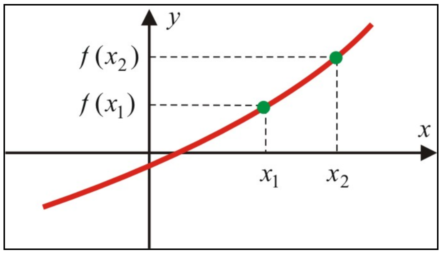
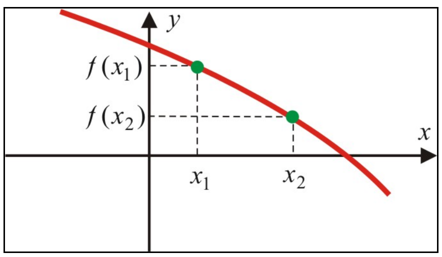
4.1 Increasing and Decreasing Functions

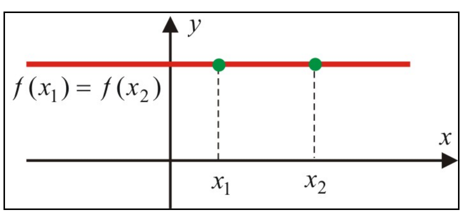
A Increasing and Decreasing Functions
A function f is *increasing* over the interval (a,b) if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in the interval (a,b).



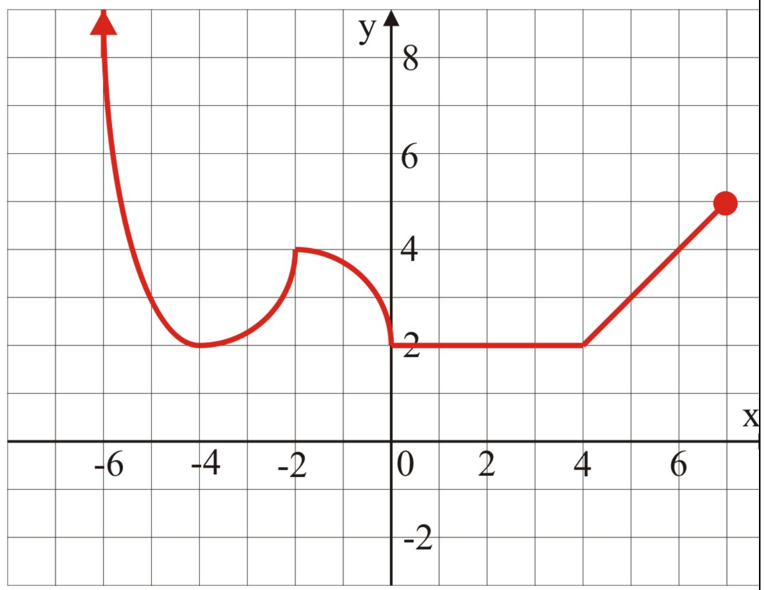
A function f is *decreasing* over the interval (a,b) if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ in the interval (a,b).



A function f is *constant* over the interval (a,b) if $f(x_1) = f(x_2)$ for every x_1 and x_2 in the interval (a,b).



Ex 1. Find the intervals where the function $y = f(x)$ is increasing, decreasing, or is constant.



f is increasing over $(-4,-2)$ and $(4,7)$.
f is decreasing over $(-6,-4)$ and over $(-2,0)$.
f is constant over $(0,4)$.

B Test for Intervals of Increase or Decrease
Let $y = f(x)$ be a differentiable function over (a,b). Then:

If $f'(x) > 0$ for all $x \in (a,b)$ then f is *increasing* over (a,b).

If $f'(x) < 0$ for all $x \in (a,b)$ then f is *decreasing* over (a,b).

If $f'(x) = 0$ for all $x \in (a,b)$ then f is *constant* over (a,b).

Ex 2. Find the intervals of increase or decrease for $f(x) = 2x^3 + 3x^2 - 12x$.

$f'(x) = 6x^2 + 6x - 12 = 6(x^2 + x - 2) = 6(x+2)(x-1)$
$f'(x) = 0 \Rightarrow 6(x+2)(x-1) = 0 \Rightarrow x = -2 \text{ or } x = 1$
$f(-2) = 2(-2)^3 + 3(-2)^2 - 12(-2) = 2(-8) + 3(4) + 24 = 20$
$f(1) = 2 + 3 - 12 = -7$

Sign Chart for f':

x		-2		1	
$f(x)$	↗	20	↘	-7	↗
$f'(x)$	+	0	-	0	+

f is increasing over $(-\infty,-2)$ and over $(1,\infty)$ and is decreasing over $(-2,1)$.

Ex 3. Find the intervals of increase or decrease for $f(x)=\frac{2}{x}-\frac{1}{x^2}$.

$f'(x)=\frac{-2}{x^2}+\frac{2}{x^3}=\frac{2(1-x)}{x^3}$

$f'(x)=0 \Rightarrow 2(1-x) \Rightarrow x=1$

$f(1)=\frac{2}{1}-\frac{1}{1^2}=2-1=1$

Sign Chart for f':

x		0		1	
$f(x)$	↘	DNE	↗	1	↘
$f'(x)$	-	DNE	+	0	-

$\therefore f$ is increasing over $(0,1)$ and is decreasing over $(-\infty,0)$ and over $(1,\infty)$.

Ex 4. Find the intervals of increase or decrease for $f(x)=\frac{x}{x^2+1}$.

$f'(x)=\frac{1(x^2+1)-x(2x)}{(x^2+1)^2}=\frac{1-x^2}{(x^2+1)^2}$

$f'(x)=0 \Rightarrow 1-x^2=0 \Rightarrow x=\pm 1$

$f(\pm 1)=\frac{\pm 1}{(\pm 1)^2+1}=\pm\frac{1}{2}$

Sign Chart for f':

x		-1		1	
$f(x)$	↘	$-\frac{1}{2}$	↗	$\frac{1}{2}$	↘
$f'(x)$	-	0	+	0	-

$\therefore f$ is increasing over $(-1,1)$ and is decreasing over $(-\infty,-1)$ and over $(1,\infty)$.

Ex 5. Find the intervals of increase or decrease for $f(x)=(x-2)\sqrt[3]{x^2}$.

$f(x)=(x-2)x^{\frac{2}{3}}$

$f'(x)=x^{\frac{2}{3}}+(x-2)\frac{2}{3}x^{-\frac{1}{3}}=\frac{3}{3}x^{-\frac{1}{3}}x^{+\frac{1}{3}}x^{\frac{2}{3}}+(x-2)\frac{2}{3}x^{-\frac{1}{3}}$

$=\frac{3x+2(x-2)}{3\sqrt[3]{x}}=\frac{5x-4}{3\sqrt[3]{x}}$

$f'(x)=0 \Rightarrow 5x-4=0 \Rightarrow x=\frac{4}{5}$

$f(4/5)=(4/5-2)\sqrt[3]{(4/5)^2}\cong -0.345$

$f'(0)\ DNE$

$f(0)=0$

Sign Chart for f':

x		0		$4/5$	
$f(x)$	↗	0	↘	-0.345	↗
$f'(x)$	+	DNE	-	0	+

$\therefore f$ is increasing over $(-\infty,0)$ and over $(4/5,\infty)$ and is decreasing over $(0,4/5)$.

Reading: Nelson Textbook, Pages 165-169
Homework: Nelson Textbook: Page 169 #1cd, 3bc, 5, 7, 8, 11