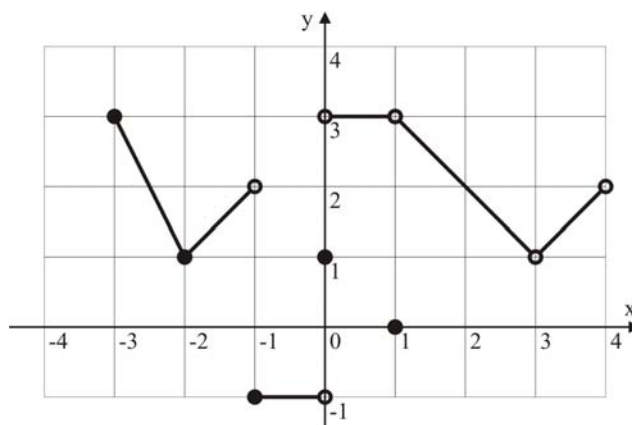


K/U	A	C	T/I
15	24	5	6

1. The function  $f$  is defined by the graph represented in the right figure. Find: [K/U 3 marks]

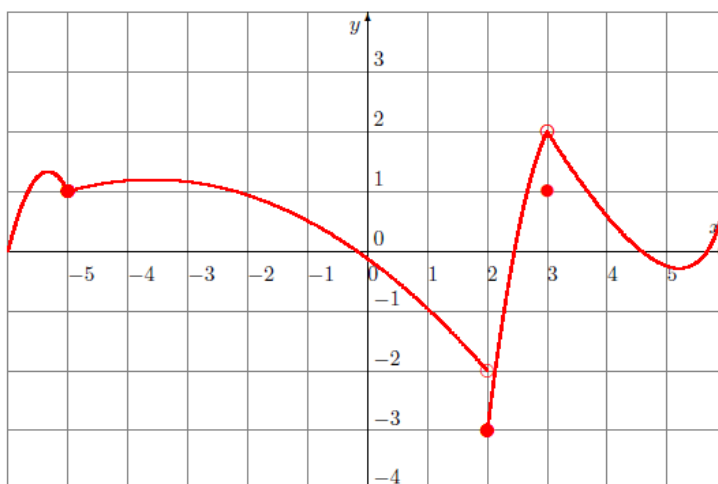
- a)  $\lim_{x \rightarrow 0^-} f(x)$
- b)  $\lim_{x \rightarrow 0^+} f(x)$
- c)  $\lim_{x \rightarrow 0} f(x)$
- d)  $\lim_{x \rightarrow 1} f(x)$
- e)  $\lim_{x \rightarrow 3} f(x)$
- f)  $\lim_{x \rightarrow -1} f(x)$



2. Consider the following function defined by its graph: [C 3 marks]

Analyze the discontinuity of this function (continuous or discontinuous) and the type of discontinuity (removable, jump or infinite discontinuity) at the following numbers. Justify your answer (explain why).

- a) at  $x = -5$



- b) at  $x = 2$

- c) at  $x = 3$

3. Analyse the limit and the continuity of the signum function  $\text{sgn}(x)$  at  $x = 0$ . Graph and explain. [K/U 4 marks]

$$\text{sgn}(x) = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

4. Find each limit.

[K/U 8 marks]

$$[1] \text{ a) } \lim_{x \rightarrow 0} \frac{x^2 - 9}{x + 3}$$

$$[1.5] \text{ b) } \lim_{x \rightarrow 4} \frac{x^2 - 6x + 8}{x^2 - 2x - 8}$$

$$[1.5] \text{ c) } \lim_{x \rightarrow -3} \frac{x^3 + 27}{x^2 - 9}$$

$$[2] \text{ d) } \lim_{t \rightarrow 0} \left( \frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right)$$

$$[2] \text{ e) } \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{2+x} - \sqrt{2-x}}$$

5. Consider the piecewise defined function below. Find the values of the constants  $a$ , and  $b$  such that the function  $y = f(x)$  to be continuous at any number. Show your work. [A 4 marks]

$$f(x) = \begin{cases} \frac{3}{x+2} - a & , x < 1 \\ 2 & , x = 1 \\ \sqrt{x+b} & , x > 1 \end{cases}$$

6. Consider the following position function:  $s(t) = 2\sqrt{t-1}$

[A 4 marks]

[1] a) Find the average velocity over the time interval  $[5,10]$

[3] b) Find the instantaneous velocity at  $t = 5$ . Show your work.

7. Find the equation of the tangent line to the graph of  $f(x) = 3x - \frac{2}{x}$  at the point  $P(2,5)$ . Show your work.

[A 4 marks]

8. Consider the function:  $f(x) = -3x^2 + 2x^3$ .

[A C 5 marks]

[3] a) Use the alternate formula  $m = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$  to find the slope of the tangent line to the graph of the curve at the generic point  $P(a, f(a))$ .

[2] b) Explain how would you find the point(s) where the tangent line is horizontal.

9. Analyse the continuity of the function. Graph the function.

[A 4 marks]

$$f(x) = \begin{cases} \frac{x|x-1|}{x-1} & , x \neq 1 \\ 0 & , x = 1 \end{cases}$$

10. An oil tank is being drained for cleaning. After  $t$  minutes there are  $V$  litres of oil left in the tank, where  $V(t) = 35(25-t)^2$ ,  $0 \leq t \leq 25$ .

[A 5 marks]

[1] a. Determine what  $V(0)$  and  $V(25)$  represent.

[1] b. Determine the average rate of change of volume during the first 15 minutes.

[3] c. Determine the rate of change of volume at the time  $t = 15$  minutes.

11. Use technology (a scientific calculator) to estimate the slope of the tangent line to the curve  $y = \sqrt{x + \sqrt{x}}$  at the point  $P(4, \sqrt{6})$  by using  $h = 0.0001$ . Show your work. [A 2 marks]

12. Find  $a$  and  $b$  such that  $\lim_{x \rightarrow 0} \frac{\sqrt{ax + b} - 2}{x} = 1$ .

[TIPS 4 marks]