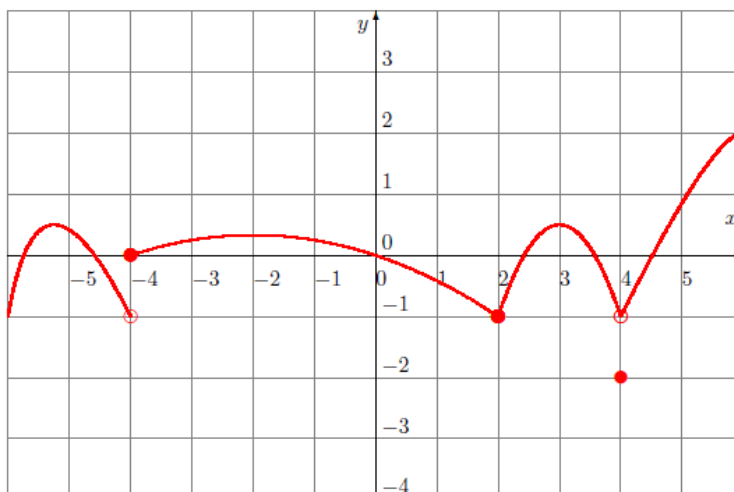


K/U	A	C	T/I
15	24	5	6

1. The function  $f$  is defined by the graph represented in the right figure. Find: [K/U 3 marks]

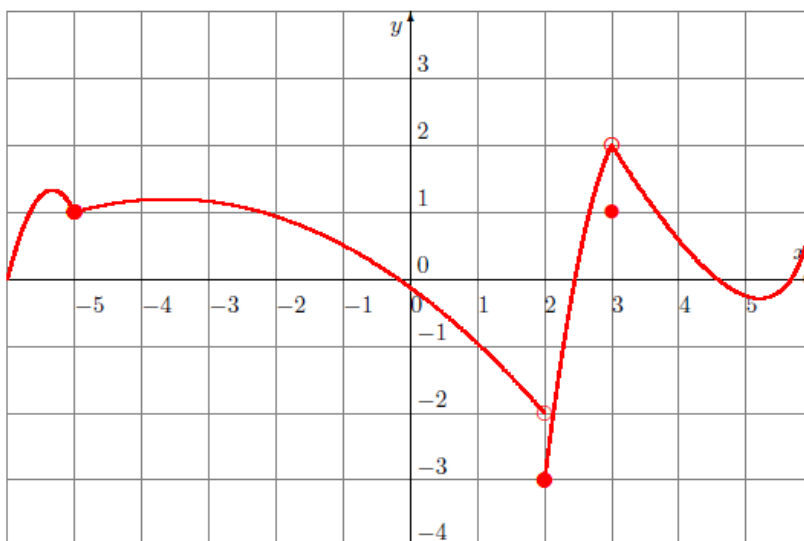
- a)  $\lim_{x \rightarrow -4^-} f(x)$
- b)  $\lim_{x \rightarrow -4^+} f(x)$
- c)  $\lim_{x \rightarrow -4} f(x)$
- d)  $\lim_{x \rightarrow 2} f(x)$
- e)  $\lim_{x \rightarrow 4} f(x)$
- f)  $\lim_{x \rightarrow 0} f(x)$



2. Consider the following function defined by its graph: [C 3 marks]

Analyze the discontinuity of this function (continuous or discontinuous) and the type of discontinuity (removable, jump or infinite discontinuity) at the following numbers. Justify your answer (explain why).

- a) at  $x = -5$



- b) at  $x = 2$

- c) at  $x = 3$

3. Analyse the limit and the continuity of the  $H(x)$  function at  $x = 0$ . Graph and explain. [K/U 4 marks]

$$H(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

4. Find each limit.

[K/U 8 marks]

$$[1] \text{ a) } \lim_{x \rightarrow -1} \frac{x^2 - 1}{x - 1}$$

$$[1.5] \text{ b) } \lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{x^2 + x - 2}$$

$$[1.5] \text{ c) } \lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 4}$$

$$[2] \text{ d) } \lim_{t \rightarrow 1} \frac{\sqrt{t} - 1}{t - 1}$$

$$[2] \text{ e) } \lim_{x \rightarrow 0} \frac{\sqrt{4+x} - 2}{3 - \sqrt{9+x}}$$

5. Consider the piecewise defined function below. Find the values of the constants  $a$ , and  $b$  such that the function  $y = f(x)$  to be continuous at any number. Show your work. [A 4 marks]

$$f(x) = \begin{cases} a + x^2 & , x < 0 \\ -1 & , x = 0 \\ -|x - b| & , x > 0 \end{cases}$$

6. Consider the following position function:  $s(t) = \frac{2t}{t+1}$

[A 4 marks]

[1] a) Find the average velocity over the time interval  $[2,9]$

[3] b) Find the instantaneous velocity at  $t = 2$ . Show your work.

7. Find the equation of the tangent line to the graph of  $f(x) = \sqrt{x-1}$  at the point  $P(5,2)$ . Show your work.

[A 4 marks]

8. Consider the function:  $f(x) = x^3 + 3x^2$ .

[A C 5 marks]

[3] a) Use the alternate formula  $m = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$  to find the slope of the tangent line to the graph of the curve at the generic point  $P(a, f(a))$ .

[2] b) Explain how would you find the point(s) where the tangent line is horizontal.

9. Analyse the continuity of the function. Graph the function.

[A 4 marks]

$$f(x) = \begin{cases} \frac{\sqrt{x^2}}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$$

10. An oil tank is being drained for cleaning. After  $t$  minutes there are  $V$  litres of oil left in the tank, where  $V(t) = 20(t-10)^2$ ,  $0 \leq t \leq 10$ .

[A 5 marks]

[1] a. Determine what  $V(0)$  and  $V(10)$  represent.

[1] b. Determine the average rate of change of volume during the first 5 minutes.

[3] c. Determine the rate of change of volume at the time  $t = 5$  minutes.

11. Use technology (a scientific calculator) to estimate the slope of the tangent line to the curve  $y = \sqrt{x^2 + \sqrt{x-1}}$  at the point  $P(2, \sqrt{5})$  by using  $h = 0.00001$ . Show your work. [A 2 marks]

12. For what value of  $c$  is the function:

[TIPS 4 marks]

$$f(x) = \begin{cases} (cx-1)^3 & \text{if } x > 2 \\ c^2x^2 - 1 & \text{if } x \leq 2 \end{cases}$$

Continuous at every number?