

SOLUTIONS

MCV4U Quiz #2 Differentiation Rules

Name

1. Use the first principle to find the derivative of $y = \frac{x}{x-2}$.

[4 marks]

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{x+h}{x+h-2} - \frac{x}{x-2}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cancel{x^2} - \cancel{2x} + \cancel{hx} - \cancel{2h} - \cancel{x^2} - \cancel{hx} + \cancel{2x}}{(x+h-2)(x-2)h} = \lim_{h \rightarrow 0} \frac{-2h}{\cancel{h}(x+h-2)(x-2)h} \end{aligned}$$

$$f'(x) = \frac{-2}{(x-2)^2}$$

2. Analyze the differentiability of $y = x^{4/5}$. $y = x^{4/5}$

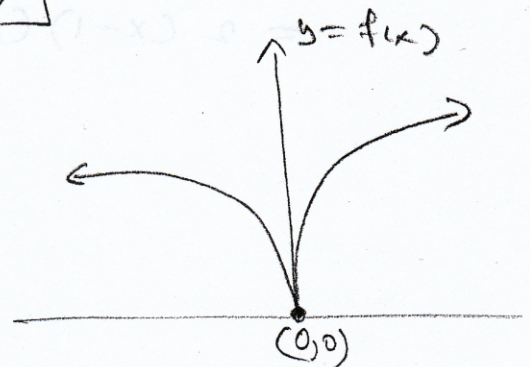
[4 marks]

$$y' = \frac{4}{5} x^{-1/5} = \frac{4}{5 \sqrt[5]{x}}$$

$y = f(x)$ is differentiable over $\mathbb{R} \setminus \{0\}$
 $y = f(x)$ is not differentiable at $x = 0$
[cusp point at $(0,0)$]

$$\lim_{x \rightarrow 0^-} f'(x) = -\infty$$

$$\lim_{x \rightarrow 0^+} f'(x) = +\infty$$



2. Differentiate. Do not simplify.

[6 marks]

[2] a) $f(x) = 5x^3 - 3x^2 + 2x + 5$

$$f'(x) = 15x^2 - 6x + 2$$

[2] b) $f(x) = \sqrt{x}(x^2 - 1)$

$$f'(x) = \frac{1}{2\sqrt{x}}(x^2 - 1) + \sqrt{x}(2x)$$

[2] c) $f(x) = (x-1)^2(x^2+2)^3$

$$\begin{aligned} f'(x) &= 2(x-1)(x^2+2)^3 + (x-1)^2(3)(x^2+2)^2(2x) \\ &= 2(x-1)(x^2+2)^3 + 6x(x-1)^2(x^2+2)^2 \end{aligned}$$