3.5 Curve Sketching

Algorithm for Curve Sketching

1. Domain
⇨ $denominator \neq 0$ (fractional functions)
⇨ $radicand \geq 0$ (even roots)
⇨ $logarithmic\ argument > 0$ (logarithmic functions)

2. Intercepts
⇨ $f(x) = 0$ (x-intercepts or zeros)
⇨ $numerator = 0$ (for fractional functions)
⇨ $y - \text{int} = f(0)$

3. Symmetry
⇨ $f(-x) = f(x)$ (even functions are symmetric about the y-axis)
⇨ $f(-x) = -f(x)$ (odd functions are symmetric about the origin)
⇨ $f(x+T) = f(x)$ (periodic functions have cycles)

4. Asymptotes
⇨ $\lim_{x \to \pm\infty} f(x)$ (horizontal asymptote)
⇨ $\lim_{x \to a} f(x)$ (vertical asymptote where a is a zero of the denominator)
⇨ long division (oblique asymptotes for rational functions)

5. First Derivative
⇨ compute $f'(x)$
⇨ find critical points ($f'(x) = 0$ or $f'(x)$ DNE)
⇨ find intervals of increase/decrease
⇨ find the local extrema (first derivative test) and global extrema

6. Second Derivative
⇨ compute $f''(x)$
⇨ find points where $f''(x) = 0$ or $f''(x)$ DNE
⇨ find points of inflection
⇨ find intervals of concavity upward/downward
⇨ check the local extrema using the second derivative test

7. Sketching
⇨ use broken lines to draw the asymptotes
⇨ plot x- and y- intercepts, extrema, and inflection points
⇨ draw the curve near the asymptotes
⇨ sketch the curve

⇨ use a graphic technology (e.g. WinPlot) to check your work

Examples

1. $f(x) = 2x^3 - 3x^2 - 36x$
2. $f(x) = 3x^5 - 5x^3$
3. $f(x) = \dfrac{x}{x^2+1}$
4. $f(x) = \dfrac{x^2+1}{x^2-4}$
5. $f(x) = \dfrac{x^3}{x^2-1}$
6. $f(x) = x^2\sqrt{x+1}$
7. $f(x) = x^{5/3} - 5x^{2/3}$

Practice Questions

1. Sketch the graph of the following polynomial functions.

a) $f(x) = (x-1)^3$
b) $f(x) = x^2(x+3)$
c) $f(x) = x(x^2-3)$
d) $f(x) = (x+1)^4$
e) $f(x) = x^3(3x-4)$
f) $f(x) = x^2(x-2)^2$
g) $f(x) = x^2(x-1)(x+1)$
h) $f(x) = (x^2-3)(x^2-5)$
i) $f(x) = 3x^5 - 5x^3 + 3$

2. Sketch the graph of the following rational functions.

a) $f(x) = \dfrac{x-1}{x+1}$
b) $f(x) = \dfrac{x}{x^2-1}$
c) $f(x) = \dfrac{x^2+1}{x^2-1}$
d) $f(x) = \dfrac{x^3}{x^2+1}$
e) $f(x) = \dfrac{x^2-1}{x^2-4}$
f) $f(x) = \dfrac{x^4-1}{x^3-1}$

3. Sketch the graph of the following radical functions.

a) $f(x) = \sqrt{x-1}$
b) $f(x) = \sqrt[3]{x+1}$
c) $f(x) = \sqrt[3]{(x-1)^2}$
d) $f(x) = x + 3x^{2/3}$
e) $f(x) = (x-4)\sqrt[3]{x}$
f) $f(x) = 2x^{1/3} - x^{2/3}$
g) $f(x) = x(6-x)^{2/3}$
i) $f(x) = x\sqrt[3]{x^2-15}$

Answers

Use a graphic technology (e.g. WinPlot) to check your work.

