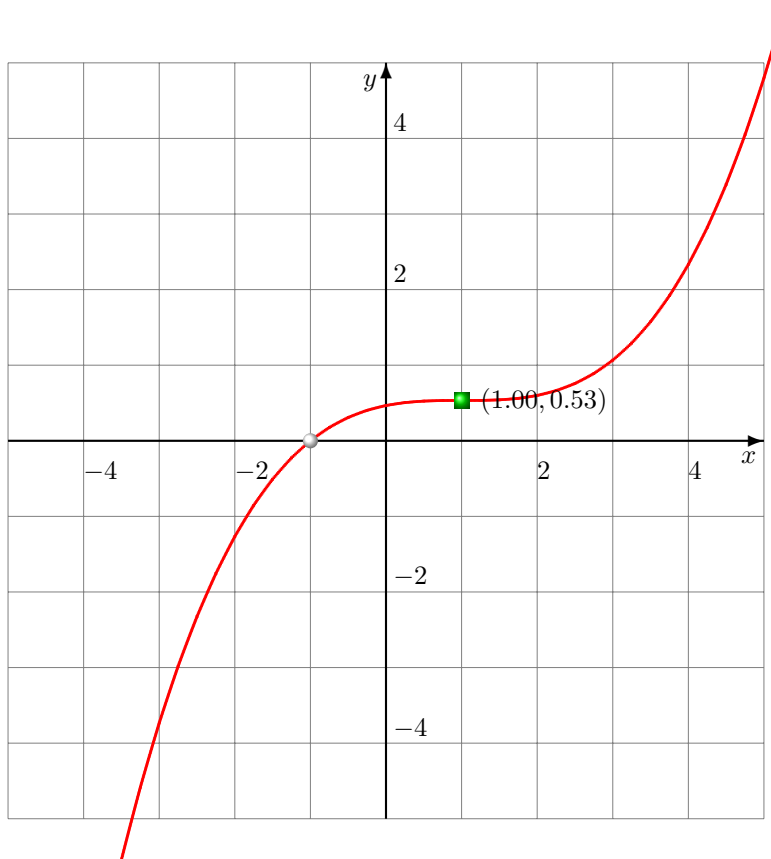
Sign Chart for the Second Derivative $f''(x)$

| | | | |
|----------|---|-------|---|
| x | | 1.000 | |
| $f(x)$ | ∩ | 0.533 | ∪ |
| $f''(x)$ | - | 0 | + |

Inflection Points

There is an inflection point at $P_6 = (1.000, 0.533)$

Graph



$$9. f(x) = \frac{4 - 18x + 12x^2 - 2x^3}{-18}$$

Domain

The function $f(x)$ is a polynomial function. Therefore the domain is $D_f = \mathbb{R}$.

Symmetry

$$f(-x) = \frac{4 - 18(-x) + 12(-x)^2 - 2(-x)^3}{-6} = \frac{4 + 18x + 12x^2 + 2x^3}{-6}$$

$$f(-x) \neq f(x) \quad f(-x) \neq -f(x)$$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros

$f(2) = 0$. Therefore $x = 2$ is a zero of $f(x)$ and $(x - 2)$ is a factor of $f(x)$. By factoring $f(x)$:

$$f(x) = \frac{1}{-18}(x - 2)(-2 + 8x - 2x^2)$$

The other zeros are given by: $-2 + 8x - 2x^2 = 0$. The discriminant of this quadratic equation is:

$$\Delta = (8)^2 - 4(-2)(-2) = 48$$

There are two more real zeros given by:

$$x = \frac{-8 \pm \sqrt{(48)}}{2(-2)}$$

$$x_2 = \frac{-8 + \sqrt{(48)}}{-4} = 0.268$$

$$x_3 = \frac{-8 - \sqrt{(48)}}{-4} = 3.732$$

The zero(s) of the function $f(x)$ is(are): $x_1 = 2$ $x_2 = 0.268$ $x_3 = 3.732$

y-intercept

$$y - int = f(0) = \frac{4}{-18} = -0.222$$

Asymptotes

The function $f(x)$ is a polynomial function of degree 3. Therefore the function does not have any kind of asymptotes.

Critical Numbers

$$f'(x) = \frac{-18 + 24x - 6x^2}{-18}$$

Critical numbers are the solutions of the equation $f'(x) = 0$ or $-18 + 24x - 6x^2 = 0$ or $-3 + 4x - x^2 = 0$

There are two critical numbers given by:

$$x = \frac{-4 \pm \sqrt{(4)}}{2(-1)}$$

$$x_4 = \frac{-4 + \sqrt{(4)}}{-2} = 1.000 \quad y_4 = f(x_4) = 0.222$$

$$x_5 = \frac{-4 - \sqrt{(4)}}{-2} = 3.000 \quad y_5 = f(x_5) = -0.222$$

Sign Chart for the First Derivative $f'(x)$

| | | | | | |
|---------|---|-------|---|--------|---|
| x | | 1.000 | | 3.000 | |
| $f(x)$ | ↑ | 0.222 | ↓ | -0.222 | ↑ |
| $f'(x)$ | + | 0 | - | 0 | + |

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(-\infty, 1.000)$ and over $(3.000, \infty)$ and is decreasing over $(1.000, 3.000)$.

Maximum and Minimum Points

The function $f(x)$ has a maximum point at $P_4(1.000, 0.222)$ and a minimum point at $P_5(3.000, -0.222)$.

Concavity Intervals

The second derivative of the function $f(x)$ is given by: $f''(x) = \frac{24 - 12x}{-18}$

The second derivative $f''(x)$ is zero when: $f''(x) = 0 \quad 24 - 12x = 0 \quad x_6 = 2 \quad y_6 = f(x_6) = 0.000$

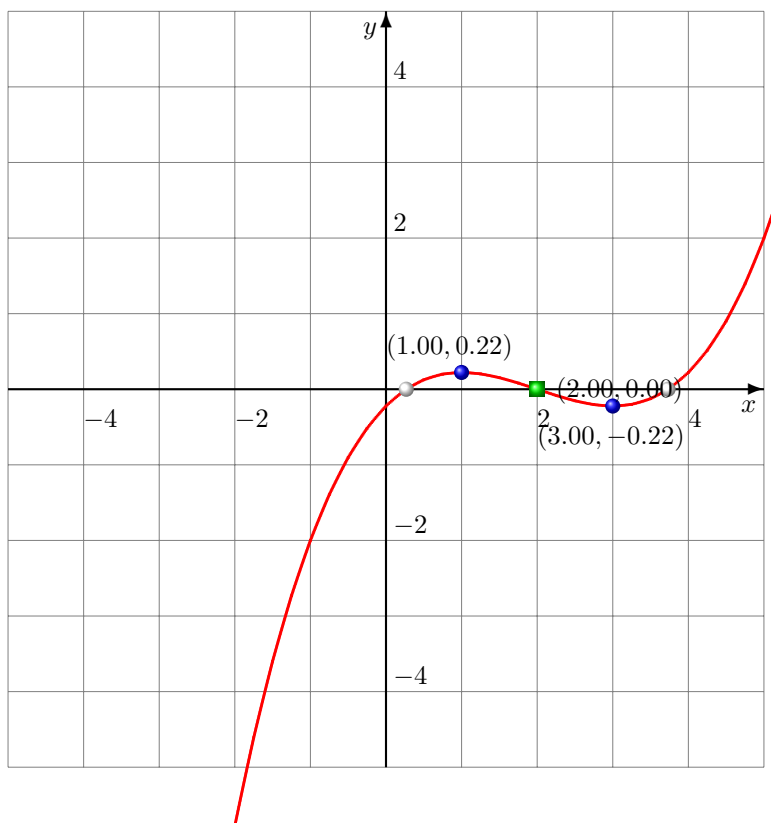
Sign Chart for the Second Derivative $f''(x)$

| | | | |
|----------|---|-------|---|
| x | | 2.000 | |
| $f(x)$ | ∩ | 0.000 | ∪ |
| $f''(x)$ | - | 0 | + |

Inflection Points

There is an inflection point at $P_6 = (2.000, 0.000)$

Graph



$$10. f(x) = \frac{12x + 12x^2 + 2x^3}{-24}$$

Domain

The function $f(x)$ is a polynomial function. Therefore the domain is $D_f = \mathbb{R}$.

Symmetry

$$f(-x) = \frac{12(-x) + 12(-x)^2 + 2(-x)^3}{6} = \frac{-12x + 12x^2 - 2x^3}{6}$$

$$f(-x) \neq f(x) \quad f(-x) \neq -f(x)$$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros

$f(0) = 0$. Therefore $x = 0$ is a zero of $f(x)$ and $(x - 0)$ is a factor of $f(x)$. By factoring $f(x)$:

$$f(x) = \frac{1}{-24}(x - 0)(12 + 12x + 2x^2)$$

The other zeros are given by: $12 + 12x + 2x^2 = 0$. The discriminant of this quadratic equation is:

$$\Delta = (12)^2 - 4(2)(12) = 48$$

There are two more real zeros given by:

$$x = \frac{-12 \pm \sqrt{(48)}}{2(2)}$$

$$x_2 = \frac{-12 - \sqrt{(48)}}{4} = -4.732$$

$$x_3 = \frac{-12 + \sqrt{(48)}}{4} = -1.268$$

The zero(s) of the function $f(x)$ is(are): $x_1 = 0 \quad x_2 = -4.732 \quad x_3 = -1.268$

y-intercept

$$y - int = f(0) = \frac{0}{-24} = 0.000$$

Asymptotes

The function $f(x)$ is a polynomial function of degree 3. Therefore the function does not have any kind of asymptotes.

Critical Numbers

$$f'(x) = \frac{12 + 24x + 6x^2}{-24}$$

Critical numbers are the solutions of the equation $f'(x) = 0$ or $12 + 24x + 6x^2 = 0$ or $2 + 4x + x^2 = 0$

There are two critical numbers given by:

$$x = \frac{-4 \pm \sqrt{(8)}}{2(1)}$$

$$x_4 = \frac{-4 - \sqrt{(8)}}{2} = -3.414 \quad y_4 = f(x_4) = -0.805$$

$$x_5 = \frac{-4 + \sqrt{(8)}}{2} = -0.586 \quad y_5 = f(x_5) = 0.138$$

Sign Chart for the First Derivative $f'(x)$

| | | | | | |
|---------|---|--------|---|--------|---|
| x | | -3.414 | | -0.586 | |
| $f(x)$ | ↓ | -0.805 | ↑ | 0.138 | ↓ |
| $f'(x)$ | - | 0 | + | 0 | - |

Increasing and Decreasing Intervals

The function $f(x)$ is decreasing over $(-\infty, -3.414)$ and over $(-0.586, \infty)$ and is increasing over $(-3.414, -0.586)$.

Maximum and Minimum Points

The function $f(x)$ has a minimum point at $P_4(-3.414, -0.805)$ and a maximum point at $P_5(-0.586, 0.138)$.

Concavity Intervals

The second derivative of the function $f(x)$ is given by: $f''(x) = \frac{24 + 12x}{-24}$

The second derivative $f''(x)$ is zero when: $f''(x) = 0 \quad 24 + 12x = 0 \quad x_6 = -2 \quad y_6 = f(x_6) = -0.333$

Sign Chart for the Second Derivative $f''(x)$

| | | | |
|----------|---|--------|---|
| x | | -2.000 | |
| $f(x)$ | ∩ | -0.333 | ∪ |
| $f''(x)$ | + | 0 | - |

Inflection Points

There is an inflection point at $P_6 = (-2.000, -0.333)$

Graph

