1. Sketch the grapf of the following polynomial function: $f(x) = \dfrac{(x+3)(x+1)(x-3)}{-4}$

2. Sketch the grapf of the following polynomial function: $f(x) = \dfrac{(x+3)(x-1)(x-2)}{-4}$

3. Sketch the grapf of the following polynomial function: $f(x) = \dfrac{(x+1)(x-0)(x-2)}{-4}$

4. Sketch the grapf of the following polynomial function: $f(x) = \dfrac{(x+3)(x+2)(x-2)}{2}$

5. Sketch the grapf of the following polynomial function: $f(x) = \dfrac{(x+1)(x-2)(x-3)}{-2}$

6. Sketch the grapf of the following polynomial function: $f(x) = \dfrac{(x+3)(x+1)(x-3)}{3}$

7. Sketch the grapf of the following polynomial function: $f(x) = \dfrac{(x+2)(x-0)(x-1)}{3}$

8. Sketch the grapf of the following polynomial function: $f(x) = \dfrac{(x+1)(x-1)(x-3)}{3}$

9. Sketch the grapf of the following polynomial function: $f(x) = \dfrac{(x+1)(x-2)(x-3)}{-5}$

10. Sketch the grapf of the following polynomial function: $f(x) = \dfrac{(x+3)(x+2)(x+1)}{-5}$

Solutions:

1. $f(x) = \dfrac{(x+3)(x+1)(x-3)}{-4}$

By expanding, $f(x) = \dfrac{-9-9x+x^2+x^3}{-4}$

Domain

The function $f(x)$ is a polynomial function. Therefore the domain is $D_f = \mathbb{R}$.

Symmetry

$$f(-x) = \frac{-9-9(-x)+(-x)^2+(-x)^3}{-4} = \frac{-9+9x+x^2-x^3}{-4}$$

$f(-x) \neq f(x) \qquad f(-x) \neq -f(x)$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros

The zeros of the function $f(x)$ are: $x_1 = -3 \qquad x_2 = -1 \qquad x_3 = 3$

y-intercept

$y - int = f(0) = \dfrac{-9}{-4} = 2.250$

Asymptotes

The function $f(x)$ is a polynomial function of degree 3. Therefore the function does not have any kind of asymptotes.

Critical Numbers

$f'(x) = \dfrac{-9+2x+3x^2}{-4}$

Critical numbers are the solutions of the equation $f'(x) = 0$ or $-9+2x+3x^2 = 0$

$$x = \frac{-2 \pm \sqrt{(-2)^2 - 4(3)(-9)}}{6}$$

$x_4 = \dfrac{-2-\sqrt{(112)}}{6} = -2.097 \qquad y_4 = f(x_4) = -1.262$

$x_5 = \dfrac{-2+\sqrt{(112)}}{6} = 1.431 \qquad y_5 = f(x_5) = 4.225$

Sign Chart for the First Derivative $f'(x)$

x		-2.097		1.431	
$f(x)$	$\downarrow$	-1.262	$\uparrow$	4.225	$\downarrow$
$f'(x)$	$-$	0	$+$	0	$-$

Increasing and Decreasing Intervals

The function $f(x)$ is decreasing over $(-\infty, -2.097)$ and over $(1.431, \infty)$ and is increasing over $(-2.097, 1.431)$.

Maximum and Minimum Points

The function $f(x)$ has a minimum point at $P_4(-2.097, -1.262)$ and a maximum point at $P_5(1.431, 4.225)$.

Concavity Intervals

The second derivative of the function $f(x)$ is given by: $f''(x) = \dfrac{2+6x}{-4}$

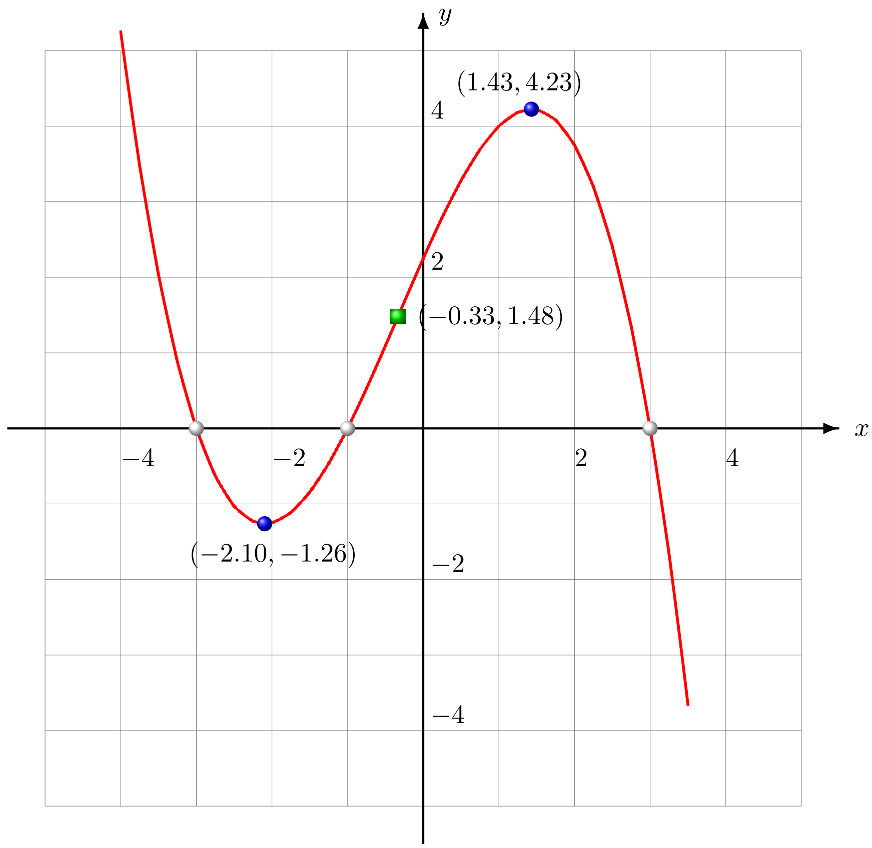
The second derivative $f''(x)$ is zero when: $f''(x) = 0 \quad 2 + 6x = 0 \quad x_6 = \frac{-1}{3} \quad y_6 = f(x_6) = 1.481$

Sign Chart for the Second Derivative $f''(x)$

x		-0.333	
$f(x)$	$\smile$	1.481	$\frown$
$f''(x)$	$+$	0	$-$

Inflection Points
There is an inflection point at $P_6 = (-0.333, 1.481)$

Graph



2. $f(x) = \dfrac{(x+3)(x-1)(x-2)}{-4}$

By expanding, $f(x) = \dfrac{6 - 7x + x^3}{-4}$

Domain

The function $f(x)$ is a polynomial function. Therefore the domain is $D_f = \mathbb{R}$.

Symmetry

$f(-x) = \dfrac{6 - 7(-x) + (-x)^3}{-4} = \dfrac{6 + 7x - x^3}{-4}$

$f(-x) \neq f(x) \qquad f(-x) \neq -f(x)$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros

The zeros of the function $f(x)$ are: $x_1 = -3 \qquad x_2 = 1 \qquad x_3 = 2$

y-intercept

$y - int = f(0) = \dfrac{6}{-4} = -1.500$

Asymptotes

The function $f(x)$ is a polynomial function of degree 3. Therefore the function does not have any kind of asymptotes.

Critical Numbers

$f'(x) = \dfrac{-7 + 3x^2}{-4}$

Critical numbers are the solutions of the equation $f'(x) = 0$ or $-7 + 3x^2 = 0$

$x = \dfrac{0 \pm \sqrt{(0)^2 - 4(3)(-7)}}{6}$

$x_4 = \dfrac{0 - \sqrt{(84)}}{6} = -1.528 \qquad y_4 = f(x_4) = -3.282$

$x_5 = \dfrac{0 + \sqrt{(84)}}{6} = 1.528 \qquad y_5 = f(x_5) = 0.282$

Sign Chart for the First Derivative $f'(x)$

x		-1.528		1.528	
$f(x)$	$\downarrow$	-3.282	$\uparrow$	0.282	$\downarrow$
$f'(x)$	$-$	0	$+$	0	$-$

Increasing and Decreasing Intervals

The function $f(x)$ is decreasing over $(-\infty, -1.528)$ and over $(1.528, \infty)$ and is increasing over $(-1.528, 1.528)$.

Maximum and Minimum Points

The function $f(x)$ has a minimum point at $P_4(-1.528, -3.282)$ and a maximum point at $P_5(1.528, 0.282)$.

Concavity Intervals

The second derivative of the function $f(x)$ is given by: $f''(x) = \dfrac{6x}{-4}$

The second derivative $f''(x)$ is zero when: $f''(x) = 0 \qquad 6x = 0 \qquad x_6 = 0 \qquad y_6 = f(x_6) = -1.500$

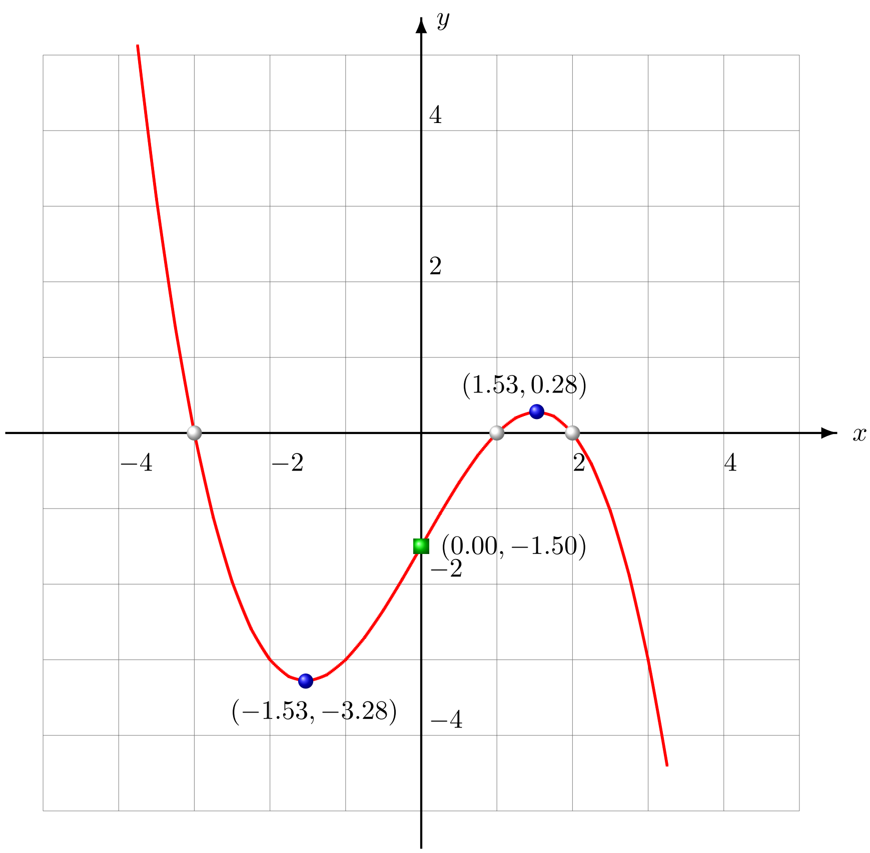
Sign Chart for the Second Derivative $f''(x)$

x		0.000	
$f(x)$	$\smile$	-1.500	$\frown$
$f''(x)$	$+$	0	$-$

Inflection Points

There is an inflection point at $P_6 = (0.000, -1.500)$

Graph



3. $f(x) = \dfrac{(x+1)(x-0)(x-2)}{-4}$

By expanding, $f(x) = \dfrac{-2x - x^2 + x^3}{-4}$

Domain

The function $f(x)$ is a polynomial function. Therefore the domain is $D_f = \mathbb{R}$.

Symmetry

$f(-x) = \dfrac{-2(-x) - (-x)^2 + (-x)^3}{-4} = \dfrac{2x - x^2 - x^3}{-4}$

$f(-x) \neq f(x) \qquad f(-x) \neq -f(x)$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros

The zeros of the function $f(x)$ are: $x_1 = -1 \qquad x_2 = 0 \qquad x_3 = 2$

y-intercept

$y - int = f(0) = \dfrac{0}{-4} = 0.000$

Asymptotes

The function $f(x)$ is a polynomial function of degree 3. Therefore the function does not have any kind of asymptotes.

Critical Numbers

$f'(x) = \dfrac{-2 - 2x + 3x^2}{-4}$

Critical numbers are the solutions of the equation $f'(x) = 0$ or $-2 - 2x + 3x^2 = 0$

$x = \dfrac{2 \pm \sqrt{(2)^2 - 4(3)(-2)}}{6}$

$x_4 = \dfrac{2 - \sqrt{(28)}}{6} = -0.549 \qquad y_4 = f(x_4) = -0.158$

$x_5 = \dfrac{2 + \sqrt{(28)}}{6} = 1.215 \qquad y_5 = f(x_5) = 0.528$

Sign Chart for the First Derivative $f'(x)$

x		-0.549		1.215	
$f(x)$	$\downarrow$	-0.158	$\uparrow$	0.528	$\downarrow$
$f'(x)$	$-$	0	$+$	0	$-$

Increasing and Decreasing Intervals

The function $f(x)$ is decreasing over $(-\infty, -0.549)$ and over $(1.215, \infty)$ and is increasing over $(-0.549, 1.215)$.

Maximum and Minimum Points

The function $f(x)$ has a minimum point at $P_4(-0.549, -0.158)$ and a maximum point at $P_5(1.215, 0.528)$.

Concavity Intervals

The second derivative of the function $f(x)$ is given by: $f''(x) = \dfrac{-2 + 6x}{-4}$

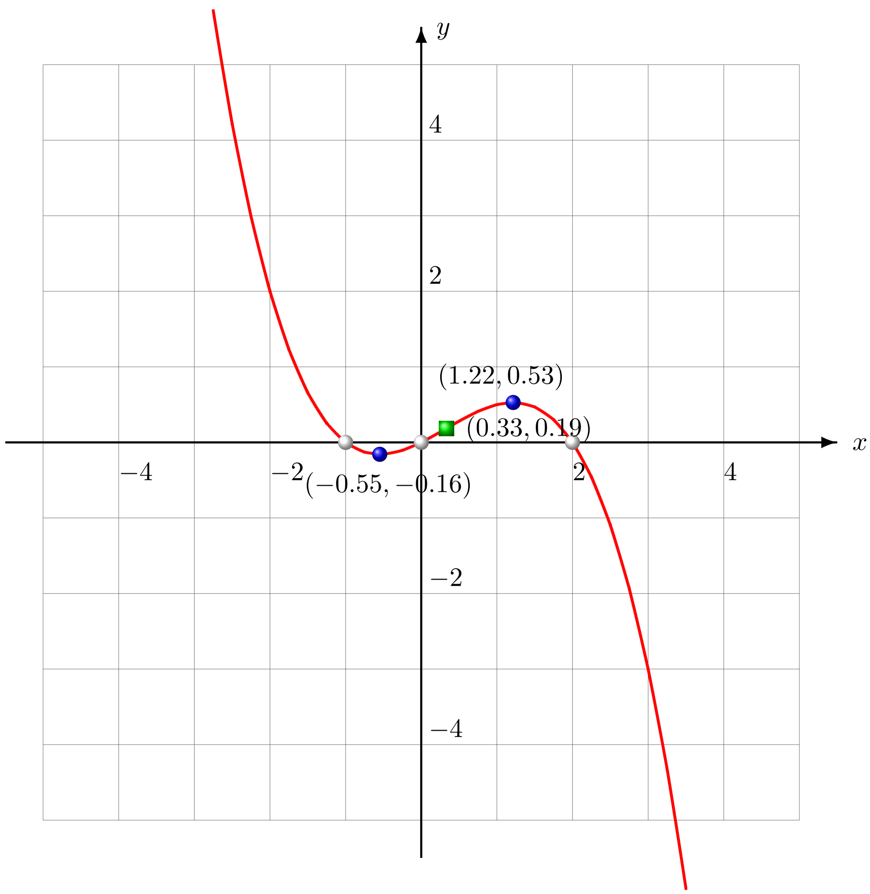
The second derivative $f''(x)$ is zero when: $f''(x) = 0 \qquad -2 + 6x = 0 \qquad x_6 = \frac{1}{3} \qquad y_6 = f(x_6) = 0.185$

Sign Chart for the Second Derivative $f''(x)$

x		0.333	
$f(x)$	$\smile$	0.185	$\frown$
$f''(x)$	$+$	0	$-$

Inflection Points
There is an inflection point at $P_6 = (0.333, 0.185)$

Graph



4. $f(x) = \dfrac{(x+3)(x+2)(x-2)}{2}$

By expanding, $f(x) = \dfrac{-12-4x+3x^2+x^3}{2}$

Domain

The function $f(x)$ is a polynomial function. Therefore the domain is $D_f = \mathbb{R}$.

Symmetry

$f(-x) = \dfrac{-12-4(-x)+3(-x)^2+(-x)^3}{2} = \dfrac{-12+4x+3x^2-x^3}{2}$

$f(-x) \neq f(x) \qquad f(-x) \neq -f(x)$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros

The zeros of the function $f(x)$ are: $x_1 = -3 \qquad x_2 = -2 \qquad x_3 = 2$

y-intercept

$y - int = f(0) = \dfrac{-12}{2} = -6.000$

Asymptotes

The function $f(x)$ is a polynomial function of degree 3. Therefore the function does not have any kind of asymptotes.

Critical Numbers

$f'(x) = \dfrac{-4+6x+3x^2}{2}$

Critical numbers are the solutions of the equation $f'(x) = 0$ or $-4+6x+3x^2 = 0$

$x = \dfrac{-6 \pm \sqrt{(-6)^2 - 4(3)(-4)}}{6}$

$x_4 = \dfrac{-6-\sqrt{(84)}}{6} = -2.528 \qquad y_4 = f(x_4) = 0.564$

$x_5 = \dfrac{-6+\sqrt{(84)}}{6} = 0.528 \qquad y_5 = f(x_5) = -6.564$

Sign Chart for the First Derivative $f'(x)$

x		-2.528		0.528	
$f(x)$	$\uparrow$	0.564	$\downarrow$	-6.564	$\uparrow$
$f'(x)$	$+$	0	$-$	0	$+$

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(-\infty, -2.528)$ and over $(0.528, \infty)$ and is decreasing over $(-2.528, 0.528)$.

Maximum and Minimum Points

The function $f(x)$ has a maximum point at $P_4(-2.528, 0.564)$ and a minimum point at $P_5(0.528, -6.564)$.

Concavity Intervals

The second derivative of the function $f(x)$ is given by: $f''(x) = \dfrac{6+6x}{2}$

The second derivative $f''(x)$ is zero when: $f''(x) = 0 \qquad 6+6x = 0 \qquad x_6 = -1 \qquad y_6 = f(x_6) = -3.000$

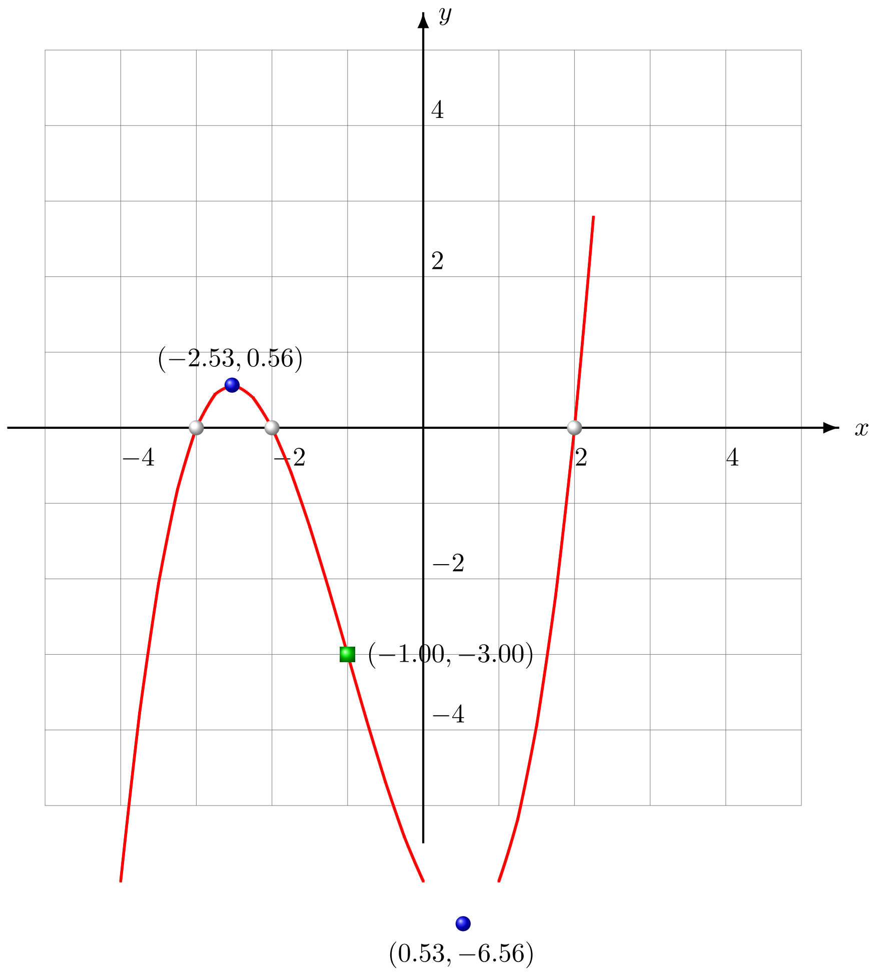
Sign Chart for the Second Derivative $f''(x)$

x		-1.000	
$f(x)$	$\frown$	-3.000	$\smile$
$f''(x)$	$-$	0	$+$

Inflection Points

There is an inflection point at $P_6 = (-1.000, -3.000)$

Graph



5. $f(x) = \dfrac{(x+1)(x-2)(x-3)}{-2}$

By expanding, $f(x) = \dfrac{6 + x - 4x^2 + x^3}{-2}$

Domain

The function $f(x)$ is a polynomial function. Therefore the domain is $D_f = \mathbb{R}$.

Symmetry

$f(-x) = \dfrac{6 + (-x) - 4(-x)^2 + (-x)^3}{-2} = \dfrac{6 - x - 4x^2 - x^3}{-2}$

$f(-x) \neq f(x) \qquad f(-x) \neq -f(x)$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros

The zeros of the function $f(x)$ are: $x_1 = -1 \qquad x_2 = 2 \qquad x_3 = 3$

y-intercept

$y - int = f(0) = \dfrac{6}{-2} = -3.000$

Asymptotes

The function $f(x)$ is a polynomial function of degree 3. Therefore the function does not have any kind of asymptotes.

Critical Numbers

$f'(x) = \dfrac{1 - 8x + 3x^2}{-2}$

Critical numbers are the solutions of the equation $f'(x) = 0 \qquad$ or $\qquad 1 - 8x + 3x^2 = 0$

$x = \dfrac{8 \pm \sqrt{(8)^2 - 4(3)(1)}}{6}$

$x_4 = \dfrac{8 - \sqrt{(52)}}{6} = 0.131 \qquad y_4 = f(x_4) = -3.032$

$x_5 = \dfrac{8 + \sqrt{(52)}}{6} = 2.535 \qquad y_5 = f(x_5) = 0.440$

Sign Chart for the First Derivative $f'(x)$

x		0.131		2.535	
$f(x)$	$\downarrow$	-3.032	$\uparrow$	0.440	$\downarrow$
$f'(x)$	$-$	0	$+$	0	$-$

Increasing and Decreasing Intervals

The function $f(x)$ is decreasing over $(-\infty, 0.131)$ and over $(2.535, \infty)$ and is increasing over $(0.131, 2.535)$.

Maximum and Minimum Points

The function $f(x)$ has a minimum point at $P_4(0.131, -3.032)$ and a maximum point at $P_5(2.535, 0.440)$.

Concavity Intervals

The second derivative of the function $f(x)$ is given by: $f''(x) = \dfrac{-8 + 6x}{-2}$

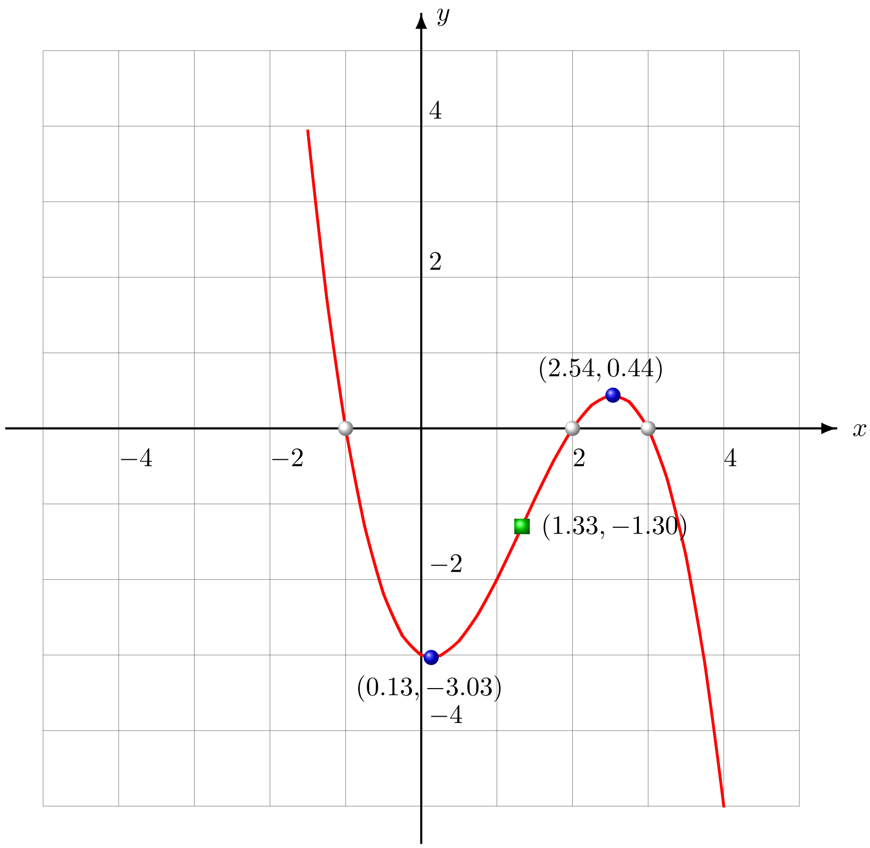
The second derivative $f''(x)$ is zero when: $f''(x) = 0 \qquad -8 + 6x = 0 \qquad x_6 = \frac{4}{3} \qquad y_6 = f(x_6) = -1.296$

Sign Chart for the Second Derivative $f''(x)$

x		1.333	
$f(x)$	$\smile$	-1.296	$\frown$
$f''(x)$	$+$	0	$-$

Inflection Points
There is an inflection point at $P_6 = (1.333, -1.296)$

Graph



6. $f(x) = \dfrac{(x+3)(x+1)(x-3)}{3}$

By expanding, $f(x) = \dfrac{-9 - 9x + x^2 + x^3}{3}$

Domain

The function $f(x)$ is a polynomial function. Therefore the domain is $D_f = \mathbb{R}$.

Symmetry

$f(-x) = \dfrac{-9 - 9(-x) + (-x)^2 + (-x)^3}{3} = \dfrac{-9 + 9x + x^2 - x^3}{3}$

$f(-x) \neq f(x) \qquad f(-x) \neq -f(x)$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros

The zeros of the function $f(x)$ are: $x_1 = -3 \qquad x_2 = -1 \qquad x_3 = 3$

y-intercept

$y - int = f(0) = \dfrac{-9}{3} = -3.000$

Asymptotes

The function $f(x)$ is a polynomial function of degree 3. Therefore the function does not have any kind of asymptotes.

Critical Numbers

$f'(x) = \dfrac{-9 + 2x + 3x^2}{3}$

Critical numbers are the solutions of the equation $f'(x) = 0$ or $-9 + 2x + 3x^2 = 0$

$x = \dfrac{-2 \pm \sqrt{(-2)^2 - 4(3)(-9)}}{6}$

$x_4 = \dfrac{-2 - \sqrt{(112)}}{6} = -2.097 \qquad y_4 = f(x_4) = 1.683$

$x_5 = \dfrac{-2 + \sqrt{(112)}}{6} = 1.431 \qquad y_5 = f(x_5) = -5.634$

Sign Chart for the First Derivative $f'(x)$

x		-2.097		1.431	
$f(x)$	$\uparrow$	1.683	$\downarrow$	-5.634	$\uparrow$
$f'(x)$	$+$	0	$-$	0	$+$

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(-\infty, -2.097)$ and over $(1.431, \infty)$ and is decreasing over $(-2.097, 1.431)$.

Maximum and Minimum Points

The function $f(x)$ has a maximum point at $P_4(-2.097, 1.683)$ and a minimum point at $P_5(1.431, -5.634)$.

Concavity Intervals

The second derivative of the function $f(x)$ is given by: $f''(x) = \dfrac{2 + 6x}{3}$

The second derivative $f''(x)$ is zero when: $f''(x) = 0 \qquad 2 + 6x = 0 \qquad x_6 = \dfrac{-1}{3} \qquad y_6 = f(x_6) = -1.975$

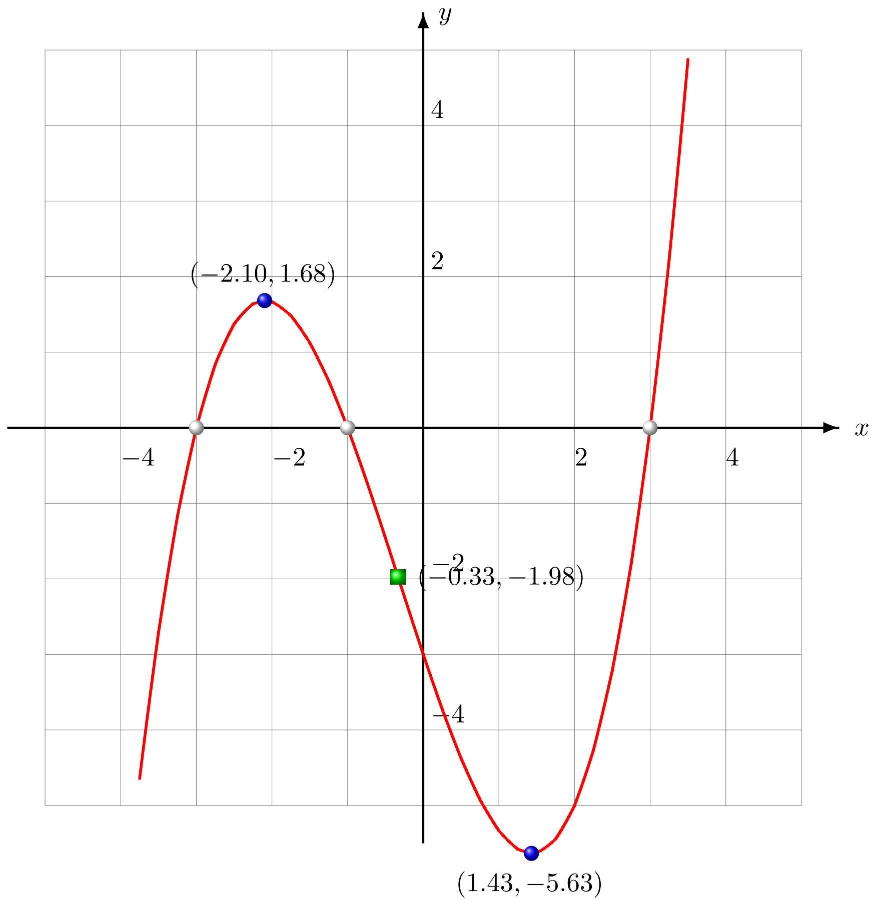
Sign Chart for the Second Derivative $f''(x)$

x		-0.333	
$f(x)$	$\frown$	-1.975	$\smile$
$f''(x)$	$-$	0	$+$

Inflection Points

There is an inflection point at $P_6 = (-0.333, -1.975)$

Graph



7. $f(x) = \dfrac{(x+2)(x-0)(x-1)}{3}$

By expanding, $f(x) = \dfrac{-2x + x^2 + x^3}{3}$

Domain

The function $f(x)$ is a polynomial function. Therefore the domain is $D_f = \mathbb{R}$.

Symmetry

$f(-x) = \dfrac{-2(-x) + (-x)^2 + (-x)^3}{3} = \dfrac{2x + x^2 - x^3}{3}$

$f(-x) \neq f(x) \qquad f(-x) \neq -f(x)$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros

The zeros of the function $f(x)$ are: $x_1 = -2 \qquad x_2 = 0 \qquad x_3 = 1$

y-intercept

$y - int = f(0) = \dfrac{0}{3} = 0.000$

Asymptotes

The function $f(x)$ is a polynomial function of degree 3. Therefore the function does not have any kind of asymptotes.

Critical Numbers

$f'(x) = \dfrac{-2 + 2x + 3x^2}{3}$

Critical numbers are the solutions of the equation $f'(x) = 0$ or $-2 + 2x + 3x^2 = 0$

$x = \dfrac{-2 \pm \sqrt{(-2)^2 - 4(3)(-2)}}{6}$

$x_4 = \dfrac{-2 - \sqrt{(28)}}{6} = -1.215 \qquad y_4 = f(x_4) = 0.704$

$x_5 = \dfrac{-2 + \sqrt{(28)}}{6} = 0.549 \qquad y_5 = f(x_5) = -0.210$

Sign Chart for the First Derivative $f'(x)$

x		-1.215		0.549	
$f(x)$	$\uparrow$	0.704	$\downarrow$	-0.210	$\uparrow$
$f'(x)$	$+$	0	$-$	0	$+$

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(-\infty, -1.215)$ and over $(0.549, \infty)$ and is decreasing over $(-1.215, 0.549)$.

Maximum and Minimum Points

The function $f(x)$ has a maximum point at $P_4(-1.215, 0.704)$ and a minimum point at $P_5(0.549, -0.210)$.

Concavity Intervals

The second derivative of the function $f(x)$ is given by: $f''(x) = \dfrac{2 + 6x}{3}$

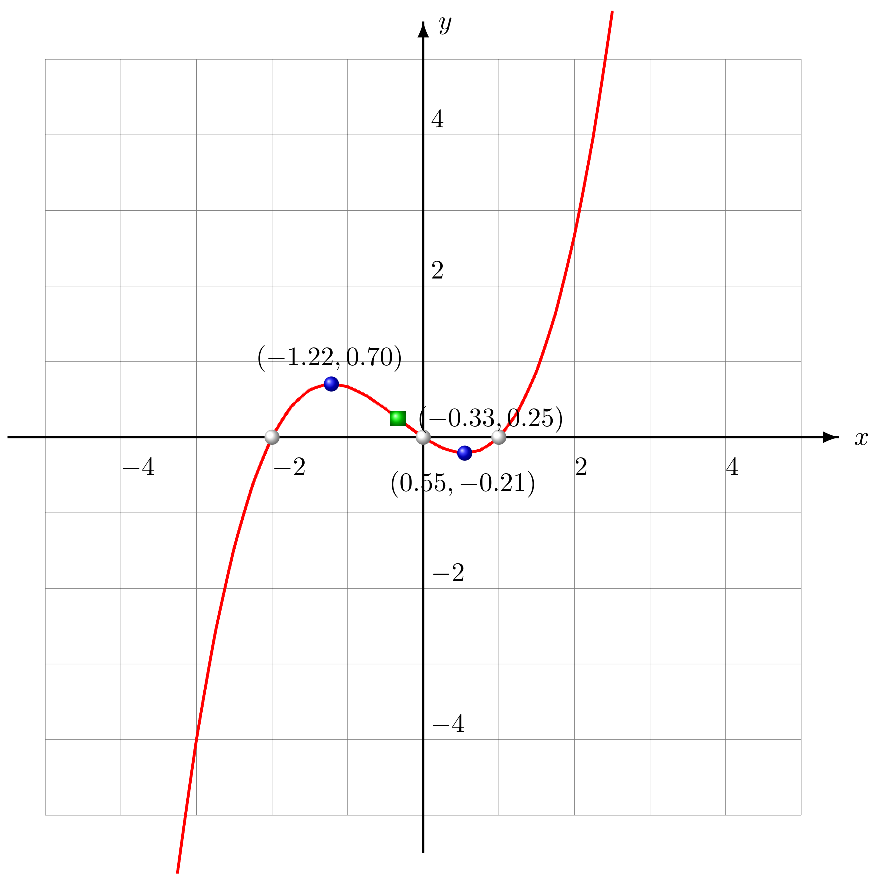
The second derivative $f''(x)$ is zero when: $f''(x) = 0 \qquad 2 + 6x = 0 \qquad x_6 = \dfrac{-1}{3} \qquad y_6 = f(x_6) = 0.247$

Sign Chart for the Second Derivative $f''(x)$

x		-0.333	
$f(x)$	$\frown$	0.247	$\smile$
$f''(x)$	$-$	0	$+$

Inflection Points
There is an inflection point at $P_6 = (-0.333, 0.247)$

Graph



8. $f(x) = \dfrac{(x+1)(x-1)(x-3)}{3}$

By expanding, $f(x) = \dfrac{3 - x - 3x^2 + x^3}{3}$

Domain

The function $f(x)$ is a polynomial function. Therefore the domain is $D_f = \mathbb{R}$.

Symmetry

$f(-x) = \dfrac{3-(-x)-3(-x)^2+(-x)^3}{3} = \dfrac{3+x-3x^2-x^3}{3}$

$f(-x) \neq f(x) \qquad f(-x) \neq -f(x)$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros

The zeros of the function $f(x)$ are: $x_1 = -1 \qquad x_2 = 1 \qquad x_3 = 3$

y-intercept

$y - int = f(0) = \dfrac{3}{3} = 1.000$

Asymptotes

The function $f(x)$ is a polynomial function of degree 3. Therefore the function does not have any kind of asymptotes.

Critical Numbers

$f'(x) = \dfrac{-1 - 6x + 3x^2}{3}$

Critical numbers are the solutions of the equation $f'(x) = 0$ or $-1 - 6x + 3x^2 = 0$

$x = \dfrac{6 \pm \sqrt{(6)^2 - 4(3)(-1)}}{6}$

$x_4 = \dfrac{6 - \sqrt{(48)}}{6} = -0.155 \qquad y_4 = f(x_4) = 1.026$

$x_5 = \dfrac{6 + \sqrt{(48)}}{6} = 2.155 \qquad y_5 = f(x_5) = -1.026$

Sign Chart for the First Derivative $f'(x)$

x		-0.155		2.155	
$f(x)$	$\uparrow$	1.026	$\downarrow$	-1.026	$\uparrow$
$f'(x)$	$+$	0	$-$	0	$+$

Increasing and Decreasing Intervals

The function $f(x)$ is increasing over $(-\infty, -0.155)$ and over $(2.155, \infty)$ and is decreasing over $(-0.155, 2.155)$.

Maximum and Minimum Points

The function $f(x)$ has a maximum point at $P_4(-0.155, 1.026)$ and a minimum point at $P_5(2.155, -1.026)$.

Concavity Intervals

The second derivative of the function $f(x)$ is given by: $f''(x) = \dfrac{-6 + 6x}{3}$

The second derivative $f''(x)$ is zero when: $f''(x) = 0 \qquad -6 + 6x = 0 \qquad x_6 = 1 \qquad y_6 = f(x_6) = 0.000$

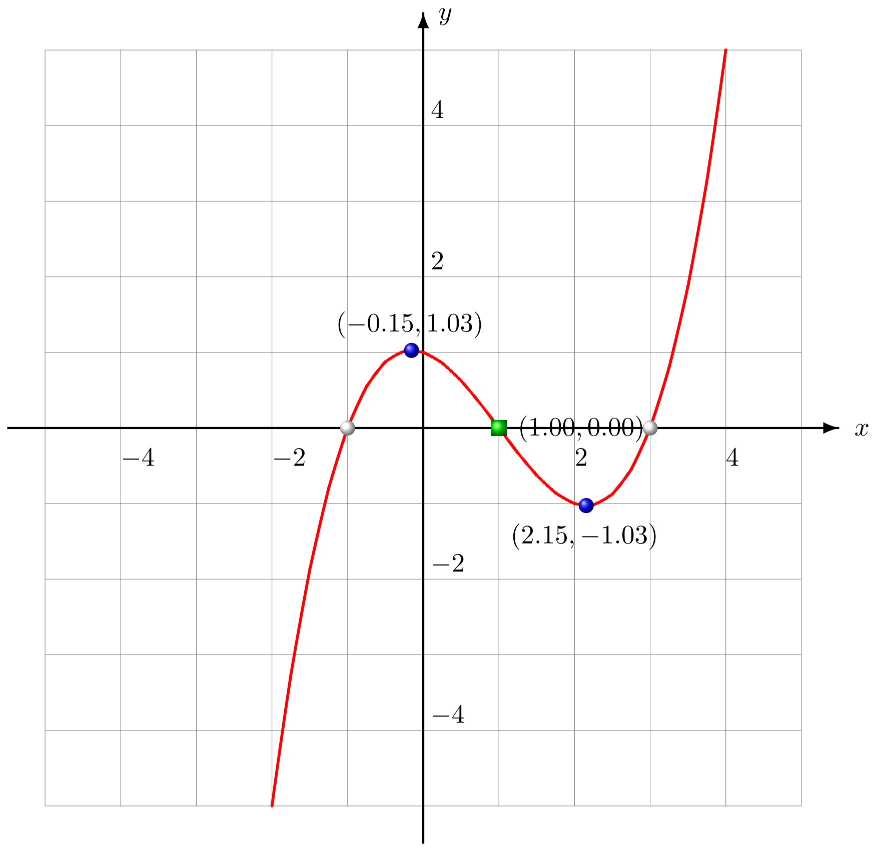
Sign Chart for the Second Derivative $f''(x)$

x		1.000	
$f(x)$	$\frown$	0.000	$\smile$
$f''(x)$	$-$	0	$+$

Inflection Points

There is an inflection point at $P_6 = (1.000, 0.000)$

Graph



9. $f(x) = \dfrac{(x+1)(x-2)(x-3)}{-5}$

By expanding, $f(x) = \dfrac{6 + x - 4x^2 + x^3}{-5}$

Domain

The function $f(x)$ is a polynomial function. Therefore the domain is $D_f = \mathbb{R}$.

Symmetry

$f(-x) = \dfrac{6 + (-x) - 4(-x)^2 + (-x)^3}{-5} = \dfrac{6 - x - 4x^2 - x^3}{-5}$

$f(-x) \neq f(x) \qquad f(-x) \neq -f(x)$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros

The zeros of the function $f(x)$ are: $x_1 = -1 \qquad x_2 = 2 \qquad x_3 = 3$

y-intercept

$y - int = f(0) = \dfrac{6}{-5} = -1.200$

Asymptotes

The function $f(x)$ is a polynomial function of degree 3. Therefore the function does not have any kind of asymptotes.

Critical Numbers

$f'(x) = \dfrac{1 - 8x + 3x^2}{-5}$

Critical numbers are the solutions of the equation $f'(x) = 0$ or $1 - 8x + 3x^2 = 0$

$x = \dfrac{8 \pm \sqrt{(8)^2 - 4(3)(1)}}{6}$

$x_4 = \dfrac{8 - \sqrt{(52)}}{6} = 0.131 \qquad y_4 = f(x_4) = -1.213$

$x_5 = \dfrac{8 + \sqrt{(52)}}{6} = 2.535 \qquad y_5 = f(x_5) = 0.176$

Sign Chart for the First Derivative $f'(x)$

x		0.131		2.535	
$f(x)$	$\downarrow$	-1.213	$\uparrow$	0.176	$\downarrow$
$f'(x)$	$-$	0	$+$	0	$-$

Increasing and Decreasing Intervals

The function $f(x)$ is decreasing over $(-\infty, 0.131)$ and over $(2.535, \infty)$ and is increasing over $(0.131, 2.535)$.

Maximum and Minimum Points

The function $f(x)$ has a minimum point at $P_4(0.131, -1.213)$ and a maximum point at $P_5(2.535, 0.176)$.

Concavity Intervals

The second derivative of the function $f(x)$ is given by: $f''(x) = \dfrac{-8 + 6x}{-5}$

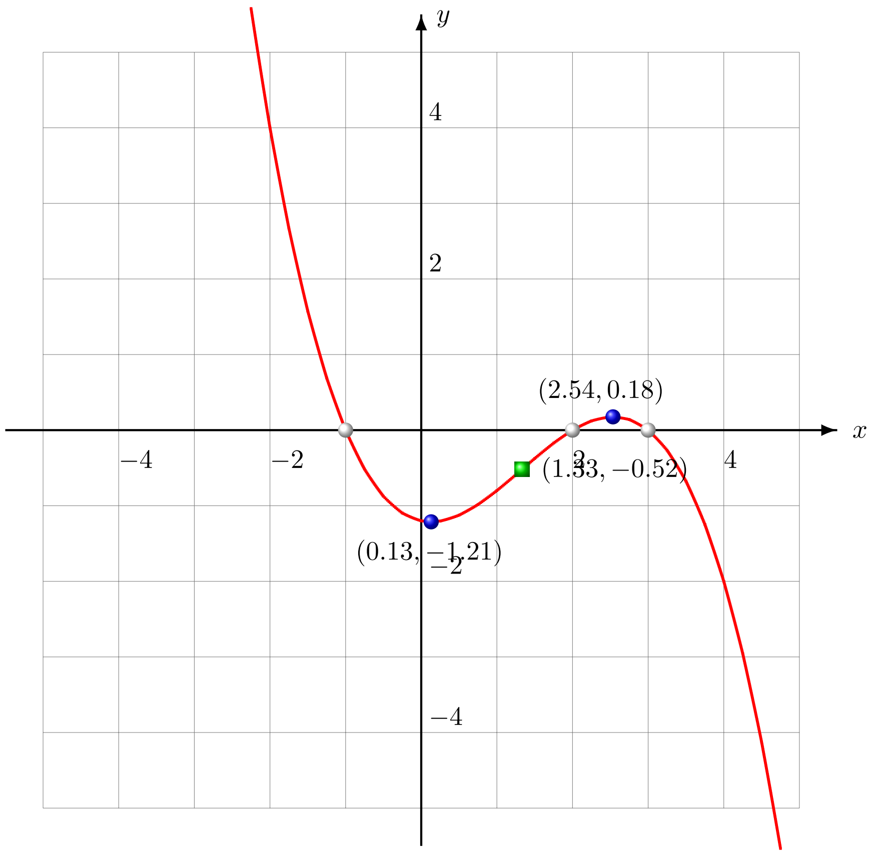
The second derivative $f''(x)$ is zero when: $f''(x) = 0 \qquad -8 + 6x = 0 \qquad x_6 = \frac{4}{3} \qquad y_6 = f(x_6) = -0.519$

Sign Chart for the Second Derivative $f''(x)$

x		1.333	
$f(x)$	$\smile$	-0.519	$\frown$
$f''(x)$	$+$	0	$-$

Inflection Points
There is an inflection point at $P_6 = (1.333, -0.519)$

Graph



10. $f(x) = \dfrac{(x+3)(x+2)(x+1)}{-5}$

By expanding, $f(x) = \dfrac{6 + 11x + 6x^2 + x^3}{-5}$

Domain

The function $f(x)$ is a polynomial function. Therefore the domain is $D_f = \mathbb{R}$.

Symmetry

$f(-x) = \dfrac{6 + 11(-x) + 6(-x)^2 + (-x)^3}{-5} = \dfrac{6 - 11x + 6x^2 - x^3}{-5}$

$f(-x) \neq f(x) \qquad f(-x) \neq -f(x)$

Therefore the function $f(x)$ is neither even nor odd function.

Zeros

The zeros of the function $f(x)$ are: $x_1 = -3 \qquad x_2 = -2 \qquad x_3 = -1$

y-intercept

$y - int = f(0) = \dfrac{6}{-5} = -1.200$

Asymptotes

The function $f(x)$ is a polynomial function of degree 3. Therefore the function does not have any kind of asymptotes.

Critical Numbers

$f'(x) = \dfrac{11 + 12x + 3x^2}{-5}$

Critical numbers are the solutions of the equation $f'(x) = 0$ or $11 + 12x + 3x^2 = 0$

$x = \dfrac{-12 \pm \sqrt{(-12)^2 - 4(3)(11)}}{6}$

$x_4 = \dfrac{-12 - \sqrt{(12)}}{6} = -2.577 \qquad y_4 = f(x_4) = -0.077$

$x_5 = \dfrac{-12 + \sqrt{(12)}}{6} = -1.423 \qquad y_5 = f(x_5) = 0.077$

Sign Chart for the First Derivative $f'(x)$

x		-2.577		-1.423	
$f(x)$	$\downarrow$	-0.077	$\uparrow$	0.077	$\downarrow$
$f'(x)$	$-$	0	$+$	0	$-$

Increasing and Decreasing Intervals

The function $f(x)$ is decreasing over $(-\infty, -2.577)$ and over $(-1.423, \infty)$ and is increasing over $(-2.577, -1.423)$.

Maximum and Minimum Points

The function $f(x)$ has a minimum point at $P_4(-2.577, -0.077)$ and a maximum point at $P_5(-1.423, 0.077)$.

Concavity Intervals

The second derivative of the function $f(x)$ is given by: $f''(x) = \dfrac{12 + 6x}{-5}$

The second derivative $f''(x)$ is zero when: $f''(x) = 0 \qquad 12 + 6x = 0 \qquad x_6 = -2 \qquad y_6 = f(x_6) = 0.000$

Sign Chart for the Second Derivative $f''(x)$

x		-2.000	
$f(x)$	$\smile$	0.000	$\frown$
$f''(x)$	$+$	0	$-$

Inflection Points
There is an inflection point at $P_6 = (-2.000, 0.000)$

Graph