6.3 Multiplication of a Vector by a Scalar

A Multiplication of a Vector by a Scalar

By multiplying a vector $\vec{v}$ by a scalar k we obtain a new vector noted $k\vec{v}$ with the following properties:

a) $k\vec{v}$ has the same direction as $\vec{v}$ if $k > 0$ and the opposite direction if $k < 0$

b) $\| k\vec{v} \| = |k| \times \| \vec{v} \|$

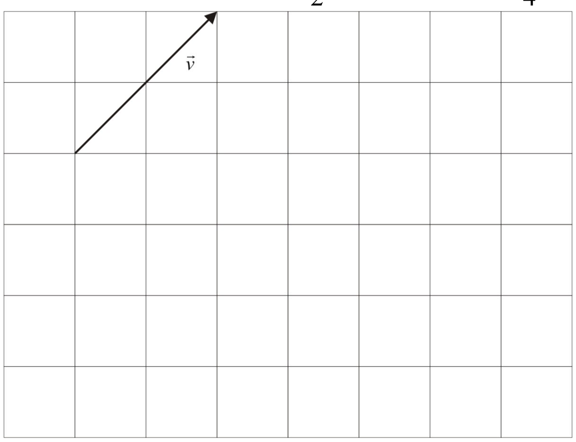
B Properties

The following properties apply for *multiplication* of a vector by a scalar:

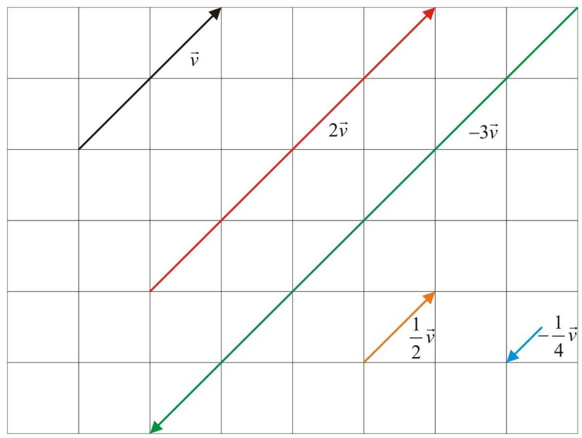
$$k(\vec{a} + \vec{b}) = k\vec{a} + k\vec{b}$$
$$k(m\vec{a}) = (km)\vec{a} = km\vec{a}$$
$$(k + m)\vec{a} = k\vec{a} + m\vec{a}$$

Ex 1. Given the vector $\vec{v}$, draw the following vectors:

a) $2\vec{v}$ b) $-3\vec{v}$ d) $\frac{1}{2}\vec{v}$ e) $-\frac{1}{4}\vec{v}$



Solution to Ex 1.



Ex 2. Given $\vec{a} = 2\vec{i} - 3\vec{j} + \vec{k}$, $\vec{b} = -\vec{i} + \vec{j} + 2\vec{k}$, write the following expressions in terms of the vectors $\vec{i}$, $\vec{j}$, and $\vec{k}$.

a) $\vec{a} + \vec{b}$

$\vec{a} + \vec{b} = (2\vec{i} - 3\vec{j} + \vec{k}) + (-\vec{i} + \vec{j} + 2\vec{k})$

$= (2-1)\vec{i} + (-3+1)\vec{j} + (1+2)\vec{k} = \vec{i} - 2\vec{j} + 3\vec{k}$

b) $2\vec{a} - 3\vec{b}$

$2\vec{a} - 3\vec{b} = 2(2\vec{i} - 3\vec{j} + \vec{k}) - 3(-\vec{i} + \vec{j} + 2\vec{k})$

$= 4\vec{i} - 6\vec{j} + 2\vec{k} + 3\vec{i} - 3\vec{j} - 6\vec{k}$

$= (4+3)\vec{i} + (-6-3)\vec{j} + (2-6)\vec{k} = 7\vec{i} - 9\vec{j} - 4\vec{k}$

C Vector Unit

An *unit vector* is a vector having a magnitude of 1.

For any vector $\vec{v}$, a unit vector parallel to $\vec{v}$ is given by:

$$\vec{u} = \frac{\vec{v}}{\| \vec{v} \|}$$

Ex 3. If $\vec{x}$ and $\vec{y}$ are two unit vectors with an angle of $30°$ between them, find the magnitude and direction of the vector $3\vec{x} - 5\vec{y}$.

$\angle(\vec{x}, \vec{y}) = \angle(3\vec{x}, 5\vec{y})$

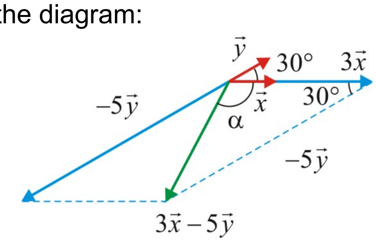
$\| 3\vec{x} - 5\vec{y} \|^2 = \| 3\vec{x} \|^2 + \| 5\vec{y} \|^2 - 2 \| 3\vec{x} \| \| 5\vec{y} \| \cos 30°$

$\| 3\vec{x} \| = 3 \| \vec{x} \| = 3, \quad \| 5\vec{y} \| = 5 \| \vec{y} \| = 5$

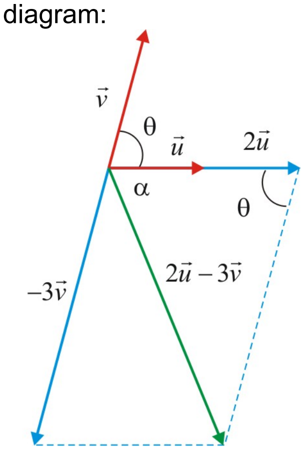
$\| 3\vec{x} - 5\vec{y} \|^2 = 3^2 + 5^2 - 2(3)(5)\cos 30° = 34 - 15\sqrt{3}$

$\| 3\vec{x} - 5\vec{y} \| = \sqrt{34 - 15\sqrt{3}} \cong 2.83$

According to the diagram:



$$\frac{\| -5\vec{y} \|}{\sin \alpha} = \frac{\| 3\vec{x} - 5\vec{y} \|}{\sin 30°} \Rightarrow \alpha = \sin^{-1}\left(\frac{5 \sin 30°}{\sqrt{34 - 15\sqrt{3}}} \right) = 61.98°$$

Ex 4. Given $\lVert \vec{u} \rVert = 8m$ and $\lVert \vec{v} \rVert = 12m$, $\lVert \vec{u}+\vec{v} \rVert = 16$, determine the magnitude and the direction of the vector $2\vec{u}-3\vec{v}$. $\theta = \angle(\vec{u},\vec{v}) = \angle(2\vec{u},3\vec{v})$ $\lVert \vec{u}+\vec{v} \rVert^2 = \lVert \vec{u} \rVert^2 + \lVert \vec{v} \rVert^2 + 2\lVert \vec{u} \rVert \lVert \vec{v} \rVert \cos\theta$ $\cos\theta = \frac{\lVert \vec{u}+\vec{v} \rVert^2 - \lVert \vec{u} \rVert^2 - \lVert \vec{v} \rVert^2}{2\lVert \vec{u} \rVert \lVert \vec{v} \rVert} = \frac{16^2-8^2-12^2}{2(8)(12)} = \frac{1}{4}$ $\lVert 2\vec{u}-3\vec{v} \rVert^2 = \lVert 2\vec{u} \rVert^2 + \lVert 3\vec{v} \rVert^2 - 2\lVert 2\vec{u} \rVert \lVert 3\vec{v} \rVert \cos\theta$ $= 4\lVert \vec{u} \rVert^2 + 9\lVert \vec{v} \rVert^2 - 2(2)(3)\lVert \vec{u} \rVert \lVert \vec{v} \rVert \cos\theta$ $= 4(8^2) + 9(12^2) - 12(8)(12)(1/4) = 1264$ $\lVert 2\vec{u}-3\vec{v} \rVert = \sqrt{1264} = 35.55$	According to the diagram:  $\frac{\lVert -3\vec{v} \rVert}{\sin\alpha} = \frac{\lVert 2\vec{u}-3\vec{v} \rVert}{\sin\theta} \Rightarrow \alpha = \sin^{-1}\left(\frac{3(12)\sqrt{1-1/16}}{\sqrt{1264}}\right) = 78.64°$

Reading: Nelson Textbook, Pages 293-298
Homework: Nelson Textbook: Page 298 #4, 9, 13, 15, 17, 18, 21, 22

