6.1 An Introduction to Vectors

**A Scalars and Vectors**

*Scalars* (in Mathematics and Physics) are quantities described completely by a number and eventually a measurement unit.

*Vectors* are quantities described by a magnitude (length, intensity or size) and direction.

Ex 1. Classify each quantity as scalar or vector.

a) time ⇒ scalar

b) position ⇒ vector

c) temperature ⇒ scalar

d) electric charge ⇒ scalar

e) mass ⇒ scalar

f) force ⇒ vector

g) displacement ⇒ vector

**B Geometric and Algebraic Vectors**

*Geometric Vectors* are vectors not related to any coordinate system.

For example, the directed line segment $\overrightarrow{AB}$:

where $A$ is called the initial (start, tail) point and $B$ is called the final (end, terminal, head or tip) point.

**C Algebraic Vectors**

*Algebraic Vectors* are vectors related to a coordinate system.

These vectors are (in general) described by their components relative to a reference system (frame).

For example $\vec{v} = (2,3,-1)$.

**D Position Vector**

The *position vector* is the directed line segment $\overrightarrow{OP}$ from the origin of the coordinate system $O$ to a generic point $P$.

Ex 2. Draw the position vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$, and $\overrightarrow{OC}$.

**E Displacement Vector**

The *displacement vector* $\overrightarrow{AB}$ is the directed line segment from the point $A$ to the point $B$.

Ex 3. Draw the displacement vectors $\overrightarrow{PQ}$ and $\overrightarrow{RQ}$.
**G Pythagorean Theorem**

In a right triangle $ABC$ with $\angle C = 90^\circ$ the following relation is true:

$$c^2 = a^2 + b^2$$

(see the figure on the right side).

**F Magnitude**

The *magnitude* is the length, size, norm or intensity of the vector. The magnitude of the vector $\vec{v}$ is denoted by $|\vec{v}|$, $\|\vec{v}\|$, or $\hat{v}$.

**Ex 4.** Consider the following diagram:

Find the magnitude of the following vectors:

a) $\overrightarrow{OA}$

$$\|\overrightarrow{OA}\| = \sqrt{2^2 + 2^2} = 2\sqrt{2}$$

b) $\overrightarrow{AB}$

$$\|\overrightarrow{AB}\| = \sqrt{3^2 + 1^2} = \sqrt{10}$$

c) $\overrightarrow{BC}$

$$\|\overrightarrow{BC}\| = \sqrt{4^2 + 2^2} = \sqrt{20} = 2\sqrt{5}$$

**G 3D Pythagorean Theorem**

In a *rectangular parallelepiped* (cuboid) the following relation is true:

$$AG^2 = d^2 = a^2 + b^2 + c^2$$

**Ex 5.** Consider the cube $ABCDEFGH$ with the side length equal to 10cm.

Find the magnitude of the following vectors:

a) $\overrightarrow{AB}$

$$\|\overrightarrow{AB}\| = 10cm$$

b) $\overrightarrow{BD}$

$$\|\overrightarrow{BD}\| = \sqrt{10^2 + 10^2} = 10\sqrt{2}cm$$

c) $\overrightarrow{BH}$

$$\|\overrightarrow{BH}\| = \sqrt{10^2 + 10^2 + 10^2} = 10\sqrt{3}cm$$

Ex 6. Consider the regular hexagon \( ABCDEF \) with the side length equal to \( 2m \), represented on the right side. Find the magnitude of the following vectors:

\( \text{a}) \quad \overrightarrow{AB} \quad \| \overrightarrow{AB} \| = 2m \)
\( \text{b}) \quad \overrightarrow{AC} \quad \| \overrightarrow{AC} \| = 2(2 \sin 60^\circ) = 2\sqrt{3}m \)
\( \text{c}) \quad \overrightarrow{AD} \quad \| \overrightarrow{AD} \| = 2(2) = 4m \)

**H Equivalent or Equal Vectors**

Two vectors are equivalent or equal if they have the same magnitude and direction.

For example \( \overrightarrow{AB} = \overrightarrow{CD} \) for the vectors represented in the next figure:

Ex 7. Find three pairs of equivalent vectors in the next diagram:

\( \overrightarrow{AB} = \overrightarrow{EF} \)
\( \overrightarrow{AF} = \overrightarrow{DG} \)
\( \overrightarrow{CA} = \overrightarrow{GE} \)

**I Opposite Vectors**

Two vectors are called opposite if they have the same magnitude and opposite direction.

The opposite vector of the vector \( \overrightarrow{v} \) is denoted by \( \overrightarrow{-v} \). Example: \( \overrightarrow{AB} = \overrightarrow{-DC} \)

Note that \( \overrightarrow{AB} = -\overrightarrow{BA} \).

Ex 8. Find three pairs of opposite vectors in the previous diagram.

\( \overrightarrow{AB} = -\overrightarrow{CD} \)
\( \overrightarrow{HF} = -\overrightarrow{BD} \)
\( \overrightarrow{CH} = -\overrightarrow{EB} \)

**J Parallel Vectors**

Two vectors are parallel if their directions are either the same or opposite.

If \( \overrightarrow{v_1} \) and \( \overrightarrow{v_2} \) are parallel, then we write \( \overrightarrow{v_1} \parallel \overrightarrow{v_2} \).

Ex 9. Use the following diagram and identify three vectors parallel to \( \overrightarrow{AG} \).

\( \overrightarrow{AG} \parallel \overrightarrow{FE} \)
\( \overrightarrow{AG} \parallel \overrightarrow{CB} \)
\( \overrightarrow{AG} \parallel \overrightarrow{GD} \)
### K Direction

To express the direction of a vector in a horizontal plane, the following standards are used. Note. Because we use a reference system, the following vectors may be considered also algebraic.

**True (Azimuth) Bearing** The direction of the vector is given by the angle between the North and the vector, measured in a clockwise direction.

Example: \( \vec{v} = 5m / s \ [120^\circ] \).

**Quadrant Bearing** The direction is given by the angle between the North-South line and the vector.

Example: \( 5m[N45^\circ E] \).

Read: \( 45^\circ \) East of North.

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<table>
<thead>
<tr>
<th>Ex 10. Draw each vector given by magnitude and true bearing.</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) ( \vec{r} = 2m ) at a true bearing of ( [60^\circ] )</td>
</tr>
<tr>
<td>b) ( \vec{a} = 5m / s^2 [225^\circ] )</td>
</tr>
</tbody>
</table>

![Diagram](image1.png)

<table>
<thead>
<tr>
<th>Ex 11. Draw each vectors given by magnitude and quadrant bearing.</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) ( \vec{d} = 2m[S60^\circ E] )</td>
</tr>
<tr>
<td>b) ( \vec{F} = 10N[W] )</td>
</tr>
</tbody>
</table>

![Diagram](image2.png)

<table>
<thead>
<tr>
<th>Ex 12. Convert each vector.</th>
</tr>
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<tbody>
<tr>
<td>a) ( \vec{v} = 5m / s [210^\circ] ) (to quadrant bearing) ( \vec{v} = 5m / s [210^\circ] = 5m / s [S30^\circ W] )</td>
</tr>
<tr>
<td>b) ( \vec{d} = 25m[N30^\circ W] ) (to true bearing) ( \vec{d} = 25m[N30^\circ W] = 25m[330^\circ] )</td>
</tr>
</tbody>
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**Reading:** Nelson Textbook, Pages 275-278

**Homework:** Nelson Textbook: Page 279 #1, 4, 6, 8, 9, 11