

### 3.5 Curve Sketching

#### Algorithm for Curve Sketching

##### 1. Domain

- ⇒ denominator  $\neq 0$  (fractional functions)
- ⇒ radicand  $\geq 0$  (even roots)
- ⇒ logarithmic argument  $> 0$  (logarithmic functions)

##### 2. Intercepts

- ⇒  $f(x) = 0$  (x-intercepts or zeros)
- ⇒ numerator = 0 (for fractional functions)
- ⇒ y-int =  $f(0)$

##### 3. Symmetry

- ⇒  $f(-x) = f(x)$  (even functions are symmetric about the y-axis)
- ⇒  $f(-x) = -f(x)$  (odd functions are symmetric about the origin)
- ⇒  $f(x+T) = f(x)$  (periodic functions have cycles)

##### 4. Asymptotes

- ⇒  $\lim_{x \rightarrow \pm\infty} f(x)$  (horizontal asymptote)
- ⇒  $\lim_{x \rightarrow a} f(x)$  (vertical asymptote where  $a$  is a zero of the denominator)
- ⇒ long division (oblique asymptotes for rational functions)

##### 5. First Derivative

- ⇒ compute  $f'(x)$
- ⇒ find critical points ( $f'(x) = 0$  or  $f'(x)$  DNE)
- ⇒ find intervals of increase/decrease
- ⇒ find the local extrema (first derivative test) and global extrema

##### 6. Second Derivative

- ⇒ compute  $f''(x)$
- ⇒ find points where  $f''(x) = 0$  or  $f''(x)$  DNE
- ⇒ find points of inflection
- ⇒ find intervals of concavity upward/downward
- ⇒ check the local extrema using the second derivative test

##### 7. Sketching

- ⇒ use broken lines to draw the asymptotes
- ⇒ plot x- and y- intercepts, extrema, and inflection points
- ⇒ draw the curve near the asymptotes
- ⇒ sketch the curve
- ⇒ use a graphic technology (e.g. WinPlot) to check your work

#### Examples

- $f(x) = 2x^3 - 3x^2 - 36x$
- $f(x) = 3x^5 - 5x^3$
- $f(x) = \frac{x}{x^2 + 1}$
- $f(x) = \frac{x^2 + 1}{x^2 - 4}$
- $f(x) = \frac{x^3}{x^2 - 1}$
- $f(x) = x^2\sqrt{x+1}$
- $f(x) = x^{5/3} - 5x^{2/3}$

#### Practice Questions

1. Sketch the graph of the following polynomial functions.

- $f(x) = (x-1)^3$
- $f(x) = x^2(x+3)$
- $f(x) = x(x^2-3)$
- $f(x) = (x+1)^4$
- $f(x) = x^3(3x-4)$
- $f(x) = x^2(x-2)^2$
- $f(x) = x^2(x-1)(x+1)$
- $f(x) = (x^2-3)(x^2-5)$
- $f(x) = 3x^5 - 5x^3 + 3$

2. Sketch the graph of the following rational functions.

- $f(x) = \frac{x-1}{x+1}$
- $f(x) = \frac{x}{x^2-1}$
- $f(x) = \frac{x^2+1}{x^2-1}$
- $f(x) = \frac{x^3}{x^2+1}$
- $f(x) = \frac{x^2-1}{x^2-4}$
- $f(x) = \frac{x^4-1}{x^3-1}$

3. Sketch the graph of the following radical functions.

- $f(x) = \sqrt{x-1}$
- $f(x) = \sqrt[3]{x+1}$
- $f(x) = \sqrt[3]{(x-1)^2}$
- $f(x) = x + 3x^{2/3}$
- $f(x) = (x-4)\sqrt[3]{x}$
- $f(x) = 2x^{1/3} - x^{2/3}$
- $f(x) = x(6-x)^{2/3}$
- $f(x) = x\sqrt[3]{x^2-15}$

#### Answers

Use a graphic technology (e.g. WinPlot) to check your work.