1.1 Radical Expressions: Rationalizing Denominators

A Radicals $\sqrt{a}\sqrt{a}=a$ $(\sqrt[n]{a})^n=a$ $(\sqrt[n]{a^m})=a^{\frac{m}{n}}=(\sqrt[n]{a})^m$ Note: If n is even, then $a\ge 0$ for $\sqrt[n]{a}$.	Ex 1. Simplify: a) $\sqrt{3}\sqrt{3}=3$ b) $(\sqrt{5})^3=(\sqrt{5})^2\sqrt{5}=5\sqrt{5}$ c) $(\sqrt[3]{7})^5=(\sqrt[3]{7})^3(\sqrt[3]{7})^2=7(\sqrt[3]{7})^2=7\sqrt[3]{7^2}=7\sqrt[3]{49}$
B Rationalizing Denominators (I) $\frac{a}{b\sqrt{c}}=\frac{a}{b\sqrt{c}}\frac{\sqrt{c}}{\sqrt{c}}=\frac{a\sqrt{c}}{bc}$	Ex 2. Rationalize: $\frac{2}{3\sqrt{5}}=\frac{2}{3\sqrt{5}}\frac{\sqrt{5}}{\sqrt{5}}=\frac{2\sqrt{5}}{3\times 5}=\frac{2\sqrt{5}}{15}$
C Conjugate Radicals $a+\sqrt{b}\Leftrightarrow a-\sqrt{b}$ $\sqrt{a}+\sqrt{b}\Leftrightarrow\sqrt{a}-\sqrt{b}$ $\sqrt{a}+b\sqrt{c}\Leftrightarrow\sqrt{a}-b\sqrt{c}$ $a\sqrt{b}+c\sqrt{d}\Leftrightarrow a\sqrt{b}-c\sqrt{d}$	Ex 3. For each expression, find the conjugate radical. a) $2+\sqrt{3}\quad\Rightarrow\quad 2-\sqrt{3}$ b) $\sqrt{2}-\sqrt{3}\quad\Rightarrow\quad\sqrt{2}+\sqrt{3}$ c) $\sqrt{3}+2\sqrt{5}\quad\Rightarrow\quad\sqrt{3}-2\sqrt{5}$ d) $2\sqrt{5}+3\sqrt{7}\quad\Rightarrow\quad 2\sqrt{5}-3\sqrt{7}$
D Difference of squares identity $(a+b)(a-b)=a^2-b^2$	Ex 4. Use the difference of squares identity to simplify: a) $(a+\sqrt{b})(a-\sqrt{b})=a^2-(\sqrt{b})^2=a^2-b$ b) $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})=(\sqrt{a})^2-(\sqrt{b})^2=a-b$ c) $(\sqrt{a}+b\sqrt{c})(\sqrt{a}-b\sqrt{c})=(\sqrt{a})^2-(b\sqrt{c})^2=a-b^2c$
E Rationalizing Denominators (II) Hint: Multiply and divide by the *conjugate radical e* of the denominator.	Ex 5. Rationalize the denominator: a) $\frac{3}{1-\sqrt{2}}=\frac{3}{1-\sqrt{2}}\frac{1+\sqrt{2}}{1+\sqrt{2}}=\frac{3(1+\sqrt{2})}{1^2-(\sqrt{2})^2}=\frac{3(1+\sqrt{2})}{1-2}=-3(1+\sqrt{2})$ b) $\frac{4}{2+3\sqrt{5}}=\frac{4}{2+3\sqrt{5}}\frac{2-3\sqrt{5}}{2-3\sqrt{5}}=\frac{4(2-3\sqrt{5})}{2^2-(3\sqrt{5})^2}=\frac{4(2-3\sqrt{5})}{4-9\times 5}=-\frac{4(2-3\sqrt{5})}{41}$ c) $\frac{2}{\sqrt{3}-\sqrt{6}}=\frac{2}{\sqrt{3}-\sqrt{6}}\frac{\sqrt{3}+\sqrt{6}}{\sqrt{3}+\sqrt{6}}=\frac{2(\sqrt{3}+\sqrt{6})}{(\sqrt{3})^2-(\sqrt{6})^2}=\frac{2(\sqrt{3}+\sqrt{6})}{3-6}=-\frac{2(\sqrt{3}+\sqrt{6})}{3}$
F Rationalizing Numerators Hint: Multiply and divide by the *conjugate radical* of the numerator.	Ex 6. Rationalize the numerator: $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{2}-1}=\frac{\sqrt{5}-\sqrt{3}}{\sqrt{2}-1}\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}=\frac{(\sqrt{5})^2-(\sqrt{3})^2}{(\sqrt{2}-1)(\sqrt{5}+\sqrt{3})}=\frac{2}{(\sqrt{2}-1)(\sqrt{5}+\sqrt{3})}$
G Equivalent Expressions Hint: You may get equivalent expressions by *rationalizing* the numerator or denominator. Note: State restrictions.	Ex 7. Find equivalent expressions by rationalizing. State restrictions. a) $\frac{x-1}{\sqrt{x}-1}=\frac{x-1}{\sqrt{x}-1}\frac{\sqrt{x}+1}{\sqrt{x}+1}=\frac{(x-1)(\sqrt{x}+1)}{x-1}=\sqrt{x}+1,\quad x\ge 0, x\ne 1$ b) $\frac{\sqrt{x+9}-3}{x}=\frac{\sqrt{x+9}-3}{x}\frac{\sqrt{x+9}+3}{\sqrt{x+9}+3}=\frac{x}{x(\sqrt{x+9}+3)}=\frac{1}{\sqrt{x+9}+3},$ $x\ge -9, x\ne 0$

	c) $\dfrac{\dfrac{1}{\sqrt{x+h}}-\dfrac{1}{\sqrt{x}}}{h}=\dfrac{\dfrac{1}{x+h}-\dfrac{1}{x}}{h\left(\dfrac{1}{\sqrt{x+h}}+\dfrac{1}{\sqrt{x}}\right)}=-\dfrac{1}{x(x+h)\left(\dfrac{1}{\sqrt{x+h}}+\dfrac{1}{\sqrt{x}}\right)}$, $x+h>0, x>0, h\neq 0$
H More algebraic identities $a^3-b^3=(a-b)(a^2+ab+b^2)$ $a^3+b^3=(a+b)(a^2-ab+b^2)$ $a^4-b^4=(a-b)(a+b)(a^2+b^2)$	Ex 8. For each case, the numerator and denominator have a common zero. Use algebraic identities to eliminate the common zero. State restrictions. a) $\dfrac{x-1}{\sqrt[3]{x}-1}=\dfrac{(\sqrt[3]{x})^3-1}{\sqrt[3]{x}-1}=\dfrac{(\sqrt[3]{x}-1)((\sqrt[3]{x})^2+\sqrt[3]{x}+1)}{\sqrt[3]{x}-1}=\sqrt[3]{x^2}+\sqrt[3]{x}+1, \quad x\neq 1$ b) $\dfrac{x^4-1}{x^3-1}=\dfrac{(x-1)(x+1)(x^2+1)}{(x-1)(x^2+x+1)}=\dfrac{(x+1)(x^2+1)}{x^2+x+1}, \quad x\neq 1$

Reading: Nelson Textbook, Pages 6-8
Homework: Nelson Textbook: Page 9, #1a, 2a, 3a, 4a, 5, 6a, 7ac

