6.1 An Introduction to Vectors

A Scalars and Vectors

Scalars (in Mathematics and Physics) are quantities *described completely by a number* and eventually a measurement unit.

Vectors are quantities described by a *magnitude* (length, intensity or size) and *direction*.

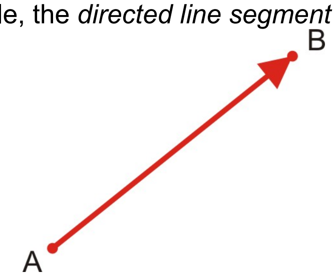
Ex 1. Classify each quantity as scalar or vector.

a) time $\Rightarrow$ scalar
b) position $\Rightarrow$ vector
c) temperature $\Rightarrow$ scalar
d) electric charge $\Rightarrow$ scalar
e) mass $\Rightarrow$ scalar
f) force $\Rightarrow$ vector
g) displacement $\Rightarrow$ vector

B Geometric and Algebraic Vectors

Geometric Vectors are vectors not related to any coordinate system.

For example, the *directed line segment* $\overrightarrow{AB}$:



where A is called the initial (start, tail) point and B is called the final (end, terminal, head or tip) point.

C Algebraic Vectors

Algebraic Vectors are vectors related to a coordinate system.
These vectors are (in general) described by their *components* relative to a reference system (frame).

For example $\vec{v} = (2,3,-1)$.

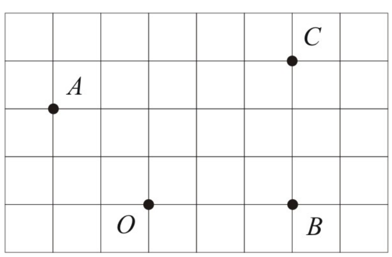
D Position Vector

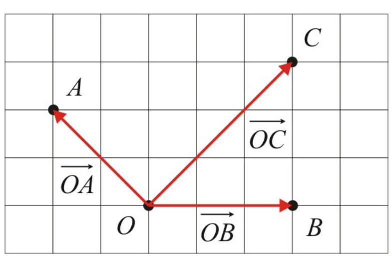
The *position vector* is the directed line segment $\overrightarrow{OP}$ from the origin of the coordinate system O to a generic point P.

E Displacement Vector

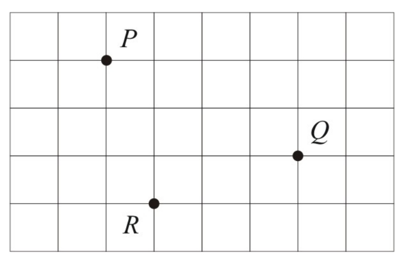
The *displacement vector* $\overrightarrow{AB}$ is the directed line segment from the point A to the point B.

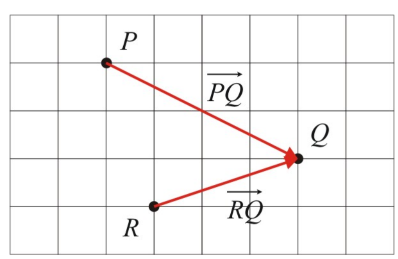
Ex 2. Draw the position vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$, and $\overrightarrow{OC}$.





Ex 3. Draw the displacement vectors $\overrightarrow{PQ}$ and $\overrightarrow{RQ}$.



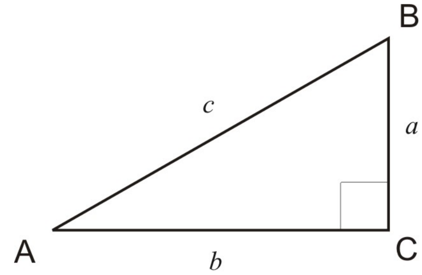


G Pythagorean Theorem

In a right triangle ABC with $\angle C = 90°$ the following relation is true:

$$c^2 = a^2 + b^2$$

(see the figure on the right side).

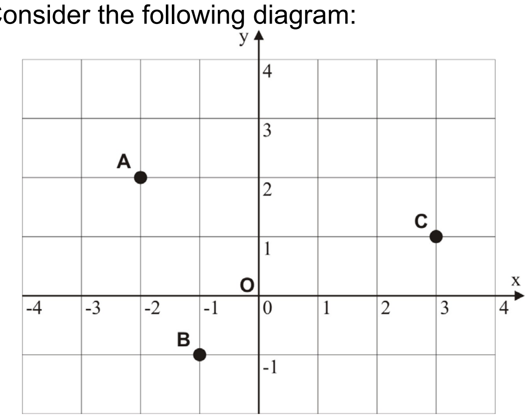


F Magnitude

The *magnitude* is the length, size, norm or intensity of the vector.

The magnitude of the vector $\vec{v}$ is denoted by $|\vec{v}|$, $\|\vec{v}\|$, or v.

Ex 4. Consider the following diagram:



Find the magnitude of the following vectors:

a) $\overrightarrow{OA}$

$\|\overrightarrow{OA}\| = \sqrt{2^2 + 2^2} = 2\sqrt{2}$

b) $\overrightarrow{AB}$

$\|\overrightarrow{AB}\| = \sqrt{3^2 + 1^2} = \sqrt{10}$

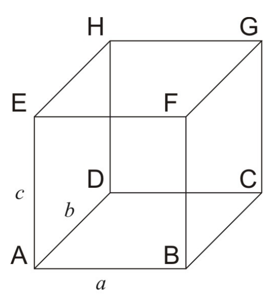
c) $\overrightarrow{BC}$

$\|\overrightarrow{BC}\| = \sqrt{4^2 + 2^2} = \sqrt{20} = 2\sqrt{5}$

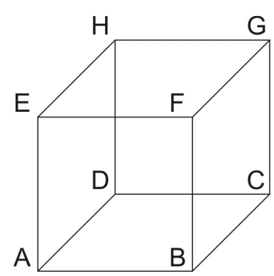
G 3D Pythagorean Theorem

In a *rectangular parallelepiped* (cuboid) the following relation is true:

$$AG^2 = d^2 = a^2 + b^2 + c^2$$



Ex 5. Consider the cube $ABCDEFGH$ with the side length equal to $10cm$.



Find the magnitude of the following vectors:

a) $\overrightarrow{AB}$

$\|\overrightarrow{AB}\| = 10cm$

b) $\overrightarrow{BD}$

$\|\overrightarrow{BD}\| = \sqrt{10^2 + 10^2} = 10\sqrt{2}cm$

c) $\overrightarrow{BH}$

$\|\overrightarrow{BH}\| = \sqrt{10^2 + 10^2 + 10^2} = 10\sqrt{3}cm$

Ex 6. Consider the regular hexagon $ABCDEF$ with the side length equal to $2m$, represented on the right side. Find the magnitude of the following vectors:

a) $\overrightarrow{AB}$

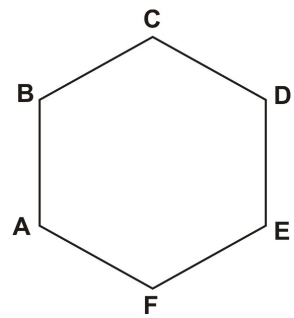
$\|\overrightarrow{AB}\| = 2m$

b) $\overrightarrow{AC}$

$\|\overrightarrow{AC}\| = 2(2\sin 60°) = 2\sqrt{3}m$

c) $\overrightarrow{AD}$

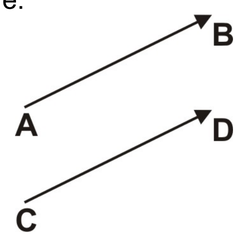
$\|\overrightarrow{AD}\| = 2(2) = 4m$



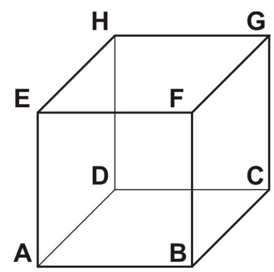
H Equivalent or Equal Vectors

Two vectors are *equivalent* or *equal* if they have the same magnitude and direction.

For example $\overrightarrow{AB} = \overrightarrow{CD}$ for the vectors represented in the next figure:



Ex 7. Find three pairs of equivalent vectors in the next diagram:



$\overrightarrow{AB} = \overrightarrow{EF}$

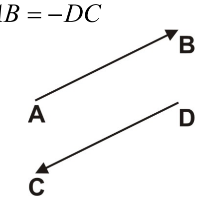
$\overrightarrow{AF} = \overrightarrow{DG}$

$\overrightarrow{CA} = \overrightarrow{GE}$

I Opposite Vectors

Two vectors are called *opposite* if they have the same magnitude and opposite direction.

The opposite vector of the vector $\vec{v}$ is denoted by $-\vec{v}$. Example: $\overrightarrow{AB} = -\overrightarrow{DC}$



Note that $\overrightarrow{AB} = -\overrightarrow{BA}$.

Ex 8. Find three pairs of opposite vectors in the previous diagram.

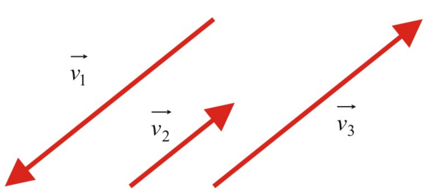
$\overrightarrow{AB} = -\overrightarrow{CD}$

$\overrightarrow{HF} = -\overrightarrow{BD}$

$\overrightarrow{CH} = -\overrightarrow{EB}$

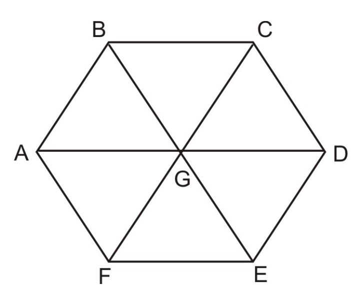
J Parallel Vectors

Two vectors are *parallel* if their directions are either the same or opposite.



If $\vec{v}_1$ and $\vec{v}_2$ are parallel, then we write $\vec{v}_1 \parallel \vec{v}_2$.

Ex 9. Use the following diagram and identify three vectors parallel to $\overrightarrow{AG}$.



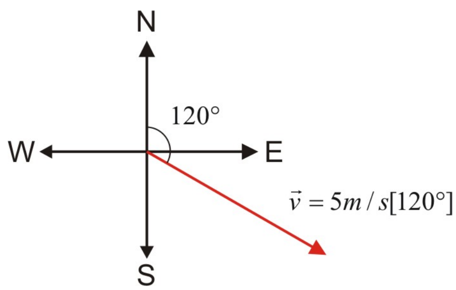
$\overrightarrow{AG} \parallel \overrightarrow{FE}$

$\overrightarrow{AG} \parallel \overrightarrow{CB}$

$\overrightarrow{AG} \parallel \overrightarrow{GD}$

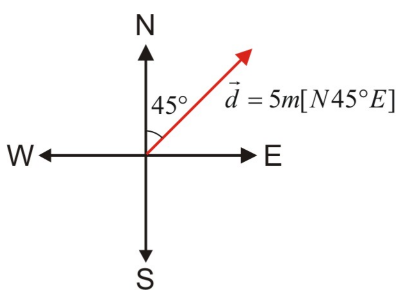
K Direction
To express the direction of a vector in a horizontal plane, the following standards are used.
Note. Because we use a reference system, the following vectors may be considered also algebraic.

True (Azimuth) Bearing The direction of the vector is given by the angle between the North and the vector, measured in a clockwise direction.
Example: $\vec{v} = 5m/s\ [120°]$.



Quadrant Bearing The direction is given by the angle between the North-South line and the vector.

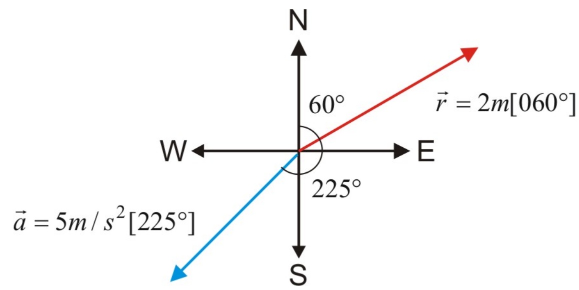
Example: $5m[N45°E]$.
Read: $45°$ East of North.



Note. $5m[N45°E] = 5m[NE]$
Read: $5m$ North-East.

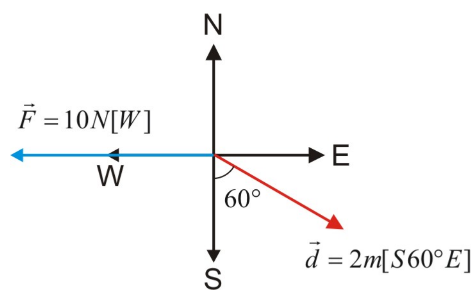
Ex 10. Draw each vector given by magnitude and true bearing.

a) $\vec{r} = 2m$ at a true bearing of $[060°]$
b) $\vec{a} = 5m/s^2[225°]$



Ex 11. Draw each vectors given by magnitude and quadrant bearing.

a) $\vec{d} = 2m[S60°E]$
b) $\vec{F} = 10N[W]$



Ex 12. Convert each vector.

a) $\vec{v} = 5m/s[210°]$ (to quadrant bearing)
$\vec{v} = 5m/s[210°] = 5m/s[S30°W]$

b) $\vec{d} = 25m[N30°W]$ (to true bearing)
$\vec{d} = 25m[N30°W] = 25m[330°]$

Reading: Nelson Textbook, Pages 275-278
Homework: Nelson Textbook: Page 279 #1, 4, 6, 8, 9, 11