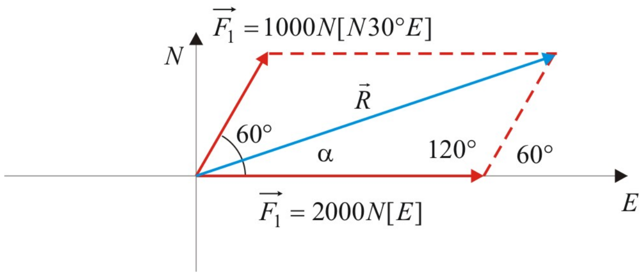
7.1 Vectors as Forces

A Vector Force
The force is a vector and the measurement unit is N (Newton).

B Resultant Force
The vector sum of a system of forces is called resultant.

$\vec{R} = \vec{F_1} + \vec{F_2} + ... + \vec{F_n}$

Ex 1. Two forces are applied to a boat $\vec{F_1} = 1000N[N30°E]$ and $\vec{F_1} = 2000N[E]$. Find the resultant force (magnitude and direction).



$\|\vec{F_1}\| = 1000N, \quad \|\vec{F_2}\| = 2000N, \quad \theta = \angle(\vec{F_1}, \vec{F_2}) = 60°$

$\vec{R} = \vec{F_1} + \vec{F_2}$

$\|\vec{R}\|^2 = \|\vec{F_1}\|^2 + \|\vec{F_2}\|^2 + 2\|\vec{F_1}\|\|\vec{F_2}\|\cos\theta$

$= 1000^2 + 2000^2 + 2(1000)(2000)\cos 60° = 7000000$

$\therefore \|\vec{R}\| = \sqrt{7000000} = 1000\sqrt{7}N = 2645.75N$

$\frac{\sin\alpha}{\|\vec{F_1}\|} = \frac{\sin(120°)}{\|\vec{R}\|} \Rightarrow \frac{\sin\alpha}{1000} = \frac{\sin 120°}{1000\sqrt{7}}$

$\therefore \alpha = \sin^{-1}\left(\frac{\sin 120°}{\sqrt{7}}\right) = 19.11°$

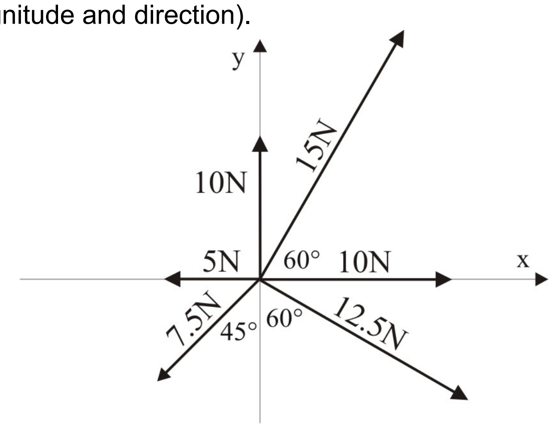
C Algebraic Resultant Force
The scalar components of the resultant force are given by:

$$R_x = F_{1x} + F_{2x} + ... + F_{nx}$$
$$R_y = F_{1y} + F_{2y} + ... + F_{ny}$$

The magnitude and the direction of the resultant force are given by:

$$\|\vec{R}\| = \sqrt{R_x^2 + R_y^2}$$
$$\tan\theta = \frac{R_y}{R_x}$$

Ex 2. Find the resultant of the following system of forces (magnitude and direction).



$R_x = 10 + 15\cos 60° + 0 - 5 - 7.5\sin 45° + 12.5\sin 60° = 18.022N$

$R_y = 0 + 15\sin 60° + 10 + 0 - 7.5\cos 45° - 12.5\cos 60° = 11.437N$

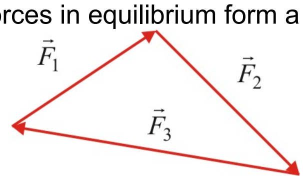
$\|\vec{R}\| = \sqrt{R_x^2 + R_y^2} = \sqrt{18.022^2 + 11.437^2} = 21.34N$

$\tan\theta = \frac{R_y}{R_x} \Rightarrow \theta = \tan^{-1}(11.437/18.022) = 32.4°$

D Equilibrium
A system of forces is the state of equilibrium if the resultant force is $\vec{0}$.

$$\vec{R} = \vec{0}$$

Note. Three forces in equilibrium form a triangle.

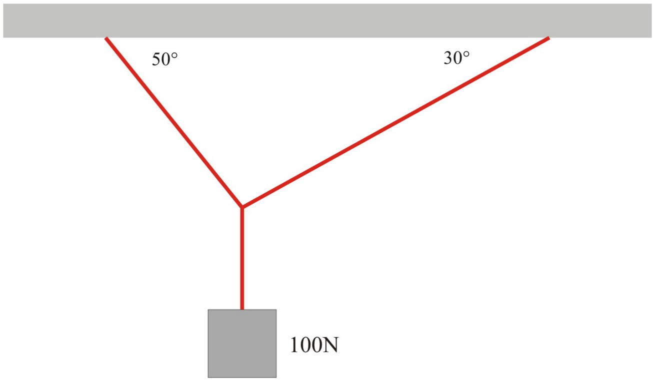


Consequently:
a) the forces are coplanar
b) the largest magnitude is less or equal to the sum of the other two magnitudes

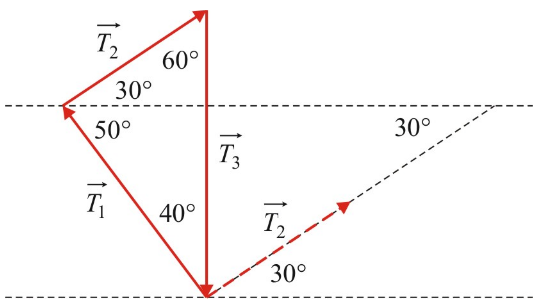
Ex 3. Find if a system of three forces with $\|\vec{F_1}\| = 12$, $\|\vec{F_2}\| = 3$ and $\|\vec{F_3}\| = 5$ may be in equilibrium.

$\|\vec{F_1}\| > \|\vec{F_2}\| + \|\vec{F_3}\|$ (because $12 > 3 + 5$)
Therefore, the system can not be in equilibrium.

Ex 4. Find the tensions in each string such that the body is at equilibrium.



Method #1 (using geometric vectors)



$T_3 = 100N$

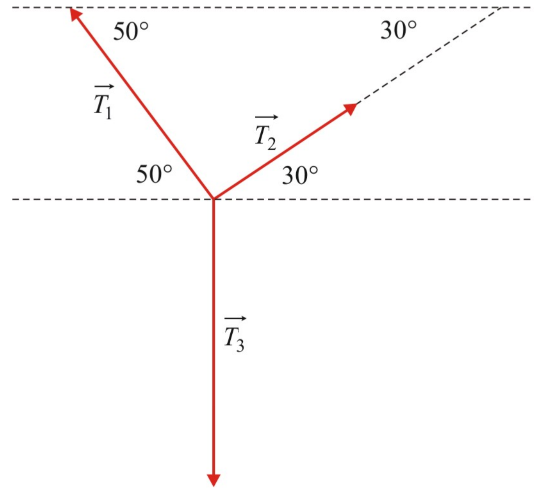
$\frac{T_3}{\sin(30° + 50°)} = \frac{T_1}{\sin 60°}$

$T_1 = \frac{100 \sin 60°}{\sin 80°} \cong 87.94N$

$\frac{T_3}{\sin(30° + 50°)} = \frac{T_2}{\sin 40°}$

$T_2 = \frac{100 \sin 40°}{\sin 80°} \cong 65.27N$

Method #2 (using scalar components)



$T_3 = 100N$

$R_x = T_2 \cos 30° - T_1 \cos 50° = 0 \Rightarrow T_2 = T_1 (\cos 50°)/(\cos 30°)$

$R_y = T_1 \sin 50° + T_2 \sin 30° - 100 = 0 \Rightarrow$

$T_1 \sin 50° + T_1 \frac{\cos 50°}{\cos 30°} \sin 30° - 100 = 0$

$T_1 = \frac{100}{\sin 50° + \frac{\cos 50°}{\cos 30°} \sin 30°} \cong 87.94N$

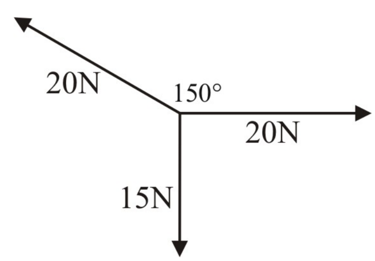
$T_2 = \frac{T_1 \cos 50°}{\cos 30°} = \frac{87.94 \cos 50°}{\cos 30°} \cong 65.27N$

E Equilibrant Force

The equilibrant force is the vector force $\vec{E}$ required to be added to a system of forces with a resultant force $\vec{R}$ such that the new system of forces is at equilibrium.

$$\vec{R} + \vec{E} = \vec{0}$$

Ex 5. Find an equilibrant for the following system of forces.



$R_x = 20 - 20 \cos 30° = 2.68N$

$R_y = 20 \sin 30° - 15 = -5N$

$\vec{R} = (2.68, -5)N \Rightarrow \therefore \vec{E} = -\vec{R} = (-2.68, 5)N$

Reading: Nelson Textbook, Pages 352-362

Homework: Nelson Textbook: Page 362 #2, 5a, 6ac, 8, 9, 11, 12, 13, 15, 16, 19

